TURN and TALK

Let

$$f(x) = x^5 - x + 1$$

(a) Prove that $f(x)$ is continuous for all real numbers.

(b) Show that $f(x) = 0$ has at least one root in the interval $[-2, -1]$.

(c) Explain why IVT does not guarantee how many roots exist.

$$f(x) = x^5 - x + 1$$

(a) Polynomial ⇒ continuous for all real x

(b)

- $f(-2) = -32 + 2 + 1 = -29$
- $f(-1) = -1 + 1 + 1 = 1$

Sign change ⇒

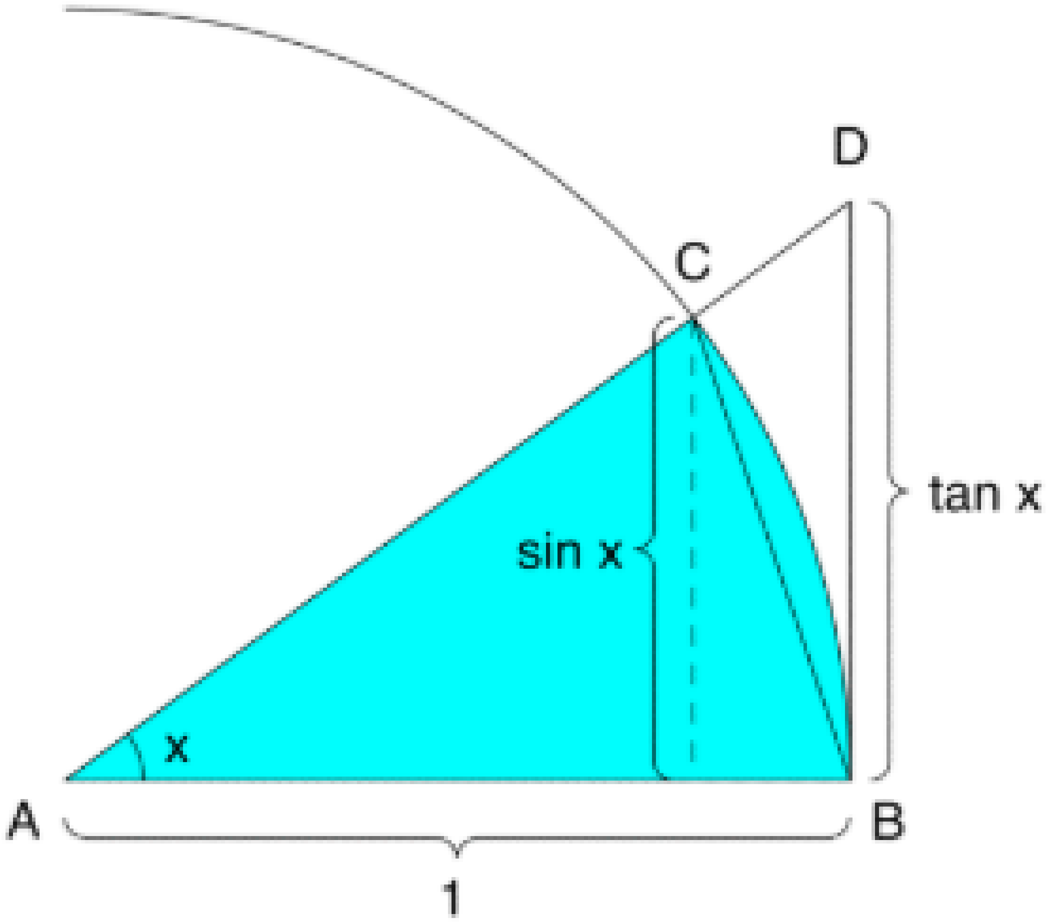
⇒ **At least one root in** $(-2, -1)$

(c)

IVT only guarantees **existence of at least one root where sign changes occur**, not the **number of roots**, because the function may cross the axis multiple times.

