1 Mark for Review

Which of these is *not* true about a survey conducted on a random sample from a population?

(A) It can establish a causal relationship. (A)

(B) The results can be generalized to a larger population. (B)

(C) It is an observational study. (C)

(D) Bias can be introduced in the way questions are asked. (D)

(E) It can be used by researchers to make estimates about the population. (E)

A

2 Mark for Review

A teacher at an elementary school notices this about two of her students:

- The probability that Isa rides her bike to school on a randomly selected day (Event A) is 0.6.
- The probability that Sarah rides her bike to school on a randomly selected day (Event B) is 0.45.
- The probability that they both ride to school on a randomly selected day (Event (A and B)) is 0.35.

Which of the following statements must be true according to this observation?

(A) Event A and Event B are mutually exclusive and are independent. (A)

(B) Event A and Event B are mutually exclusive and are not independent. (B)

(C) Event A and Event B are not mutually exclusive and are independent. (C)

(D) Event A and Event B are not mutually exclusive and are not independent. (D)

(E) Event A and Event B are not mutually exclusive and independence can not be determined with the information given. (E)

C

3 Mark for Review

A slot machine is programmed to give players an expected value of -$0.85 per spin. What is the meaning of the expected value, in this context?

(A) The players lose $0.85 every time they spin the slot machine.

(B) The players win $0.85 every time they spin the slot machine.

(C) The players lose an average of $0.85 per spin on the slot machine.

(D) The players win an average of $0.85 per spin on the slot machine.

(E) A player has an 85 percent chance of losing money when playing the slot machine.

D

Interference and Experiments

1. Inference — 统计推断
2. Population — 总体
3. Sample — 样本
4. Parameter — 参数
5. Statistic — 统计量
6. Confidence interval — 置信区间
7. Hypothesis test — 假设检验
8. p-value — p值
9. Random assignment — 随机分配
10. Confounding variable — 混杂变量

- **Interference** occurs when the response of one experimental unit is affected by another unit's treatment.
  - It violates the assumption that experimental units act independently.
  - Interference can distort results and lead to incorrect conclusions.
- Researchers reduce interference through **careful design and control** of experimental conditions.
  - Proper experimental design helps ensure that observed effects are due to the treatments alone.

FRQ: Randomized Experiment & Inference

A researcher wants to investigate whether a new revision method improves AP Statistics test scores. She selects **60 students** from a large school and randomly assigns them to two groups:

- **Group A (30 students):** uses the traditional revision method
- **Group B (30 students):** uses the new revision method

After four weeks, both groups take the same standardized test. The researcher records the mean scores:

- Group A: mean = 72, standard deviation = 8
- Group B: mean = 76, standard deviation = 10

Assume conditions for inference are met.

(a)

Explain why this study is an **experiment** rather than an observational study.

(b)

Identify the **treatment** and the **response variable**.

(c)

State appropriate **null and alternative hypotheses** to test whether the new method improves scores.

(d)

Explain what a **Type I error** would mean in this context.

(e)

The p-value for the test is 0.03. Interpret this value in context.

(f)

At a significance level of $\alpha = 0.05$, what conclusion should the researcher make? Justify your answer.

(a) Why is this an experiment?

This is an experiment because the researcher **actively assigns students randomly** to one of two revision methods. Random assignment allows the researcher to **control the treatment**, which is the key feature of an experiment.

(b) Treatment and response variable

- **Treatment:** Revision method (traditional vs new method)
- **Response variable:** AP Statistics test score

(c) Hypotheses

Let μ_A = mean score for traditional method
Let μ_B = mean score for new method

- **Null hypothesis (H_0):** $\mu_B = \mu_A$ (no difference in mean scores)
- **Alternative hypothesis (H_1):** $\mu_B > \mu_A$ (new method improves scores)

(d) Type I error

A Type I error would occur if the researcher **concludes that the new revision method improves scores when in reality it does not** (there is actually no difference between the methods).

