PRECALCULUS · PROBABILITY

Introduction to Probability

The probability scale · theoretical and experimental probability · complements · the long run

28	5	8
questions worked	sections	true/false checks

70 marks of practice + 5 starter marks · the worksheet questions plus the challenge and stretch tier

How this lesson runs

Starter, hook, key words — then two concept blocks, each with a worked example and exercises — true or false at the end

Starter	A mid-term exam question reworked as FIND THE ERROR — hunt the mistakes in Ravi's working.
Hook	A short puzzle to think about — no marks, just intuition. Then key words for the lesson.
Concepts	Two teaching blocks: key concept, worked example, then exercise questions for each.
Exercise	Worksheet questions W1–W10 and drills A1–A8 sit inside the concept blocks; the Stretch and challenge section gathers P1–P8 and coin-streak C1, and a final CHALLENGE closes the lesson.
Solution	The next slide gives the condensed solution: the answer, the key steps, and a trap to avoid.
True or false	The lesson ends with eight statements — one per slide, answer on the slide that follows.

S

Starter · From the mid-term · the percentile question

The question only 31% got right — reworked as an error hunt

- The November 2026 mid-term asked for the 90th percentile of a normal distribution
- One student's working hides planted errors — find every line that went wrong

Key points · the starter ideas

Everything you need for the exam-question starter, in five lines

1	**Percentile question**	“90th percentile” asks for the x with 90% of the distribution below it: find the z first, then convert.
2	**The z-table step**	z = 1.28 leaves about 90% below (exactly 0.8997). The percentile IS the left-tail area — read z from the table, never guess it.
3	**Unstandardise**	Convert back with $x = \mu + z\sigma$. Multiply z by σ FIRST, then add μ: $1.28 \times 15 = 19.2$, so $x = 100 + 19.2 = 119.2$.
4	**Order of operations**	$x = \mu + z\sigma$ has one correct reading: $\mu + (z \times \sigma)$. Adding before multiplying gives a wrong value that looks plausible.
5	**Choose the nearest**	With answer options, pick the value closest to your computed x. 119.2 sits closest to 120, not to 100 or 140.

Question S1

FROM THE MID-TERM

3 marks

Starter · normal percentile from the mid-term

FIND THE ERROR · WHICH LINES WENT WRONG?

Mid-Term Examination · November 2026 · Question 5

Tortoise shell lengths are approximately normal with mean $\mu = 100$ mm and standard deviation $\sigma = 15$ mm. Ravi is finding the 90th percentile. His working:

Line 1 The 90th percentile is the value with 90% of shells below it

Line 2 From the table, the z-score for 90% is about 1.28

Line 3 $x = \mu + z + \sigma = 100 + 1.28 + 15 = 116.28$

Line 4 So the 90th percentile is about 116 mm

Which line(s) went wrong? State each error and write the correct line.

Question S1 — solution

3 marks

Starter · normal percentile from the mid-term

CONDENSED SOLUTION

- Lines 1 and 2 are correct: 90% below, $z \approx 1.28$.
- Line 3 is wrong: the formula is $x = \mu + z\sigma$, so $100 + 1.28 \times 15 = 119.2$.
- Ravi added z and σ instead of multiplying — 116.28 is not on any option.
- Correct line 4: the 90th percentile is about 119 mm (nearest option 120).

TRAP The mid-term's big trap: answering 100 (the mean). The mean is the 50th percentile, not the 90th. Multiply z by σ before adding μ.

Question S2

FROM THE MID-TERM

2 marks

Starter · normal percentile from the mid-term

FINISH THE WORKING · COMPLETE PRIYA'S SOLUTION

Mid-Term Examination · November 2026 · Question 5, continued
The same tortoises: shell lengths ~ N(100, 15).

Priya is finding the 20th percentile. Her working so far:

> From the table, the z-score with 20% below it is about −0.84
> $x = \mu + z\sigma = 100 + (-0.84)(15) = \ldots$

Finish Priya's working: compute x, and state why the percentile is below the mean.

Question S2 — solution

2 marks

Starter · normal percentile from the mid-term

CONDENSED SOLUTION

- $x = 100 + (-0.84)(15) = 100 - 12.6 = 87.4$.
- A negative z means 20% sits BELOW the value, so x is below the mean.
- Check: 87.4 is 12.6 mm under $\mu = 100$ — about 0.84 SDs, as the table says.

TRAP Below-mean percentiles use negative z-scores. Dropping the minus sign would put the 20th percentile above the mean — impossible.

HOOK · BEFORE WE START

The quiet coin

Amara flips a fair coin 10 times and gets heads every single time. At the back of the room, Jay has been quietly flipping his own fair coin all lesson — around 700 flips — and his longest head streak was just 9.

