F1

Annie Bai	Franklin Chen	Charlie Cheng	Zoey Cheng	Harry Cui	King Deng	Aisling Fu	Ken He	Ivy Jing
Lee Li	Ziheng Li	Lewis Liu	Sam Liu	Ava Peng	Peter Shen	Sylvia Song	Coco Wang	Eileen Wang
Jason Wang	Luna Wang	Cathy Yan	Melody You	Allen Zhang	Lucas Zhang	Lynette Zhang	Ryder Zhang	Flora Zhou
Cyntina Zuo								

F2

Angelina Wang	Azura Cheng	Bonnie Zhao	Cecilia Jia	Diana Li	Oscar Liu	Zavier Liu	Tom Ou	Mia Peng
Joshua Hui	Julius Lv	Justin Jia	Lucas Li	Lyra Zhang	Iris Shao	Dylan Suo	PeytonAson Wang	Struck Wang
Selene Hou	Serena Feng	Silas Lv	Simon Wang	Stella Sun	Yola Wang	Mark Xiao	Elsa Ye	Emily Zhang
Zinnia Dong	Flora Zhang							

F3

Allen Peng	Aurora Yuan	Brittney Wei	Cynthia Liu	Erya Hu	Eva Gai	Felicity Pan	Fiona Ding	Freya Fan
Gordon Yao	Helios Luo	Honey Ruan	Iris Xie	Jasper Wu	Kaven Zhang	Kevin Gao	Leo Yang	Linger Li
Lydia Wei	Maggie Gao	Micheal Zhao	Ray Meng	Rose Jiang	Ross Ma	Roy Liu	Ryan Wang	Sky Bai
Star Su	Stella Xi							

F4

Aris Cheng	Tracy Dang	Jason Du	Alanna Fan	Iris Gao	Eason Gong	Vardy Hai	Edmund He	Liora Li
Miranda Li	Leo Liu	Francis Lv	Dorothy Qian	Dary Song	Jerry Tu	Belinda Wang	Winnie Wang	Elena Wei
James Xu	Mike Yan	Simon Yang	Coco Zhang	Patrick Zhang	Sophia Zhang	Molly Zheng	Carrie Zhou	Rudy Zhu

F5

Abbie Dong	Alan Chen	Angela An	Annette Zhao	Asher Tian	Jove Bai	Camellia Long	Candice Li	Carmen Du
Ricardo Lian	Emma Zhang	Ethan Yu	Ethen Li	Jack Spark Men	Jarry Wang	Cressen Liang	Leo Liu	Lucas Bai
Lycia Liu	Nina Dang	August Mao	Robin Mi	Vicky Yang	Victor Li	Yolanda Wang	Zephyra Hu	

F6

Lucas Chang	Chloe Fang	Bruce Gao	Wyatt Hou	Jackson Jiang	Ivan Lei	Eric Li	Keira Li	Vivian Li
Jeremy Lin	Orange Liu	Serika Ren	Yumiko Shi	Hannah Si	Eric Song	Eric Tan	Silas Wang	Niki Wei
Kiki Wen	Miyu Wu	Tia Wu	Kumi Yuan	Estelle Zhang	Soren Zhang	Viakey Zhang	Jack Zhao	

Turn and Talk

Function:

$$f(x) = \frac{\sqrt{x+1}}{\sqrt{x^2 - 5x + 6}}$$

Domain restrictions:

Turn and Talk

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$$f(x) = \frac{\sqrt{x+1}}{\sqrt{x^2 - 5x + 6}}$$

Domain restrictions:

$$\boxed{[-1, 2) \cup (3, \infty)}$$

Basic Classes of Functions

- function — 函数
- constant function — 常数函数
- linear function — 线性函数
- quadratic function — 二次函数
- cubic function — 三次函数
- polynomial function — 多项式函数
- rational function — 有理函数
- absolute value function — 绝对值函数
- square root function — 平方根函数
- exponential function — 指数函数
- logarithmic function — 对数函数
- sine function — 正弦函数
- cosine function — 余弦函数
- tangent function — 正切函数
- piecewise function — 分段函数

Reminders:
Q2 Gradebook has now closed
Posters and Notes will contribute to Q3 gradebook
Desmos (A3 or A4) or any other Mathematics Posters sent to me by Friday 6th February