(e) Interpretation of p-value (0.03)

If there is truly **no difference between the revision methods**, then there is a **3% probability** of obtaining a difference in sample means at least as extreme as the one observed (or more extreme) just by random chance.

(f) Conclusion at $\alpha = 0.05$

Since **p-value = 0.03 < 0.05**, we **reject the null hypothesis**.

There is **statistically significant evidence** that the new revision method **improves AP Statistics test scores.**

TWO- SAMPLE T-TEST

1. Two-sample t-test — 双样本 t 检验
2. Two-sample t-interval — 双样本 t 置信区间
3. Difference in means — 均值差
4. Independent samples — 独立样本
5. Sample mean — 样本均值
6. Standard error — 标准误差
7. Degrees of freedom — 自由度
8. Null hypothesis — 原假设
9. p-value — p值
10. Random assignment — 随机分配

AP Statistics Stretch & Challenge (Two-Sample t Procedures)

A education researcher investigates whether **sleep duration affects AP Statistics performance**.

A sample of **40 students** is randomly selected from a large population. They are divided into two groups based on sleep patterns over a month:

- **Group A:** 20 students who sleep **less than 6 hours per night**
- **Group B:** 20 students who sleep **8 or more hours per night**

At the end of the month, both groups take the same AP Statistics exam.

Summary statistics:

- Group A: $\bar{x}_A = 71$, $s_A = 9$, $n_A = 20$
- Group B: $\bar{x}_B = 78$, $s_B = 11$, $n_B = 20$

Assume all conditions for inference are met.

(a)

Identify the **appropriate inference procedure** and justify your choice.

(b)

State the **null and alternative hypotheses** in words and symbols.

(c)

Explain whether a **pooled or unpooled t-procedure** should be used, and why.

(d)

Interpret what a **Type II error** would mean in this context.

(e)

Suppose the 95% confidence interval for $\mu_A - \mu_B$ is:

$$(-12.4,\ -2.6)$$

Interpret this interval in context.

(f) Stretch question

Based on the confidence interval, discuss whether it is reasonable to claim that **sleep causes higher AP Statistics scores**, and explain your reasoning using both **statistical inference and experimental design principles**.

(a) Inference procedure

The appropriate procedure is a **two-sample t-test for the difference in means**.

This is because we are comparing the **mean AP Statistics scores of two independent groups** (less than 6 hours sleep vs 8+ hours sleep) using sample data.

(b) Hypotheses

Let μ_A = mean score for students who sleep < 6 hours

Let μ_B = mean score for students who sleep ≥ 8 hours

- **Null hypothesis (H_0):** $\mu_A = \mu_B$ (no difference in mean scores)
- **Alternative hypothesis (H_1):** $\mu_A < \mu_B$ (students who sleep more score higher)

(c) Pooled or unpooled?

Use an **unpooled two-sample t-procedure**.

Reason: The population standard deviations are **unknown and not assumed equal**, so we do not pool variances (this is the standard AP Stats approach unless explicitly told otherwise).

(d) Type II error

A Type II error would occur if the researcher **fails to detect a real difference**, meaning they conclude there is **no difference in scores based on sleep**, when in reality students who sleep more **do perform better**.

(e) Interpretation of confidence interval

The interval $(-12.4,\ -2.6)$ is for $\mu_A - \mu_B$.

This means we are **95% confident that students who sleep less than 6 hours score between 2.6 and 12.4 points lower on average** than students who sleep 8 or more hours.

Since the entire interval is negative, it suggests **Group B performs better**.

(f) Stretch question (causation + inference)

It would **not be fully reasonable to claim that sleep causes higher scores**.

Although this is a **random sample**, the groups are based on **sleep behaviour, not random assignment**, so this is an **observational study**, not a true experiment.

- The **confidence interval shows a statistically significant difference**
- However, **confounding variables** (e.g. study habits, stress, ability, time management) may explain the difference

Therefore:

- We can conclude there is a **significant association between sleep and performance**
- But we **cannot confidently infer causation** due to lack of random assignment

TRUE or FALSE

- A randomised experiment allows us to establish causation more confidently than an observational study.