- **Which streak is more surprising — and should the coin be blamed in either case?**
- Talk to the person next to you for a minute. There is no formula yet — just intuition.
- Keep your answer in mind: this lesson gives you the language of probability to settle it.

The quiet coin — solution

no marks

Two streaks, one idea: chance makes streaks, and the long run judges them

CONDENSED SOLUTION

- Amara: P(10 heads in 10 flips) = (1/2)^10 = 1/1024 ≈ 0.001 — unusual, but one chance in 1024.
- Jay: in about 700 flips the longest streak you'd EXPECT is roughly log2(700) ≈ 9.45 — a 9-streak is completely ordinary.
- So neither coin should be blamed: improbable events are normal given enough trials — the short run wobbles, the long run settles.

TRAP A streak is not evidence of bias. Only the long run — many flips compared with the model — can judge a coin. Keep this in mind when we reach short-run variation.

Vocabulary

Key terms with Chinese translations

probability	**概率**	a number from 0 to 1 measuring how likely an event is
random	**随机的**	unpredictable in the short run, predictable in the long run
outcome	**结果**	one possible result of a chance experiment
event	**事件**	a set of outcomes we are interested in
sample space (S)	**样本空间**	the list of all possible outcomes
trial	**试验**	one repetition of the experiment
theoretical probability	**理论概率**	favourable ÷ total, assuming equally likely outcomes
experimental probability	**实验概率**	relative frequency from actual trials
complement	**补事件**	the event does not happen: P(not A) = 1 − P(A)
relative frequency	**相对频率**	occurrences ÷ trials, the long-run estimate of P
law of large numbers	**大数定律**	relative frequency approaches the true probability as trials increase

01

Key concept 1 · The probability scale

How probability is measured, written and bounded

- 0 = impossible · 1 = certain · 1/2 = even chance
- Notation: P(A) — never negative, never more than 1

Key points · the probability scale

Where every probability sits, and how to write it

1	**The scale**	Probability measures how likely an event is: always a number from 0 (impossible) to 1 (certain).
2	**The endpoints**	P(an impossible event) = 0, e.g. rolling a 7 on a fair die. P(a certain event) = 1, e.g. rolling a number from 1 to 6.
3	**Landmarks**	1/2 = even chance (a fair coin). Near 0 = very unlikely. Near 1 = very likely. Fractions, decimals and percentages all describe the same value.
4	**The bounds**	$0 \le P(A) \le 1$. A probability can never be negative and never more than 1 — a value like 1.2 is impossible.
5	**Notation**	P(A) reads as “the probability of event A”. An event is any set of outcomes from the sample space S.

Worked example · the probability scale

the question

Question first — attempt it before the working

THE QUESTION

A spinner has 8 equal sectors: 3 red, 4 blue and 1 green. Amara spins once.

Sort these events by their probability, least to most likely: the spinner lands green · the spinner lands blue · the spinner lands red or green · the spinner lands a colour at all

Worked example · the probability scale — working

the working

The key steps, in order

THE WORKING

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Sample space: 8 equally likely sectors
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Green: 1/8 = 0.125 ·  Blue: 4/8 = 1/2 ·  Red or green: (3+1)/8 = 4/8 = 1/2 ·
A colour at all: 8/8 = 1
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Order: green (0.125)  <  blue = red-or-green (1/2)  <  a colour at all (1)
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Blue and red-or-green tie: different events, same probability
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Probabilities of 1 mean certainty: spinning "a colour" must happen. Check every probability sits between 0 and 1 before comparing.

Question A1

1 mark

Probability scale

THE QUESTION

Which of the following CANNOT be a probability?

(A) 0 (B) 0.75 (C) 1.2 (D) 1/3

Question A1 — solution

1 mark

Probability scale

CONDENSED SOLUTION

- (C) 1.2 — probabilities never exceed 1.
- 0, 0.75 and 1/3 all sit on the scale.

TRAP An event being “very likely” does not push its probability past 1. The most likely an event can be is P = 1, certain.

Question A2

1 mark

Theoretical probability · fair die

THE QUESTION

A fair six-sided die is rolled once.

What is P(six)?

(A) 1/6 (B) 1/5 (C) 1/2 (D) 1/3

Question A2 — solution

1 mark

Theoretical probability · fair die

CONDENSED SOLUTION

- (A) 1/6
- One favourable outcome out of six equally likely outcomes.

TRAP “Fair” is the keyword: it guarantees the six outcomes are equally likely, so counting is enough.

Question A3

1 mark

Theoretical probability · fair die

THE QUESTION

A fair six-sided die is rolled once.

What is P(an even number)?