1. Derivative — 导数
2. Sine function — 正弦函数
3. Cosine function — 余弦函数
4. Tangent function — 正切函数
5. Secant function — 正割函数
6. Cosecant function — 余割函数
7. Cotangent function — 余切函数
8. Chain rule — 链式法则
9. Product rule — 乘积法则
10. Radian measure — 弧度制

TURN and TALK



Prove using the diagram below and squeeze theorem

$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1$$

UNIT 2 KNOWLEDGE - CALCULUS 12 – DIFFERENTIATION: DEFINITION AND FUNDAMENTAL PROPERTIES

- **2.1** Defining Average and Instantaneous Rates of Change at a Point ✓
- **2.2** Defining the Derivative of a Function and Using Derivative Notation ✓
- **2.3** Estimating Derivatives of a Function at a Point ✓
- **2.4** Connecting Differentiability and Continuity: Determining When Derivatives Do and Do Not Exist ✓
- **2.5** Applying the Power Rule ✓
- **2.6** Derivative Rules: Constant, Sum, Difference, and Constant Multiple ✓
- **2.7** Derivatives of cos x, sin x, e^x , and ln x
- **2.8** The Product Rule
- **2.9** The Quotient Rule
- **2.10** Finding the Derivatives of Tangent, Cotangent, Secant, and/or Cosecant Functions

What Will We Learn?

- We'll look at the basic rules for differentiating four of the most common transcendental functions - $\cos x, \sin x, e^x, and \ln x$.

Recall from Precalculus: the Sum-Angle Formulas from Precalculus

ADDITION AND SUBTRACTION FORMULAS

Formulas for sine:

$$\sin(s + t) = \sin s \cos t + \cos s \sin t$$

$$\sin(s - t) = \sin s \cos t - \cos s \sin t$$

Formulas for cosine:

$$\cos(s + t) = \cos s \cos t - \sin s \sin t$$

$$\cos(s - t) = \cos s \cos t + \sin s \sin t$$

Formulas for tangent:

$$\tan(s + t) = \frac{\tan s + \tan t}{1 - \tan s \tan t}$$

$$\tan(s - t) = \frac{\tan s - \tan t}{1 + \tan s \tan t}$$

Recall: From Unit 1

Recall from Unit 1

$$\lim_{h\to 0}\left(\frac{\cos(h)-1}{h}\right) = 0$$

$$\lim_{h\to 0}\frac{\sin(h)}{h} = 1$$

Four Most Common Transcendental Derivative Formulas

$$\frac{d}{dx}[\sin x] =$$

$$\frac{d}{dx}[\cos x] =$$

These four functions are the four most common transcendental functions that you will encounter in calculus!

Let’s see a proof!

Proving $\frac{d}{dx}[\cos x] = -\sin x$ Using the Limit Definition of Derivative

Formulas for cosine:

$$\cos(s+t) = \cos s \cos t - \sin s \sin t$$
$$\cos(s-t) = \cos s \cos t + \sin s \sin t$$

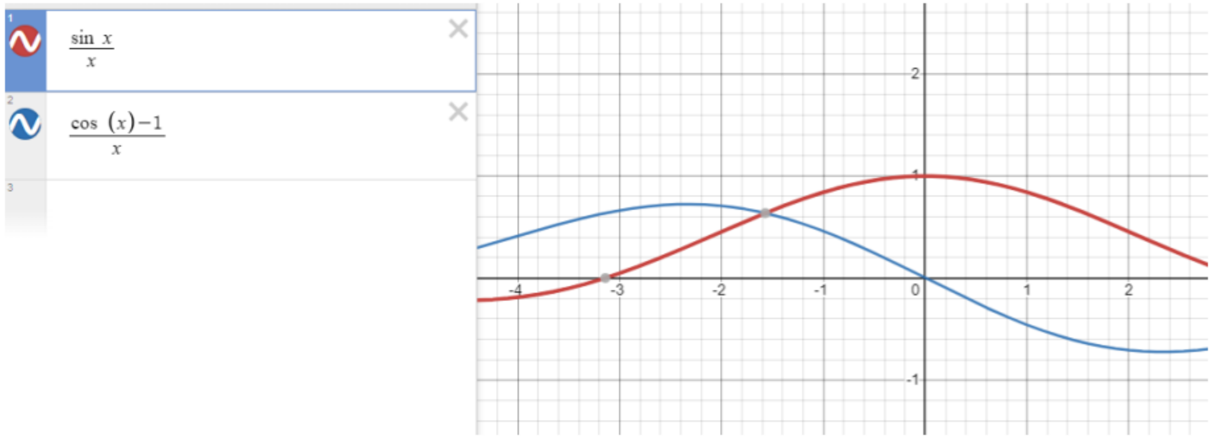
We'll use the limit definition of the derivative and replace it with $\cos x$