TRUE

TRUE or FALSE

- The larger the sample size, the less variability there is in a sample statistic.

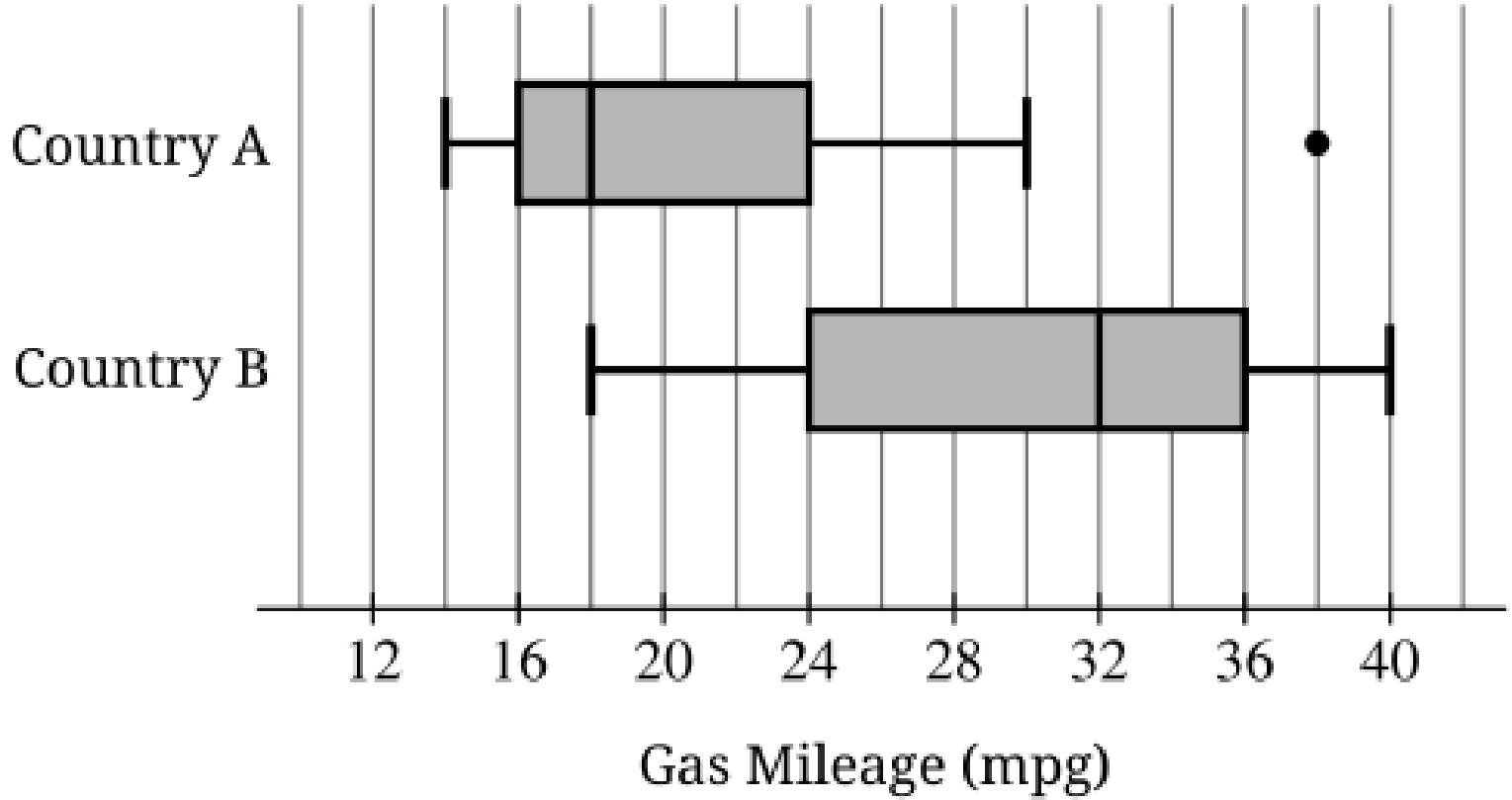
TRUE

TRUE or FALSE

- A placebo group is used to reduce bias in an experiment.