SLOPE

DEFINITION

Consider line L passing through points (x_1, y_1) and (x_2, y_2). Let $\Delta y = y_2 - y_1$ and $\Delta x = x_2 - x_1$ denote the changes in y and x, respectively. The **slope** of the line is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}. \tag{1.3}$$

https://openstax.org/books/calculus-volume-1/pages/1-1-review-of-functions

DEFINITION

Consider a line passing through the point (x_1, y_1) with slope m. The equation

$$y - y_1 = m(x - x_1)$$

is the **point-slope equation** for that line.

Consider a line with slope m and y-intercept $(0, b)$. The equation

$$y = mx + b$$

is an equation for that line in **slope-intercept form.**

The **standard form of a line** is given by the equation

$$ax + by = c,$$

where a and b are not both zero. This form is more general because it allows for a vertical line, $x = k$.

Finding the Slope and Equations of Lines

Consider the line passing through the points $(11, -4)$ and $(-4, 5)$, as shown in Figure 1.17.

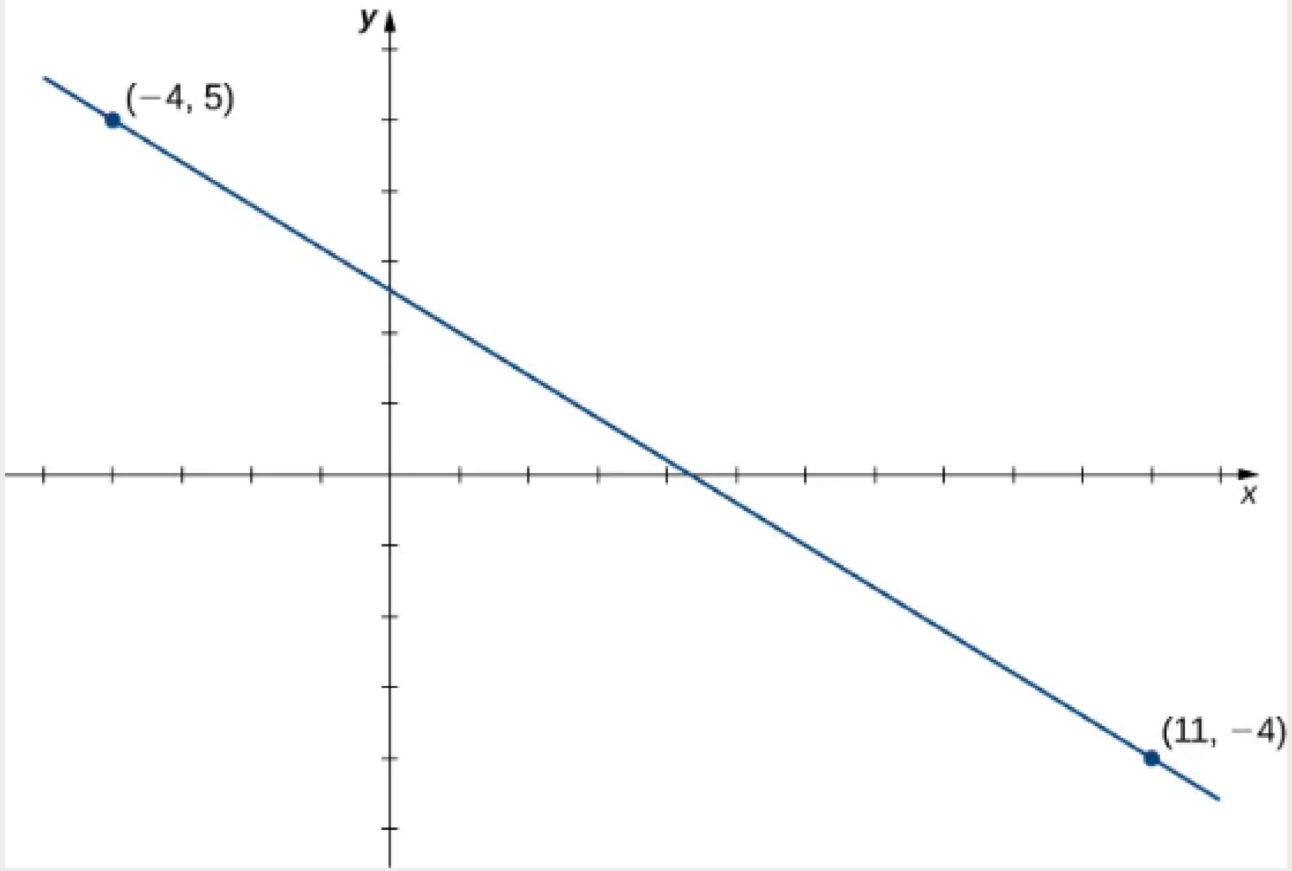

Figure 1.17 Finding the equation of a linear function with a graph that is a line between two given points.

a. Find the slope of the line.
b. Find an equation for this linear function in point-slope form.
c. Find an equation for this linear function in slope-intercept form.

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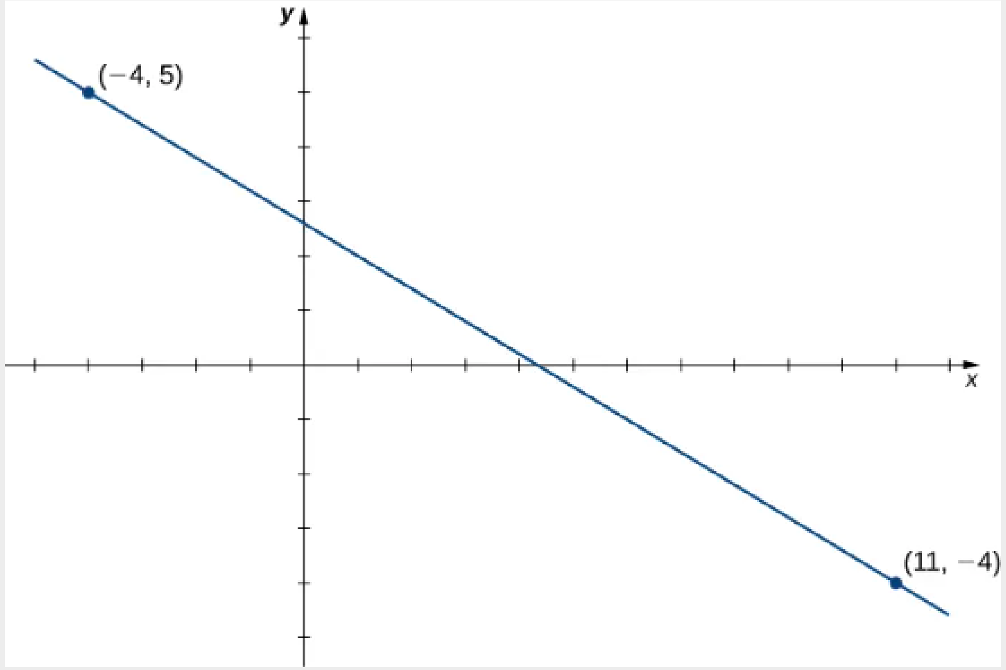