$$\frac{d}{dx}[\cos x] =$$

Next, we'll use trigonometric identity (sum-angle identity for cosine)

Proving $\frac{d}{dx}[\cos x] = -\sin x$ Using the Limit Definition of Derivative

$$\frac{d}{dx}[\cos x] =$$



Example: Find the derivative of each of the following.

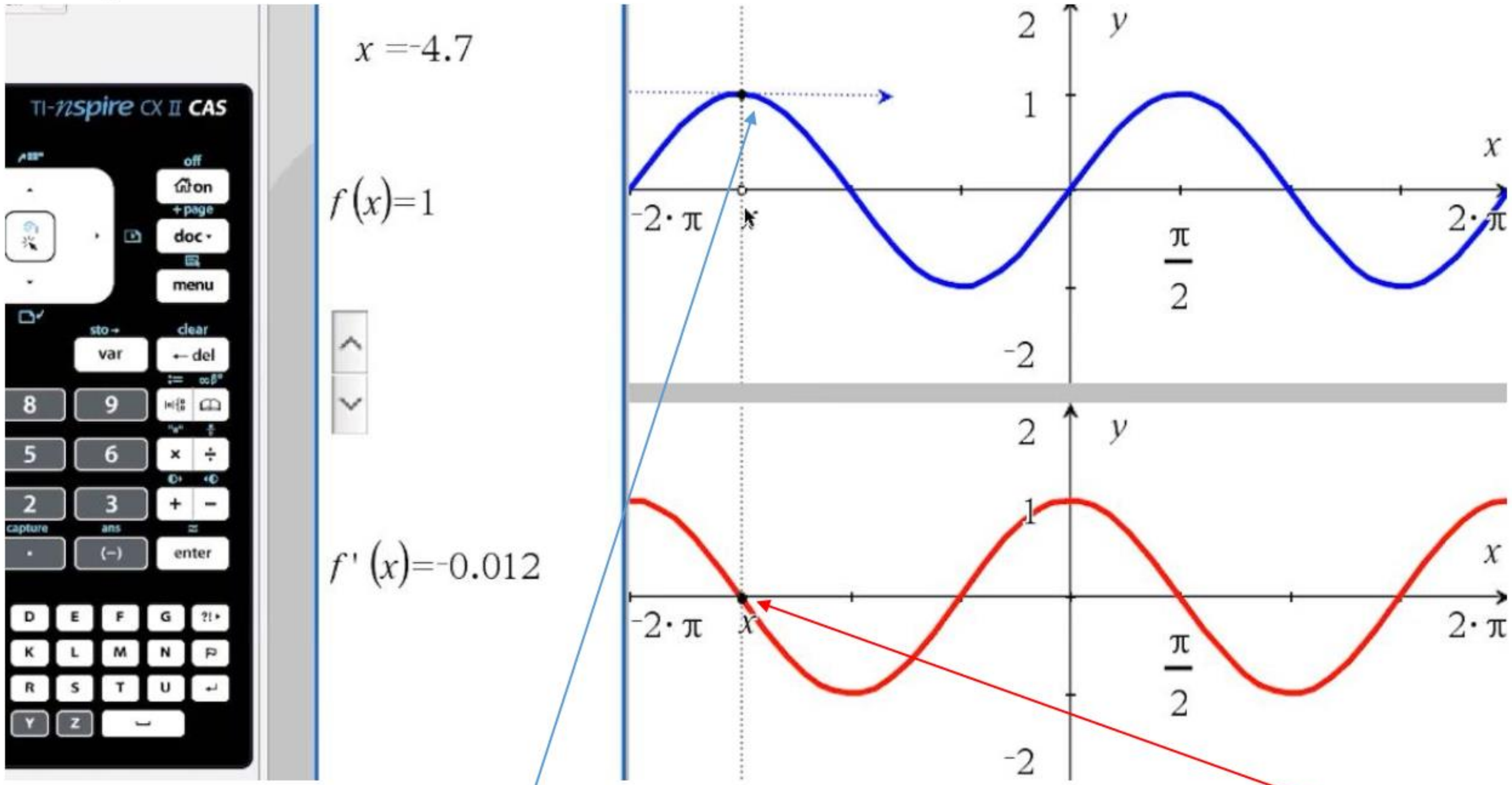
a) $f(x) = 5e^x$

b) $g(x) = e^x - 4x$

c) $h(x) = 7e^x + \sin x$

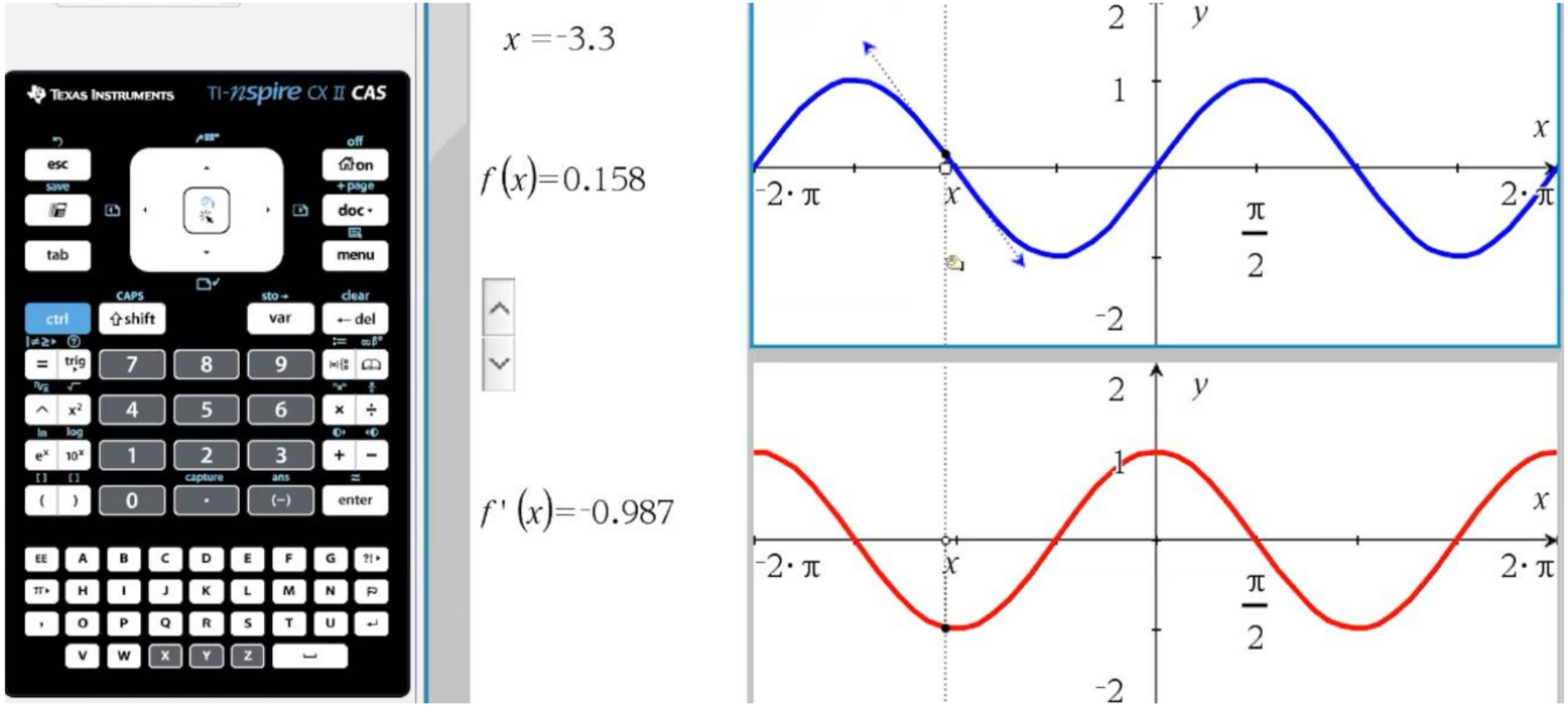
d) $j(x) = 3\ln x - \cos x$

Why the derivative of $\sin x = \cos x$



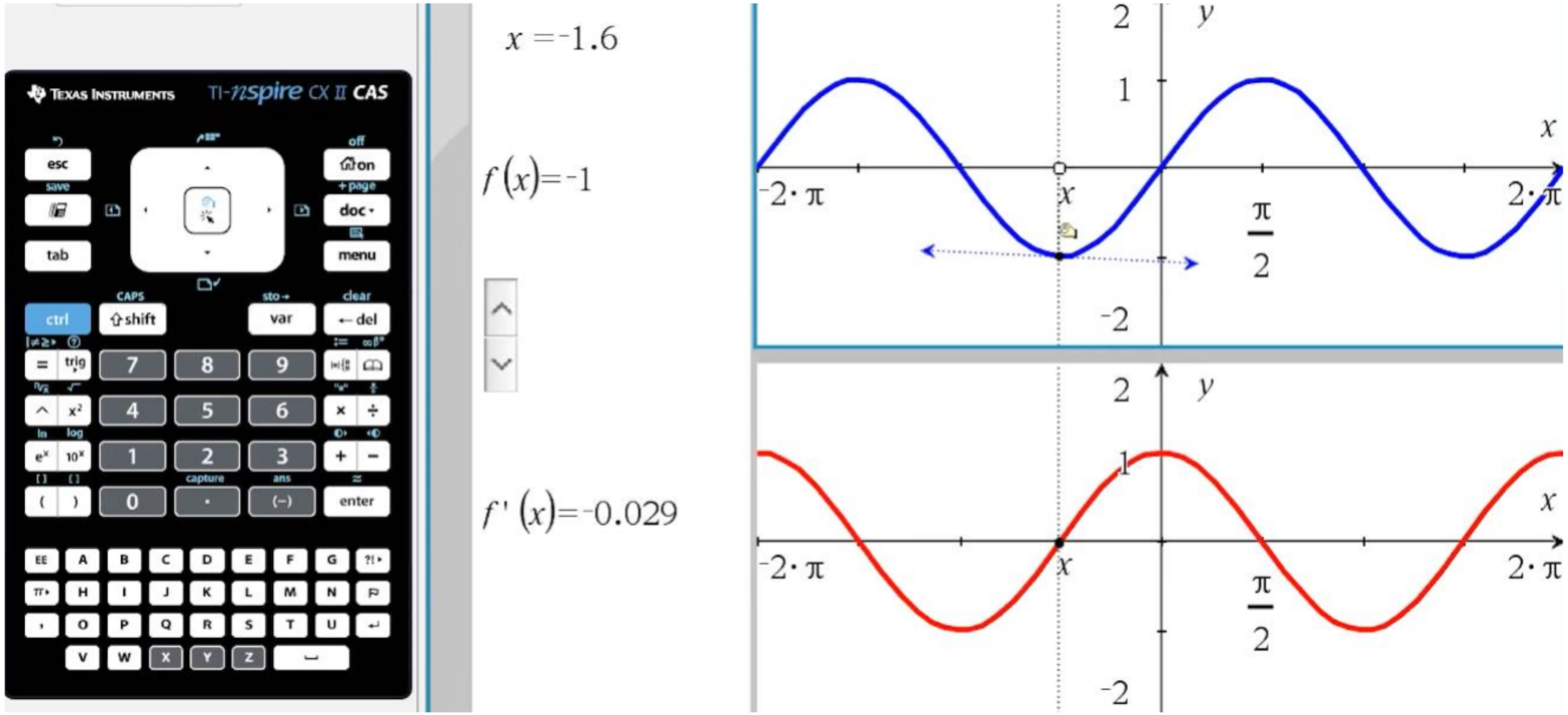
- Looking at the tangent drawn to $\boldsymbol{sin\ x}$ (blue), at the current position, the slope of the tangent line is 0.
- For $\boldsymbol{cos\ x}$ (in red) at the same $\boldsymbol{x - position}$, we have a y-value of 0 because the value of the derivative is 0.