TRUE

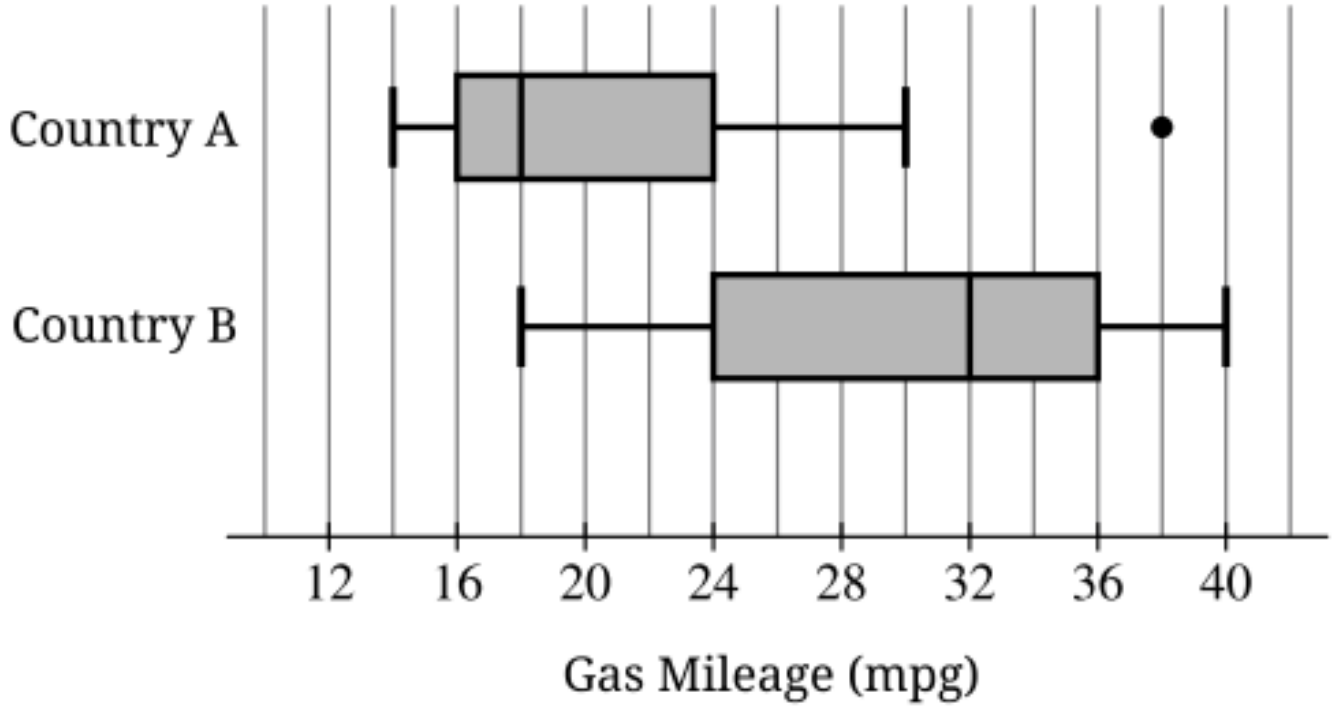
Boxplots of Gas Mileage for Each Country



A. Compare the distributions of gas mileage for the sample of cars manufactured in Country A and the sample of cars manufactured in Country B.

B. For the distribution of gas mileage for the sample of cars manufactured in Country A, would you expect the mean to be greater than 18 mpg, less than 18 mpg, or equal to 18 mpg? Justify your answer.

Boxplots of Gas Mileage for Each Country



The manager will create a new boxplot with the combined data from the sample of cars manufactured in Country A and the sample of cars manufactured in Country B.

i. What is the range of the combined data set? Justify your answer.

ii. What is a possible value of the median of the combined data set? Justify your answer by referencing the boxplots shown.