Figure 1.17 Finding the equation of a linear function with a graph that is a line between two given points.

a. Find the slope of the line.
b. Find an equation for this linear function in point-slope form.
c. Find an equation for this linear function in slope-intercept form.

Solution

a. The slope of the line is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-4)}{-4 - 11} = -\frac{9}{15} = -\frac{3}{5}.$$

b. To find an equation for the linear function in point-slope form, use the slope $m = -3/5$ and choose any point on the line. If we choose the point $(11, -4)$, we get the equation

$$f(x) + 4 = -\frac{3}{5}(x - 11).$$

c. To find an equation for the linear function in slope-intercept form, solve the equation in part b. for $f(x)$. When we do this, we get the equation

$$f(x) = -\frac{3}{5}x + \frac{13}{5}.$$

Polynomials

RULE: THE QUADRATIC FORMULA

Consider the quadratic equation

$$ax^2 + bx + c = 0,$$

where $a \neq 0$. The solutions of this equation are given by the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \tag{1.8}$$

If the discriminant $b^2 - 4ac > 0$, this formula tells us there are two real numbers that satisfy the quadratic equation. If $b^2 - 4ac = 0$, this formula tells us there is only one solution, and it is a real number. If $b^2 - 4ac < 0$, no real numbers satisfy the quadratic equation.