(A) 1/6 (B) 1/3 (C) 1/2 (D) 2/3

Question A3 — solution

1 mark

Theoretical probability · fair die

CONDENSED SOLUTION

- (C) 1/2
- Even outcomes: {2, 4, 6} → 3 of 6 = 1/2.

TRAP Count the outcomes IN the event first ({2, 4, 6}), then divide — never divide by the outcomes NOT in the event.

Question W1

1 mark

The probability scale · worksheet Q1

THE QUESTION

A bag contains 10 counters of which 3 are red. One counter is drawn at random.

What is P(red)?

(A) 3/7 (B) 7/10 (C) 3/10 (D) 1/3

Question W1 — solution

1 mark

The probability scale · worksheet Q1

CONDENSED SOLUTION

- (C) 3/10
- 3 red counters out of 10 counters in total.

TRAP The denominator is the TOTAL bag (10), not the other counters (10 − 3 = 7). 3/7 has the right numerator and the wrong total.

Question W2

1 mark

Theoretical probability · worksheet Q2

THE QUESTION

A fair six-sided die is rolled once.

What is P(a number less than 4)?

(A) 1/3 (B) 1/2 (C) 1/6 (D) 2/3

Question W2 — solution

1 mark

Theoretical probability · worksheet Q2

CONDENSED SOLUTION

- (B) 1/2
- Less than 4: {1, 2, 3} → 3 of 6 = 1/2.

TRAP Count the outcomes IN the event first ({1, 2, 3}), then divide — “less than 4” does not include 4 itself.

Question W5

1 mark

Theoretical or experimental? · worksheet Q5

THE QUESTION

In a class of 20 students, 7 walk to school.

Which statement is correct?

(A) P(walks) = 7/13, an experimental probability
(B) P(walks) = 7/20, an experimental probability
(C) P(walks) = 7/20, a theoretical probability
(D) P(walks) = 7/13, a theoretical probability

Question W5 — solution

1 mark

Theoretical or experimental? · worksheet Q5

CONDENSED SOLUTION

- (C) P(walks) = 7/20, a theoretical probability
- 7 walkers out of the 20 students in the class — a known composition, not recorded trials.

TRAP Two decisions in one mark: divide by the TOTAL class (20), and classify the type. No experiment was repeated, so 7/20 is theoretical.

Question W9

5 marks

Counters · full working · worksheet Q9

THE QUESTION

A bag contains 5 red, 8 blue and 2 white counters. One counter is drawn at random.

(a) Find P(blue) and P(white). [2]
(b) Find P(not red). [1]
(c) Find P(blue or white), and explain how your answer connects to your answer in part (b). [1]
(d) The counter is replaced and the experiment is repeated 60 times. About how many blue counters would you expect? [1]

Question W9 — solution

5 marks

Counters · full working · worksheet Q9

CONDENSED SOLUTION

- Total = 5 + 8 + 2 = 15
- (a) P(blue) = 8/15 · P(white) = 2/15 (2)
- (b) P(not red) = 1 − 5/15 = 10/15 = 2/3 (1)
- (c) P(blue or white) = (8 + 2)/15 = 10/15 = 2/3 — the same event: not red IS blue or white (1)
- (d) 60 × 8/15 = 32 blue counters (1)

TRAP Simplify fully: 10/15 must become 2/3. In (c) the complement of “red” is blue-and-white — the same 10 counters — so the two answers match exactly.

Question A4

1 mark

Theoretical probability · cards

THE QUESTION

One card is drawn at random from a standard 52-card deck.

What is P(a face card: Jack, Queen or King)?

(A) 3/52 (B) 1/13 (C) 3/13 (D) 1/4

Question A4 — solution

1 mark

Theoretical probability · cards

CONDENSED SOLUTION

- (C) 3/13
- 12 face cards (3 per suit × 4 suits), so 12/52 = 3/13.

TRAP Count face cards as J, Q and K only — aces are NOT face cards. Using 16 instead of 12 gives the wrong fraction.

Question A5

1 mark

Theoretical probability · counters

THE QUESTION

A bag contains 5 red and 3 green counters. One counter is drawn at random.

What is P(green)?

(A) 3/5 (B) 3/8 (C) 5/8 (D) 1/3

Question A5 — solution

1 mark

Theoretical probability · counters

CONDENSED SOLUTION

- (B) 3/8
- 3 green of the 5 + 3 = 8 counters in total.

TRAP The denominator is the TOTAL bag (5 + 3 = 8), not the other colour. 3/5 has the right numerator and the wrong total.

