PRECALCULUS · STATISTICS

Introducing Statistics

Random and non-random patterns — telling structure apart from chance

10
questions worked

3
sections

8
true/false checks

Built from the source lesson "Introducing statistics · random and non-random patterns"

How this lesson runs

Key points, worked examples, questions to try — then true or false

Key points	Each section opens with a summary of the ideas you need, main point highlighted first.
Worked example	A full example is split across two slides: the question first, then the working.
Question	Questions appear on their own slides. Attempt each before turning the page.
Solution	The next slide gives the condensed solution: the answer and the key reasoning only.
True or false	The lesson ends with eight statements — one per slide, answer on the slide that follows.

Key topic · Random and non-random patterns

What this lesson is about, in one page

Random patterns	Natural variation with no predictable order. Outcomes are unpredictable in advance, yet each event still has a probability.
Non-random patterns	Structure, trends and predictability. Often explained by an underlying factor — time, treatment, a rule — not by chance.
Why it matters	Statistics separates meaningful patterns from random noise. Before acting on a pattern, ask: could chance alone produce this?
Staying fair	Randomness actually helps sampling: it keeps bias out. Controlled conditions keep other variables constant so the factor being studied stands out.

Vocabulary

Key terms for this lesson

random pattern	outcomes are unpredictable, but each event has a probability
natural variation	the spread caused purely by chance, with no underlying cause
non-random pattern	structure or a trend: values change in a predictable way
trend	a long-term direction in data, e.g. values rising steadily over time
underlying factor	the cause behind a non-random pattern (time, treatment, rule)
probability	a number from 0 to 1 giving the long-run chance of an event
simulation	repeating a chance process many times to study its variation
expected variation	the spread that chance alone typically produces in a sample
sampling bias	a systematic tilt in how data is collected that distorts results
controlled conditions	keeping other variables constant so one factor can be studied

01

Random and non-random patterns

Two kinds of pattern — and the question that tells them apart

- Random: unpredictable outcomes, each with a probability
- Non-random: structure, trend, predictability — an underlying factor

Worked example · classifying a dataset

the question

Question first — test scores from two marking methods

THE QUESTION

A teacher records the number of students absent each day for two weeks.

Week 1 (before a flu outbreak): 3, 4, 2, 3, 5

Week 2 (after the outbreak): 8, 9, 7, 10, 8

Is the change between the two weeks best explained as a random pattern or a non-random pattern? Justify your answer.

Worked example · classifying a dataset — working

the working

Answer with a decision, then the evidence that forces it

THE WORKING

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Step 1  Decision: a NON-RANDOM pattern.
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Step 2  Evidence: every Week 2 value sits ABOVE every Week 1 value.
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Step 3  A likely cause: the flu outbreak raised absence levels.
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Step 4  Random variation within a week is small; the jump between weeks is
systematic, not chance.
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The justify marks come from evidence + cause. “It went up” alone is not enough: say why chance is an unlikely explanation.

Key points · random vs non-random

The definitions you will use in every question

1	**Random pattern**	Outcomes are unpredictable in advance, but each event has a probability. Individual results vary; nothing is fixed.
2	**Natural variation**	Even a fair, random process produces different results each time — coin flips, queues, defects. Difference alone proves nothing.
3	**Non-random pattern**	Structure: values follow a trend or rule, often driven by an underlying factor such as time, treatment or a design choice.
4	**The key question**	Could chance alone produce what I see? If yes, it may be random. If the pattern is too systematic, look for a cause.
5	**Say both parts**	A full justification names the classification AND gives evidence: how the data behave, and what factor could explain them.

Question A1

3 marks

Random vs non-random · classify

THE QUESTION

A café counts the number of customers arriving between 9 am and 10 am each morning.

Mon 23 · Tue 31 · Wed 26 · Thu 29 · Fri 24

Is this daily change best described as a random pattern or a non-random pattern? Justify your answer.

Question A1 — solution

3 marks

Random vs non-random · classify

CONDENSED SOLUTION

- Random pattern.
- The values fluctuate around a similar level (mid-20s to low 30s).
- Day-to-day differences are the natural variation of arrivals — no trend and no single cause is acting.

TRAP “The numbers are different every day” does NOT make a pattern non-random. Different results are exactly what randomness produces.

Question A2

3 marks

Random vs non-random · classify

THE QUESTION

A student practises typing and records words per minute each week.

Week 1: 32 Week 4: 41 Week 7: 49 Week 10: 58

Is the upward movement across weeks a random pattern or a non-random pattern? Justify your answer.

Question A2 — solution

3 marks

Random vs non-random · classify

CONDENSED SOLUTION

- Non-random pattern.
- Steady upward trend: the score rises by about 8–9 wpm every 3 weeks.
- A plausible underlying factor: practice is improving the skill — the improvement is systematic, not chance.