Graphing Polynomial Functions

For the following functions a. and b., i. describe the behavior of $f(x)$ as $x \to \pm\infty$, ii. find all zeros of f, and iii. sketch a graph of f.

a. $f(x) = -2x^2 + 4x - 1$
b. $f(x) = x^3 - 3x^2 - 4x$

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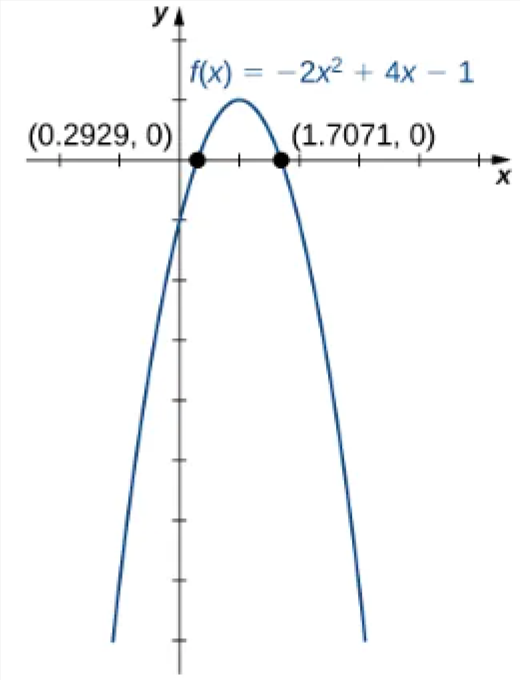

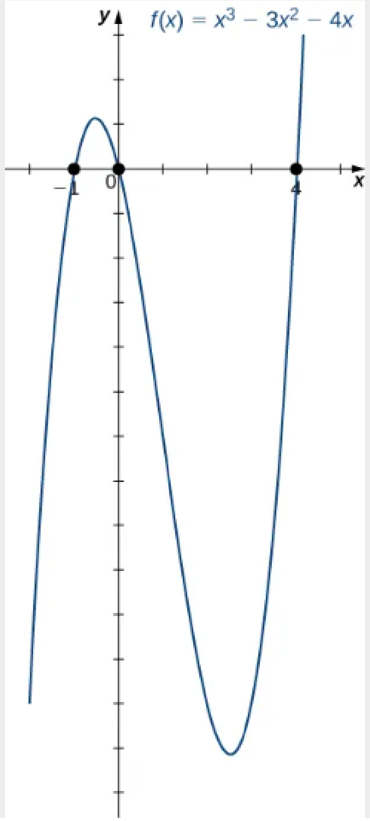

Mathematical Models

Maximizing Revenue

A company is interested in predicting the amount of revenue it will receive depending on the price it charges for a particular item. Using the data from Table 1.6, the company arrives at the following quadratic function to model revenue R (in thousands of dollars) as a function of price per item p:

$$R(p) = p \cdot (-1.04p + 26) = -1.04p^2 + 26p$$

for $0 \leq p \leq 25$.

a. Predict the revenue if the company sells the item at a price of $p = \$5$ and $p = \$17$.
b. Find the zeros of this function and interpret the meaning of the zeros.
c. Sketch a graph of R.
d. Use the graph to determine the value of p that maximizes revenue. Find the maximum revenue.