03

Key concept 2 · The complement rule

A new idea: sometimes the fastest path is the event that does NOT happen

- P(not A) = 1 − P(A)
- A and not A together fill the whole sample space

Key points · the complement rule

One subtraction replaces a long count

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P(not A)  =  1  −  P(A)        and always        P(A) + P(not A) = 1
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1	**What “not” means**	The complement of A is everything in the sample space EXCEPT the outcomes of A. Rolling not-a-six: five of the six outcomes.
2	**When it saves work**	When “not” is easier to count: P(at least one six) is hard, but P(not A) from 1 − P(A) can be one subtraction.
3	**Same scale**	If P(A) = 1/4, then P(not A) = 3/4 — the complement is still a probability between 0 and 1.
4	**Check your pair**	Always verify P(A) + P(not A) = 1. If your two answers add to something else, one of them is wrong.

Worked example · complement

the question

Question first — attempt it before the working

THE QUESTION

A spinner has 15 equal sectors. 4 of them are purple.

Find: (a) P(purple) (b) P(not purple) (c) P(not purple) as a percentage

Worked example · complement — working

the working

The key steps, in order

THE WORKING

(a) P(purple) = 4/15 ≈ 0.267

(b) P(not purple) = 1 − 4/15 = 11/15 ≈ 0.733

(c) 11/15 × 100 ≈ 73.3%

TRAP Subtract the PROBABILITY from 1: 1 − 4/15. Writing P(not purple) as 15 − 4 = 11 leaves a raw count, not a probability.

Question A6

1 mark

Complement rule

THE QUESTION

On a different spinner, P(purple) = 2/5.

What is P(not purple)?

(A) 2/5 (B) 3/5 (C) 5/3 (D) 1/5

Question A6 — solution

1 mark

Complement rule

CONDENSED SOLUTION

- (B) 3/5
- P(not purple) = 1 − 2/5 = 3/5.

TRAP Subtract from 1, then keep fractions: 1 − 2/5 = 3/5. An answer of 5/3 came from flipping the fraction instead of subtracting.

Question A7

1 mark

Experimental probability

THE QUESTION

A die is rolled 90 times and lands on six 18 times.

What is the experimental probability of rolling a six?

(A) 1/6 (B) 0.18 (C) 0.2 (D) 0.9

Question A7 — solution

1 mark

Experimental probability

CONDENSED SOLUTION

- (C) 0.2
- 18/90 = 0.2 — from the data, not from the die model.

TRAP Divide occurrences by TRIALS: 18/90 = 0.2. Answer (A) is the theoretical probability; answer (B) is a bare 18 ÷ 100 slip.

Question W3

1 mark

The complement rule · worksheet Q3

THE QUESTION

The probability of event A is 0.3.

What is P(not A)?

(A) 0.3 (B) 0.03 (C) 1.3 (D) 0.7

Question W3 — solution

1 mark

The complement rule · worksheet Q3

CONDENSED SOLUTION

- (D) 0.7
- P(not A) = 1 − 0.3 = 0.7.

TRAP Subtract from 1, then check the pair: 0.3 + 0.7 = 1. An answer of 1.3 came from adding instead of subtracting.

Question W8

1 mark

The complement rule · worksheet Q8

THE QUESTION

A spinner has 8 equal sectors and 3 of them are shaded.

What is P(not shaded)?

(A) 3/8 (B) 1/2 (C) 8/5 (D) 5/8

Question W8 — solution

1 mark

The complement rule · worksheet Q8

CONDENSED SOLUTION

- (D) 5/8
- Not shaded: 8 − 3 = 5 of the 8 sectors.

TRAP Subtract the shaded sectors FROM the total: 5/8. An answer of 8/5 flips the fraction — and no probability can exceed 1.

Question W4

1 mark

Experimental probability · worksheet Q4

THE QUESTION

A weather station recorded 40 days, and it rained on 9 of them.

What is the experimental probability of rain from this data?

(A) 9/31 (B) 9/40 (C) 31/40 (D) 1/9

Question W4 — solution

1 mark

Experimental probability · worksheet Q4

CONDENSED SOLUTION

- (B) 9/40
- 9 rainy days out of 40 recorded trials.

TRAP Divide occurrences by TRIALS: 9/40. Answer (A) divides by the days it did NOT rain — the denominator is always the number of trials.

Question W6

1 mark

Expected outcomes · worksheet Q6

THE QUESTION

A fair die is rolled 200 times.

Roughly how many sixes would you expect?

(A) 20 (B) 33 (C) 50 (D) 100

Question W6 — solution

1 mark

Expected outcomes · worksheet Q6

CONDENSED SOLUTION

- (B) 33
- Expected sixes = 200 × 1/6 ≈ 33.3, so roughly 33.

TRAP Multiply the trials by the probability: 200 × 1/6. Answer (C) used 1/4 and (D) used 1/2 — only the fair die's 1/6 is justified.

Question W7

1 mark

Short-run variation · worksheet Q7

THE QUESTION

A fair coin has just shown six heads in a row.