TRAP A consistent trend is the signature of a non-random pattern. Practice builds skill in one direction; chance does not climb steadily for 10 weeks.

Question A3

4 marks

Random vs non-random · spot the fake

THE QUESTION

A factory records the number of defective parts in samples of 100. A machine fault is suspected.

Sample 1: 2 Sample 2: 1 Sample 3: 9 Sample 4: 8 Sample 5: 9 Sample 6: 1

(a) Do the low and high samples suggest the defect count is random overall?
(b) Which samples point to a non-random cause, and what is it?

Question A3 — solution

4 marks

Random vs non-random · spot the fake

CONDENSED SOLUTION

- (a) Partly — small counts like 1 and 2 are within natural variation of the process.
- (b) Samples 3, 4 and 5: nine-ish defects in a row is too high to be chance.
- A non-random cause: the machine fault, starting around sample 3.
- Random variation is roughly stable over time; a sudden sustained shift signals a cause.

TRAP Do not average the whole list and miss the shift. Randomness is judged by how the data behave OVER TIME — a run of unusual values is structure, not luck.

02

Trends, structure and simulation

Asking “could chance alone do this?” — and testing it

- Simulations show what random variation normally looks like
- A pattern far outside that range points to a non-random cause

Key points · variation and chance

How statisticians separate signal from noise

1	**What chance looks like**	A fair coin flips about half heads, but not exactly: 47 or 54 heads in 100 flips are all normal random results.
2	**Simulation**	Repeat the chance process many times to see the range of results that randomness alone produces.
3	**Compare, then conclude**	If the observed result sits inside the simulated range, chance is a good explanation. Far outside it, suspect structure.
4	**Too regular to be random**	Perfect alternation or unbroken runs are suspicious: truly random data wobble, they do not repeat a neat cycle.
5	**Trend vs wobble**	A single spike can be chance. A steady direction repeated across many points is evidence of an underlying factor.

Worked example · simulation

the question

Question first — coin flips and the range of chance

THE QUESTION

Students flip a fair coin 50 times. Most classes record between 18 and 32 heads.

One student claims: “I got 29 heads, which is more than 25, so my coin is not random.”

Explain the mistake, using the idea of natural variation.

Worked example · simulation — working

the working

Randomness never repeats the same numbers twice

THE WORKING

Step 1 Expected value: 25 heads — but randomness does not hit the target exactly.

Step 2 Natural variation: repeats of the 50 flips give a RANGE, about 18–32 heads.

Step 3 29 heads lies inside that range, so it is an ordinary random outcome.

Step 4 Conclusion: the claim confuses “different from 25” with “impossible by chance”.

Chance results scatter around the expected value — only results OUTSIDE the simulated range are suspicious.

Question B1

3 marks

Simulation · coin flips

THE QUESTION

A fair coin is flipped 100 times and lands heads 43 times.

(a) How many heads would be expected?
(b) Explain why 43 does not prove the coin is unfair.
(c) What extra evidence WOULD suggest the coin is not random?

Question B1 — solution

3 marks

Simulation · coin flips

CONDENSED SOLUTION

- (a) Expected: 50 heads.
- (b) 43 is close to 50 — natural variation makes every sample differ from the expectation.
- (c) Many repeated flips consistently landing far from 50, e.g. near 40 every time.

TRAP One sample never proves a bias. Non-randomness shows up as a CONSISTENT deviation, repeated across many trials — not as one unusual result.

Question B2

4 marks

Simulation · dice

THE QUESTION

A die is rolled 60 times. After a simulation of many sets of 60 rolls, the number of sixes usually falls between 6 and 15.

(a) How many sixes are expected in 60 rolls?
(b) A set of rolls gives 14 sixes. Is that evidence of a non-random pattern?
(c) Another set gives 30 sixes. What should you now suspect, and why?

Question B2 — solution

4 marks

Simulation · dice

CONDENSED SOLUTION

- (a) Expected: 60 × (1/6) = 10 sixes.
- (b) No — 14 lies inside the simulated range 6 to 15, so chance explains it.
- (c) Suspect a non-random pattern (possibly a loaded die).
- Reason: 30 is far outside the range that randomness alone produced.

TRAP “Higher than expected” is not the test. “Outside the range chance produces” is the test — that is what the simulation gives you.

Question B3

4 marks

Too regular to be random

THE QUESTION

A queue watcher records whether each arriving customer is male (M) or female (F):

M F M F M F M F M F M F

(a) Explain why this exact alternating pattern is unlikely to be random.
(b) What would a genuinely random list of 12 arrivals look like?