Mathematical Models

Maximizing Revenue

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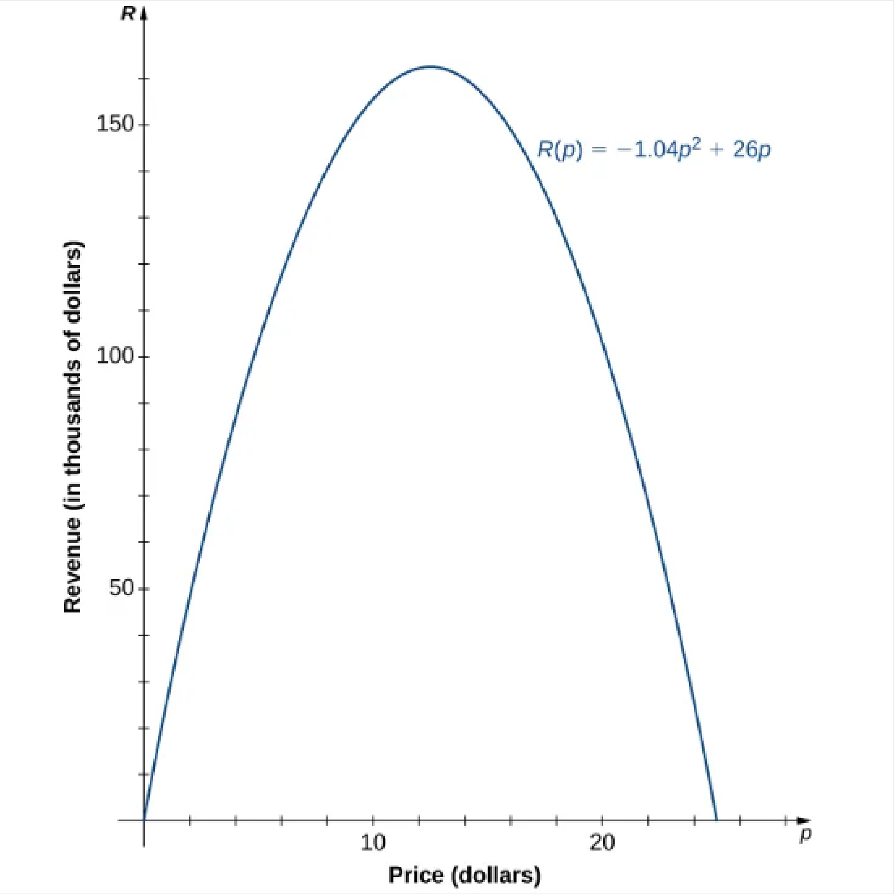
quadratic function to model revenue R (in thousands of dollar

$$R(p) = p \cdot (-1.04p + 26) = -1$$

for $0 \leq p \leq 25$.

a. Predict the revenue if the company sells the item at a pri
b. Find the zeros of this function and interpret the meaning
c. Sketch a graph of R.
d. Use the graph to determine the value of p that maximize



d. The function is a parabola with zeros at $p = 0$ and $p = 25$, and it is symmetric about the line $p = 12.5$, so the maximum revenue occurs at a price of $p = \$12.50$ per item. At that price, the revenue is $R(p) = -1.04(12.5)^2 + 26(12.5) = \$162,500$.

Algebraic Functions

algebraic function is one that involves addition, subtraction, multiplication, division, rational powers, and roots. Two types of algebraic functions are rational functions and root functions.

Finding Domain and Range for Algebraic Functions

For each of the following functions, find the domain and range.

a. $f(x) = \frac{3x-1}{5x+2}$

b. $f(x) = \sqrt{4-x^2}$

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a. $f(x) = \frac{3x-1}{5x+2}$
b. $f(x) = \sqrt{4 - x^2}$

$$x > 2 \text{ and } x < -2$$

There is no value of x that can satisfy both inequalities.

The domain of the function is $\{x| -2 \leq x \leq 2\}$.

To identify the range, note that if $-2 \leq x \leq 2$, then $0 \leq 4 - x^2 \leq 4$. Therefore, $0 \leq \sqrt{4 - x^2} \leq 2$. The range of f is $\{y|0 \leq y \leq 2\}$.

Transcendental Functions

beyond, algebra. The most common transcendental functions are trigonometric, exponential, and logarithmic functions. A *trigonometric function* relates the ratios of two sides of a right triangle. They are $\sin x, \cos x, \tan x, \cot x, \sec x, \text{and } \csc x$. (We discuss trigonometric functions later in the chapter.) An exponential function is a function of the form $f(x) = b^x$, where the base $b > 0, b \neq 1$. A **logarithmic function** is a function of the form $f(x) = \log_b(x)$ for some constant $b > 0, b \neq 1$, where $\log_b(x) = y$ if and only if $b^y = x$.

Which functions are transcendental?