The probability that the next flip is heads is:

(A) more than 1/2, heads is on a streak
(B) less than 1/2, tails is due
(C) exactly 1/2
(D) 1/64

Question W7 — solution

1 mark

Short-run variation · worksheet Q7

CONDENSED SOLUTION

- (C) exactly 1/2
- The coin has no memory: every flip of a fair coin is P = 1/2.

TRAP Tails is NOT “due” and heads is NOT “hot”. Believing past flips change the next one is the gambler’s fallacy — the trap itself.

Question W10

5 marks

Long-run relative frequency · worksheet Q10

THE QUESTION

The traffic lights on a student's route are green 45% of the time and amber 5% of the time (long-run values).

(a) Find P(the lights are red). [1]
(b) In one week the student crosses on 40 occasions. About how many times would you expect the lights to be red? [1]
(c) In one particular week the lights were red only 12 times out of 40. Explain why this does not contradict the 50% value from part (a). [2]
(d) What evidence would suggest the true probability of red is no longer 50%? [1]

Question W10 — solution

5 marks

Long-run relative frequency · worksheet Q10

CONDENSED SOLUTION

- (a) P(red) = 1 − 0.45 − 0.05 = 0.5 (1)
- (b) 0.5 × 40 = 20 times (1)
- (c) 20 is a long-run average — short-run variation means any particular week can differ; 12/40 = 0.3 is within chance wobble (2)
- (d) Over many more crossings, the red rate stays far from 50% (1)

TRAP A probability describes what happens over MANY repetitions, never one short stretch. “Only 12 this week” judges the short run — the law of large numbers needs many trials.

Question A8

1 mark

Short-run variation

THE QUESTION

A fair coin has landed heads 5 times in a row.

What is the probability that the NEXT flip is heads?

(A) less than 1/2 (B) 1/2 (C) more than 1/2 (D) 1/32

Question A8 — solution

1 mark

Short-run variation

CONDENSED SOLUTION

- (B) 1/2
- The coin has no memory: every flip of a fair coin is P = 1/2.

TRAP Tails is NOT “due” after 5 heads. Believing past flips change the next one is the gambler’s fallacy — the trap itself.

04

Stretch and challenge

The stretch tier: multi-mark challenges that go beyond the drills

- Full-working questions P1–P8: counters, experiments, the long run, short-run reasoning
- Challenge C1: the coin-streak question — how likely is a streak of heads?

Question P1

3 marks

Counters · full working

THE QUESTION

A bag contains 6 blue, 4 red and 11 green counters. One counter is drawn at random.

(a) Find P(blue). Give your answer in lowest terms.
(b) Find P(red or green).
(c) Find P(not green).

Question P1 — solution

3 marks

Counters · full working

CONDENSED SOLUTION

- Total = 6 + 4 + 11 = 21
- (a) P(blue) = 6/21 = 2/7
- (b) P(red or green) = (4 + 11)/21 = 15/21 = 5/7
- (c) P(not green) = 1 − 11/21 = 10/21

TRAP Simplify fully: 6/21 must become 2/7. And in (c) the complement of “green” is blue-and-red — 10 of the 21 counters.

Question P2

4 marks

Die experiment · theory vs data

THE QUESTION

A fair die is rolled 30 times. It lands on five 7 times.

(a) State the theoretical probability of rolling a five.
(b) Find the experimental probability from this data.
(c) Give one reason the two values differ.
(d) How many fives would you expect in 60 rolls?

Question P2 — solution

4 marks

Die experiment · theory vs data

CONDENSED SOLUTION

- (a) Theoretical: 1/6 ≈ 0.167
- (b) Experimental: 7/30 ≈ 0.233
- (c) Short-run variation — 30 trials is a small sample
- (d) 60 × 1/6 = 10 fives

TRAP Theoretical comes from the fair-die model (1/6); experimental comes from the recorded data (7/30). Mixing the two is the error examiners watch for.

Question P3

3 marks

Ordering events on the scale

THE QUESTION

Put these four events in order from LEAST to MOST likely, and give the probability of each:

- rolling a 7 on a fair die
- drawing a heart from a standard 52-card deck
- a flipped fair coin landing heads
- the Sun rising tomorrow (treat as certain)

Question P3 — solution

3 marks

Ordering events on the scale

CONDENSED SOLUTION

- Rolling a 7: P = 0 (impossible)
- Heart: 13/52 = 1/4; Heads: 1/2; Sunrise: P = 1 (certain)
- Order: 7 on a die < heart < heads < sunrise

TRAP Convert every event to a probability before comparing: 0, 1/4, 1/2, 1. Comparing the events by “how they feel” instead of by their probabilities loses the marks.