Question B3 — solution

4 marks

Too regular to be random

CONDENSED SOLUTION

- (a) Perfect alternation is extremely unlikely by chance — every one of 12 arrivals matched the cycle. Random data wobble.
- (b) A random list shows streaks and uneven mixes, e.g. M F F M M M F M F M F F.
- Unpredictability is the hallmark of randomness — a fixed cycle is predictable, so it is structure.

TRAP "Too neat" is a red flag. Real random sequences contain runs and clumps; a pattern this regular points to an underlying rule instead.

Question B4

4 marks

Real context · plant growth

THE QUESTION

Two groups of five plants are grown for 4 weeks. Group 1 gets a new fertiliser; Group 2 gets none.

Group 1 heights: 22, 24, 21, 23, 25 cm
Group 2 heights: 17, 19, 16, 18, 17 cm

(a) Is the difference between the groups likely to be random?
(b) Identify the underlying factor.
(c) Why were Group 2 plants included at all?

Question B4 — solution

4 marks

Real context · plant growth

CONDENSED SOLUTION

- (a) Unlikely to be random: every fertilised plant outgrew every unfertilised plant.
- (b) The fertiliser — the deliberate difference between the groups.
- (c) Group 2 is the control: it shows the growth that no fertiliser would give.
- Without the control, the taller plants could not be linked to the fertiliser.

TRAP Two wobbly datasets can still be a real effect. What matters is the SYSTEMATIC gap between the groups, not the wobble inside each group.

03

Sampling, fairness and control

Randomness is a tool — when it helps and when it hides the truth

- Random sampling keeps bias out of a study
- Controls and randomisation let one factor show its effect

Key points · sampling and fairness

Why randomness is deliberately built into good studies

1	**Random sampling**	Choosing people by chance gives every member the same probability of selection — that is what keeps a sample fair.
2	**Bias**	A non-random, systematic tilt in how data are collected. A volunteer-only survey over-represents people with strong opinions.
3	**Controlled conditions**	Keep every other variable constant so the factor being studied is the only thing that differs between groups.
4	**Random assignment**	Assigning subjects to groups by chance stops lurking variables from piling up on one side.
5	**Variation is honest**	Random samples still differ from each other. That spread is expected — it is not a flaw in the method.

Question C1

3 marks

Sampling and bias

THE QUESTION

A school wants to know whether students are happy with the canteen. It surveys only the students who queue at the salad bar.

(a) Why is this sample not random?
(b) In which direction is the result likely to be biased?
(c) Describe a random way to choose the sample.

Question C1 — solution

3 marks

Sampling and bias

CONDENSED SOLUTION

- (a) Students are chosen by their own behaviour (choice of queue), not by chance — selection is systematic.
- (b) Upward bias: salad-bar users are probably happier with the canteen than average.
- (c) Number every student, then use a random draw (e.g. random numbers) to pick respondents.

TRAP Voluntary or self-selected samples are never random: joining is a choice, and choices are systematic. Randomness must come from the sampler, not the subjects.

Question C2

4 marks

Random or non-random? · extend

THE QUESTION

For each scenario, decide random or non-random, and give one piece of evidence:

(a) The last digit of the jersey number of each player sampled from a league squad list.
(b) A shop's sales rise every December for 6 years in a row.
(c) The exact second at which each email arrives in an inbox during one hour.

Question C2 — solution

4 marks

Random or non-random? · extend

CONDENSED SOLUTION

- (a) Random — digits have no order and each outcome had a probability of selection.
- (b) Non-random — a repeated seasonal trend points to a cause (holiday shopping).
- (c) Random — arrival times are unpredictable one by one, with natural variation.
- Each answer pairs the label with evidence: no order vs trend vs unpredictability.

TRAP “Random” does not mean meaningless. It means each outcome is unpredictable in advance yet follows a probability — like email arrival times.

Question C3

4 marks

Fairness · experiment design

THE QUESTION

A study tests whether a new sleep app improves exam scores. Twenty volunteers are split into two groups of ten.

(a) Why should students be assigned to the two groups RANDOMLY rather than by letting them choose?
(b) State one other variable to hold constant.
(c) If Group A (app) scores higher, why is random assignment important before crediting the app?

Question C3 — solution

4 marks

Fairness · experiment design

CONDENSED SOLUTION

- (a) Self-selection is non-random: keen students might all pick the app, biasing the groups before the test starts.
- (b) Hold constant, e.g. same revision time, same exam, same sleep schedule.
- (c) Random assignment makes the two groups comparable — any ability gap is reduced, so the score gap is more credibly due to the app.
- Randomness here is not noise; it is the tool that keeps the comparison fair.

TRAP Non-random assignment creates bias BEFORE any data appear. The comparison then measures who chose each group, not the effect being studied.