Why the derivative of $\sin x = \cos x$



- ➢ As we slide, the slope of the tangent line is negative, so that indicates that the value of the derivative is negative.
- ➢ So the graph of the derivative should be negative (part of the graph below the x-axis)

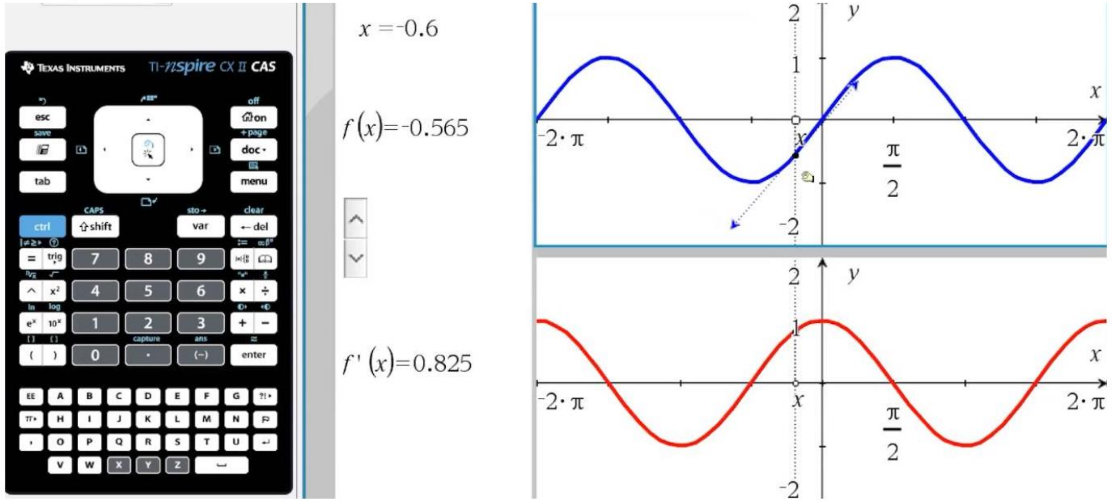
Why the derivative of $\sin x = \cos x$

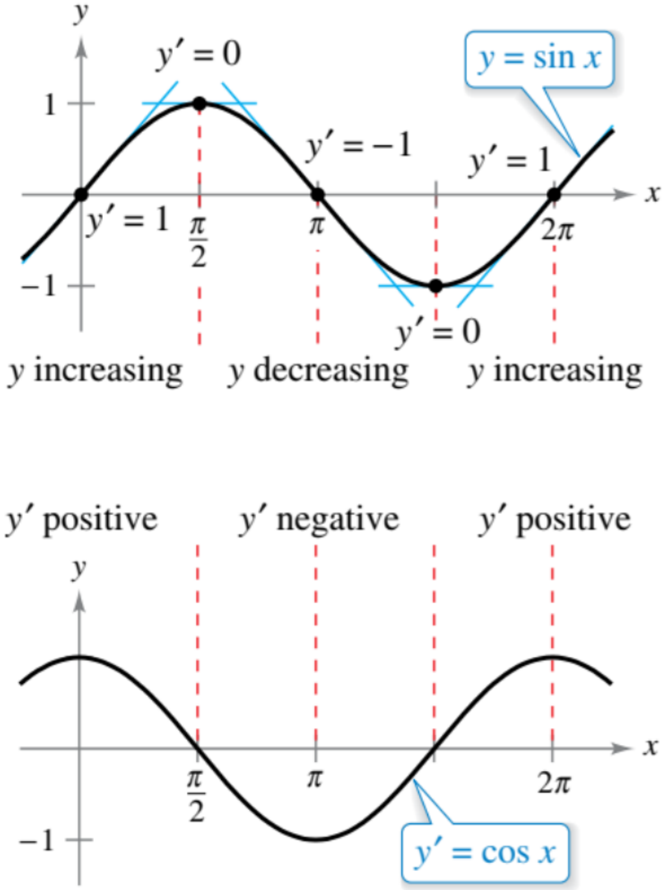


- ➢As we slide, when we get to the point where it has a horizontal tangent line, we get back to the y-value of 0 on the cosine curve.

Why the derivative of $\sin x = \cos x$

Why the derivative of $\sin x = \cos x$





The derivative of the sine function is the cosine function.

- As we slide, when we get to a positive slope, it means the graph is going to lie above the x-axis until we get to another horizontal tangent.

Key Takeaways

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\frac{d}{dx}[\cos x] = -\sin x$$

$$\frac{d}{dx}\left[e^{x}\right] = e^{x}$$

$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$

These four functions are the four most common transcendental functions that you will encounter in calculus!

Basic

1. Find $\frac{d}{dx}(\sin x)$
2. Find $\frac{d}{dx}(\cos x)$
3. Find $\frac{d}{dx}(4\sin x)$