Question P4

4 marks

Long-run relative frequency

THE QUESTION

The probability that the school bus is late on any given day is 0.4.

(a) Interpret 0.4 using long-run relative frequency.
(b) About how many late days would you expect in 40 school days?
(c) Will the bus be late on exactly 16 of the next 40 days? Explain.

Question P4 — solution

4 marks

Long-run relative frequency

CONDENSED SOLUTION

- (a) Over many days under the same conditions, about 40% of days the bus is late
- (b) $0.4 \times 40 = 16$ late days (expected, long-run average)
- (c) Not necessarily — 16 is a long-run average; any particular 40 days can differ

TRAP A probability describes what happens over MANY repetitions, not a guarantee for one short stretch. “Exactly 16” confuses the long run with the short run.

Question P5

4 marks

Short-run variation

THE QUESTION

A student flips a fair coin 5 times and gets 5 heads in a row.

(a) Explain why this result does not prove the coin is biased.
(b) Find the probability that the next flip is heads.
(c) What further evidence WOULD suggest the coin really is biased?

Question P5 — solution

4 marks

Short-run variation

CONDENSED SOLUTION

- (a) Short-run variation: 5-head streaks can happen by chance (P = 1/32 ≈ 3%)
- (b) 1/2 — the coin has no memory
- (c) Over many flips, heads keeps landing far from 50% of them

TRAP The coin does not “ome” owed tails. Past flips do not change the next one — believing they do is the gambler’s fallacy.

Question P6

4 marks

Experimental vs theoretical

THE QUESTION

A basketball player makes 14 of her first 20 free throws.

(a) Find her experimental probability of making a free throw.
(b) Is this a theoretical or an experimental probability? Why?
(c) Estimate how many free throws she would make in 40 attempts.
(d) How could she improve this estimate?

Question P6 — solution

4 marks

Experimental vs theoretical

CONDENSED SOLUTION

- (a) 14/20 = 0.7
- (b) Experimental — it comes from recorded trials, not equally likely reasoning
- (c) 0.7 × 40 = 28 free throws
- (d) More attempts: a larger sample settles closer to the long-run value

TRAP Free throws are not “equally likely” outcomes like a fair die, so there is no theoretical formula — only the data can estimate her rate.

Question P7

4 marks

Short-run reasoning · extended

THE QUESTION

A vending machine is supposed to dispense a winning ticket 50% of the time. In one lunch hour it dispensed 8 tickets and 8 were winners.

(a) Explain why 8 wins out of 8 does not disprove the 50% model.
(b) What is the probability that the next ticket is a winner?
(c) Describe the evidence that WOULD cast doubt on the 50% model.

Question P7 — solution

4 marks

Short-run reasoning · extended

CONDENSED SOLUTION

- (a) Short-run variation: an 8-win streak can happen by chance (P = 1/256)
- (b) 0.5 — the model still says each ticket is a 50% chance
- (c) Over many more tickets, the win rate stays far from 50%

TRAP “All 8 won, so the machine is broken” jumps from a tiny sample to a conclusion. Small stretches are noisy; the long run is the judge.

Question P8

4 marks

Bring it together

THE QUESTION

A four-sector spinner is suspected of favouring red. The maker claims P(red) = 1/4. In 60 spins, red came up 24 times.

(a) Find the experimental probability of red.
(b) Compare it with the claimed theoretical probability.
(c) Explain why 60 spins alone may not settle the argument.

Question P8 — solution

4 marks

Bring it together

CONDENSED SOLUTION

- (a) Experimental P(red) = 24/60 = 0.4
- (b) Claimed theoretical = 1/4 = 0.25 — the data sit well above it
- (c) 60 trials is still a fairly small sample; more spins are needed to separate chance wobble from genuine bias

TRAP 0.4 vs 0.25 looks like strong evidence, but short-run variation can produce gaps this big. Many more trials is the honest answer.

Question C1

4 marks

Challenge · coin streaks

THE QUESTION

A fair coin is flipped 5 times.

Find the probability that at least 3 heads in a row occur somewhere in the five flips.

Give your answer as an exact fraction.

Question C1 — solution

4 marks

Challenge · coin streaks

CONDENSED SOLUTION

- List all 32 sequences or use the complement.
- P(no run of 3 heads) = 24/32: sequences avoiding HHH.
- P(at least 3 heads in a row) = 1 − 24/32 = 8/32 = 1/4.
- Check: exactly 8 of the 32 sequences contain HHH — HHHHH, HHHHT, HHHTH, HHHTT, HTHHH, THHHH, THHHT, TTHHH.

TRAP 1/8 is only P(first three flips are heads). “Somewhere in the five flips” catches overlaps like HHHTH — overlapping events must NOT be added; enumerate or use the complement.