Mark scheme

Marks by question, with what earns each mark

A1	3	1 label random · 1 evidence (fluctuation about a level) · 1 cause ruled out / variation named
A2	3	1 label non-random · 1 trend evidence (steady ~8–9 wpm rise) · 1 underlying factor (practice)
A3	4	1 small counts random · 1 identifies samples 3–5 · 1 cause (machine fault) · 1 stable-vs-shift reasoning
B1	3	1 expected 50 · 1 explanation via natural variation · 1 consistent deviation as real evidence
B2	4	1 expected 10 · 1 14 is inside range · 1 suspicion at 30 · 1 far-outside-range reasoning
B3	4	1 why alternation is unlikely · 1 random lists wobble/have runs · 1 example-type description · 1 predictability point
B4	4	1 unlikely random · 1 factor = fertiliser · 1 purpose of control group · 1 link gap to factor
C1	3	1 not random (self-selected) · 1 direction of bias · 1 valid random selection method
C2	4	1 per scenario: correct label WITH evidence (a, b, c, d)
C3	4	1 why self-selection biases · 1 constant named · 1 random assignment → comparability · 1 credit the app reasoning

Main lesson total: 33 marks.

Answer key

One line per question

ANSWERS

- A1 random — fluctuates about a level, no cause
- A2 non-random — steady trend, practice is the factor
- A3 small counts random; samples 3–5 point to the fault
- B1 50 expected; 43 is natural variation; consistency would be evidence
- B2 10 expected; 14 inside range = random; 30 outside = suspect

THE GOLDEN RULE

- B3 alternation too regular; random lists have runs
- B4 gap systematic; fertiliser is the factor; control shows baseline
- C1 self-selected, biased upward; use a random draw
- C2 (a) random (b) non-random (c) random — with evidence
- C3 random assignment keeps groups comparable

True or false · 1

T / F

Decide, then justify your decision — answer on the next slide

In a random pattern, the exact outcome can be predicted in advance.

True or false · 1 — answer

FALSE

FALSE

Unpredictability is the definition of random. What IS known in advance is the probability of each event — not the outcome itself.

True or false · 2

T / F

Decide, then justify your decision — answer on the next slide

For a random process to be fair, every sample must show outcomes in exactly equal proportions.

True or false · 2 — answer

FALSE

FALSE

Chance never spreads results evenly. A fair die can give 8, 11 and 9 sixes in three sets of 60 rolls — uneven samples are natural variation, and the process is still random.

True or false · 3

T / F

Decide, then justify your decision — answer on the next slide

A consistent upward trend across many data points suggests a non-random pattern.

True or false · 3 — answer

TRUE

TRUE

Chance wobbles in no particular direction. A steady, repeated direction points to an underlying factor such as time or treatment.

True or false · 4

T / F

Decide, then justify your decision — answer on the next slide

If two samples give different results, at least one of them must be non-random.

True or false · 4 — answer

FALSE

FALSE

Natural variation makes every random sample differ. Two different samples from the same random process are exactly what randomness produces.

True or false · 5

T / F

Decide, then justify your decision — answer on the next slide

Each event in a random pattern has a probability.

True or false · 5 — answer

TRUE

TRUE

That is the second half of the definition: outcomes are unpredictable in advance, yet each event has a probability describing its long-run chance.

True or false · 6

T / F

Decide, then justify your decision — answer on the next slide

Choosing survey respondents as whoever volunteers produces a random sample.

True or false · 6 — answer

FALSE

FALSE

Volunteering is a choice, and choices are systematic — often over-representing people with strong opinions. That is bias, not randomness.

True or false · 7

T / F

Decide, then justify your decision — answer on the next slide

In a truly random sequence of coin flips, some streaks of the same result are expected.

True or false · 7 — answer

TRUE

TRUE

Genuine randomness contains runs and clumps. A sequence that never repeats a result — perfectly alternating — is the suspicious one.

True or false · 8

T / F

Decide, then justify your decision — answer on the next slide

Holding other variables constant helps show the effect of the factor being studied.

True or false · 8 — answer

TRUE

TRUE

If everything else stays the same, any systematic difference between groups is credibly due to the one factor that changed.

FIVE THINGS TO DO IN THE EXAM

Do these and random vs non-random will not cost you marks

- Give BOTH parts of a justify answer: the classification and the evidence behind it.
- Remember that different results are normal — chance produces variation, not identical repeats.
- Use a simulated range as your yardstick: inside it is chance, far outside it is structure.
- Treat a steady trend or a sustained shift as evidence of an underlying factor.
- Check for bias first: how data were collected decides whether a sample can be trusted.

Built from the source lesson “Introducing statistics · random and non-random patterns”.