4. Find $\frac{d}{dx}(7\cos x)$
5. Find $\frac{d}{dx}(3\sin x - 5\cos x)$

Basic

1. $\cos x$
2. $-\sin x$
3. $4\cos x$
4. $-7\sin x$
5. $3\cos x + 5\sin x$

Derivative of an exponential function

Let $f(x) = e^x$

Then by definition

$$f^1(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

The derivative of ln *x*

ave

Basic derivatives

1. Find $\frac{d}{dx}(e^x)$
2. Find $\frac{d}{dx}(\ln x)$
3. Find $\frac{d}{dx}(5e^x)$
4. Find $\frac{d}{dx}(3\ln x)$
5. Find $\frac{d}{dx}(-2e^x)$

Basic derivatives

1. $\frac{d}{dx}(e^x) = e^x$
2. $\frac{d}{dx}(\ln x) = \frac{1}{x}$
3. $\frac{d}{dx}(5e^x) = 5e^x$
4. $\frac{d}{dx}(3\ln x) = \frac{3}{x}$
5. $\frac{d}{dx}(-2e^x) = -2e^x$

Simple power/exponential/log combinations

6. Find $\frac{d}{dx}(e^x + 4)$
7. Find $\frac{d}{dx}(\ln x - 7)$
8. Find $\frac{d}{dx}(2e^x + 3)$
9. Find $\frac{d}{dx}(5\ln x + 1)$
10. Find $\frac{d}{dx}(e^x - \ln x)$

Simple combinations

6. $\frac{d}{dx}(e^x + 4) = e^x$
7. $\frac{d}{dx}(\ln x - 7) = \frac{1}{x}$
8. $\frac{d}{dx}(2e^x + 3) = 2e^x$
9. $\frac{d}{dx}(5\ln x + 1) = \frac{5}{x}$
10. $\frac{d}{dx}(e^x - \ln x) = e^x - \frac{1}{x}$

TRUE or FALSE

$\frac{d}{dx}(\ln x)$ is undefined at $x = 0$.

TRUE