True or false · 1

T / F

Decide, then justify your decision — answer on the next slide

A probability of 0.25 means the event happens exactly once in every four trials.

True or false · 1 — answer

FALSE

FALSE

A probability is a long-run average, not a schedule. In any particular four trials the event may occur 0, 1, 2, 3 or 4 times — about once per four trials is the long-run pattern.

True or false · 2

T / F

Decide, then justify your decision — answer on the next slide

For any event A, P(not A) = 1 − P(A).

True or false · 2 — answer

TRUE

TRUE

A and its complement cover the whole sample space, so their probabilities add to exactly 1. If P(A) = 0.3, then P(not A) = 0.7.

True or false · 3

T / F

Decide, then justify your decision — answer on the next slide

If an event is impossible, its probability is 1.

True or false · 3 — answer

FALSE

FALSE

Impossible means P = 0. A probability of 1 means the event is certain. Rolling a 7 on a fair die is impossible: P(7) = 0.

True or false · 4

T / F

Decide, then justify your decision — answer on the next slide

An experimental probability based on 500 trials is usually closer to the true probability than one based on 20 trials.

True or false · 4 — answer

TRUE

TRUE

More trials mean the relative frequency settles toward the true probability — the law of large numbers. Small samples wobble; large samples stabilise.

True or false · 5

T / F

Decide, then justify your decision — answer on the next slide

Getting 5 heads in a row proves that a coin is not fair.

True or false · 5 — answer

FALSE

FALSE

A fair coin produces a 5-head streak with probability 1/32 ≈ 3% — unusual, but entirely possible. Judge fairness over many flips, not one streak.

True or false · 6

T / F

Decide, then justify your decision — answer on the next slide

A probability can be 1.5 if the event is extremely likely.

True or false · 6 — answer

FALSE

FALSE

Probabilities never exceed 1. “Extremely likely” tops out at P = 1, certain. A value like 1.5 shows a counting or arithmetic error.

True or false · 7

T / F

Decide, then justify your decision — answer on the next slide

In the long run, the relative frequency of an event approaches its theoretical probability.

True or false · 7 — answer

TRUE

TRUE

This is the law of large numbers. As the number of trials grows, the observed proportion gets closer and closer to P(A) — the long-run meaning of probability.

True or false · 8

T / F

Decide, then justify your decision — answer on the next slide

Theoretical probability assumes that all outcomes in the sample space are equally likely.

True or false · 8 — answer

TRUE

TRUE

The favourable ÷ total formula only works for equally likely outcomes — a fair die, equal spinner sectors, a well-shuffled deck. Without that assumption, the formula fails.

Mark scheme · 1 of 2

Starter, worksheet questions and exercises — what earns each mark

S1	3	Line 3 error: x = μ + zσ (1) · correct value 119.2 (1) · nearest option 120 (1)
W1	1	(C) 3/10 · 3 red of 10 total
W3	1	(D) 0.7 · 1 − 0.3
W5	1	(C) 7/20 theoretical · total class · no trials recorded
W7	1	(C) exactly 1/2 · no memory
W9	5	1 total 15 · 1 P(blue) 8/15, P(white) 2/15 · 1 P(not red) = 2/3 · 1 link: not red = blue or white · 1 expect 32
A1	1	identifies 1.2 as impossible (over 1)
A3	1	P(even) = 3/6 = 1/2
A5	1	3 green of 8 total · 3/8
A7	1	18/90 = 0.2 (relative frequency from the data)

S2	2	x = 87.4 (1) · below the mean because z is negative / 20% below (1)
W2	1	(B) 1/2 · {1, 2, 3} of 6
W4	1	(B) 9/40 · occurrences ÷ trials
W6	1	(B) 33 · 200 × 1/6 ≈ 33.3
W8	1	(D) 5/8 · 1 − 3/8
W10	5	1 P(red) = 0.5 · 1 expect 20 · 2 short-run variation vs long run · 1 red rate far from 50%
A2	1	P(six) = 1/6
A4	1	12 face cards · 12/52 = 3/13
A6	1	P(not purple) = 1 − 2/5 = 3/5
A8	1	1/2 — the coin has no memory

Subtotal so far: 5 starter practice marks + worksheet W1–W10, 18 marks + exercises A1–A8, 8 marks.

Mark scheme · 2 of 2

Stretch and challenge — what earns each mark

C1	4	1 method (complement or systematic listing) · 1 P(no run) = 24/32 · 1 answer 1 − 24/32 = 1/4 · 1 verification / list of the 8 favourable sequences
C2	6	1 set-up of $E_0/E_1/E_2$ states · 2 correct equations (reset structure) · 1 solving to $E_0 = 14$ · 1 formula/teaser check 2^(k+1) − 2 · 1 exact-answer presentation
P1	3	1 total 21 · 1 P(blue) = 6/21 = 2/7 · 1 P(red or green) = 5/7 · 1 P(not green) = 10/21
P2	4	1 theoretical 1/6 · 1 experimental 7/30 · 1 reason: small sample/variation · 1 expectation 10
P3	3	1 probabilities 0 · 1/4 · 1/2 · 1 · 1 correct ordering · 1 justification
P4	4	1 interpretation · 1 expected 16 · 1 explanation that 16 is not guaranteed · 1 long-run language
P5	3	1 streak possible by chance · 1 next flip 1/2 · 1 no-memory reasoning · 1 many-trials evidence
P6	3	1 experimental 0.7 · 1 identified as experimental · 1 long-run evidence · 28 · 1 more attempts
P7	4	1 chance explanation · 1 next ticket 0.5 · 1 long-run evidence · 1 no conclusion from small sample
P8	4	1 experimental 0.4 · 1 comparison with 0.25 · 1 small-sample caveat · 1 more-trials conclusion

Assessed total: W1–W10, 18 + A1–A8, 8 + P1–P8, 29 + C1, 4 + end-of-deck C2, 6 = 65 marks (+ 5 starter practice marks). Worked examples are for teaching and carry no tariff.

Answer key

One line per question

ANSWERS

- S1 line 3 wrong · x = 100 + 1.28 × 15 = 119.2 · nearest 120
- S2 x = 100 − 0.84 × 15 = 87.4 · below the mean (z negative)
- W1 (C) 3/10
- W2 (B) 1/2
- W3 (D) 0.7
- W4 (B) 9/40
- W5 (C) 7/20, theoretical
- W6 (B) 33
- W7 (C) exactly 1/2
- W8 (D) 5/8

THE GOLDEN RULE

- W9 8/15 · 2/15 · 2/3 · not red = blue or white · 32
- W10 0.5 · 20 · short-run variation · red rate far from 50% long-run
- A1 (C) 1.2 · A2 (A) 1/6 · A3 (C) 1/2 · A4 (C) 3/13
- A5 (B) 3/8 · A6 (B) 3/5 · A7 (C) 0.2 · A8 (B) 1/2
- C1 1 − 24/32 = 8/32 = 1/4 (brute-force verified, 32 sequences)
- P1 2/7 · 5/7 · 10/21 · P2 1/6 · 7/30 · 10
- P3 0 < 1/4 < 1/2 < 1 · P4 40% · 16 · not guaranteed
- P5 chance streak · 1/2 · many flips · P6 0.7 · 28 · larger sample
- P7 1/256 chance · 0.5 · long run far from 50%
- P8 0.4 vs 0.25 · 60 spins too few to settle it
- C2 14 flips (E_0 = 14; E(k) = 2^(k+1) − 2)

FIVE THINGS TO DO IN THE EXAM

Do these and probability will not cost you marks

- Write the sample space first, then divide: P(A) = favourable ÷ total, and simplify the fraction.
- Check every probability sits between 0 and 1 — an answer of 1.2 or −0.3 is a red flag.
- Use the complement: P(not A) = 1 − P(A), and verify P(A) + P(not A) = 1.
- Label which probability you are using: theoretical (from the model) or experimental (from the trials).
- Interpret probabilities as long-run relative frequency — and never read a short streak as proof of bias.
- Remember the hook: Amara's 10-head streak and Jay's 9-streak were both pure chance — streaks happen, and only the long run judges a coin.

Built from the Introduction to Probability lesson materials.

Question C2

6 marks

CHALLENGE — expected number of flips

THE QUESTION

A fair coin is flipped repeatedly.

Find the expected number of flips needed to first get 3 heads in a row.

Give your answer exactly.

Question C2 — solution

6 marks

CHALLENGE — expected number of flips

CONDENSED SOLUTION

- Set up E_i = expected additional flips when you already have i consecutive heads.
- $E_0 = 1 + \frac{1}{2}E_1 + \frac{1}{2}E_0$ $E_1 = 1 + \frac{1}{2}E_2 + \frac{1}{2}E_0$ $E_2 = 1 + 0 + \frac{1}{2}E_0$
- A head moves you up a state, a tail resets you to E_0; the third head completes ($E_3 = 0$).
- Solving the three equations: $E_0 = 14$ flips.
- Formula check: E(k) = 2^(k+1) − 2, so k = 3 gives 2^4 − 2 = 14. Same logic: k = 2 → 6 flips, k = 4 → 30, k = 5 → 62.

TRAP P(3 heads in a row) = 1/8 does NOT mean 8 expected flips. The naive multiplication ignores that a tail resets all progress to zero — the true expectation is 14, nearly double the naive guess.