1. $f(x) = x^3 - 2x + 5$
2. $g(x) = \sqrt{x^2 + 1}$
3. $h(x) = e^x$
4. $p(x) = \ln(x)$
5. $q(x) = \sin x$
6. $r(x) = \dfrac{1}{x^2 - 4}$
7. $s(x) = \cos(2x)$
8. $t(x) = \sqrt[3]{x - 1}$

1. $x^3 - 2x + 5$ — Algebraic
2. $\sqrt{x^2 + 1}$ — Algebraic
3. e^x — Transcendental
4. $\ln(x)$ — Transcendental
5. $\sin x$ — Transcendental
6. $\frac{1}{x^2 - 4}$ — Algebraic
7. $\cos(2x)$ — Transcendental
8. $\sqrt[3]{x - 1}$ — Algebraic

Piecewise-Defined Function

Sometimes a function is defined by different formulas on different parts of its domain. A function with this property is known as a **piecewise-defined function.** The absolute value function is an example of a piecewise-defined function because the formula changes with the sign of x:

$$f(x) = \begin{cases} -x, x < 0 \\ x, x \geq 0 \end{cases}.$$

Graphing a Piecewise-Defined Function

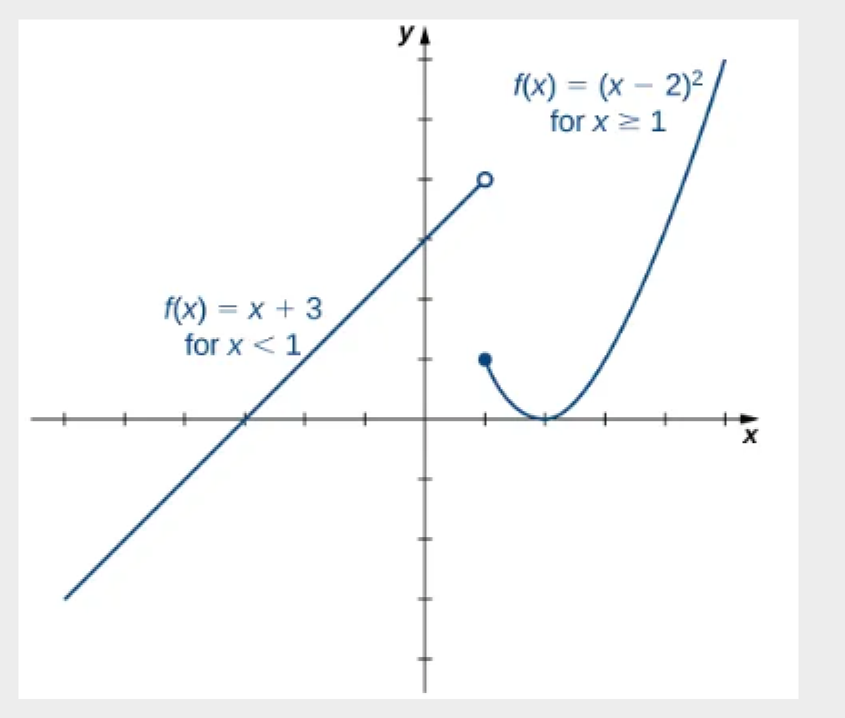
Sketch a graph of the following piecewise-defined function:

$$f(x) = \begin{cases} x+3, & x < 1 \\ (x-2)^2, & x \geq 1 \end{cases}.$$

Graphing a Piecewise-Defined Function

Sketch a graph of the following piecewise-defined function:

$$f(x) = \begin{cases} x+3, & x<1 \\ (x-2)^2, & x\geq 1 \end{cases}.$$



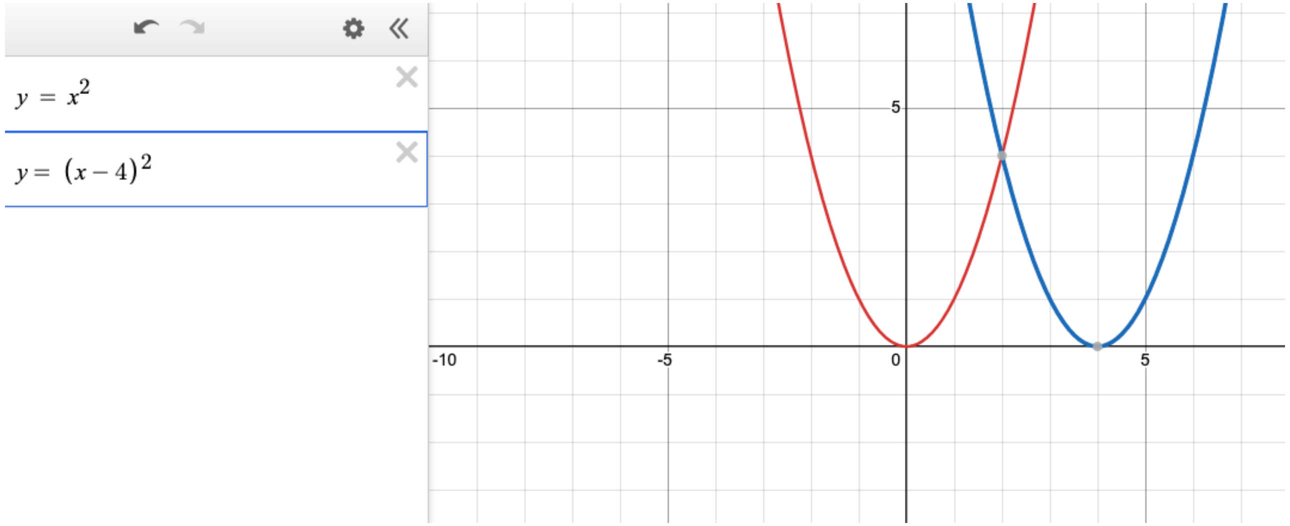
Transformation of Functions

Transformation of $f\ (c>0)$	**Effect on the graph of f**
$f(x)+c$	Vertical shift up c units
$f(x)-c$	Vertical shift down c units
$f(x+c)$	Shift left by c units
$f(x-c)$	Shift right by c units
$cf(x)$	Vertical stretch if $c>1$; vertical compression if $0<c<1$
$f(cx)$	Horizontal stretch if $0<c<1$; horizontal compression if $c>1$
$-f(x)$	Reflection about the x-axis
$f(-x)$	Reflection about the y-axis

True or False: Stretch by scale factor 3 parallel to x axis

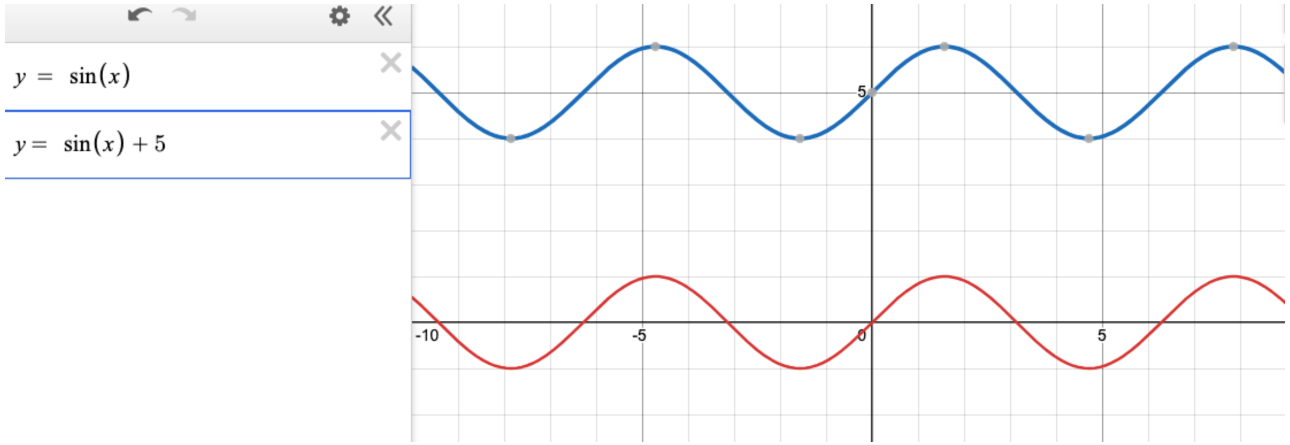
False

True or False: Shift 4 units to the right parallel to x axis



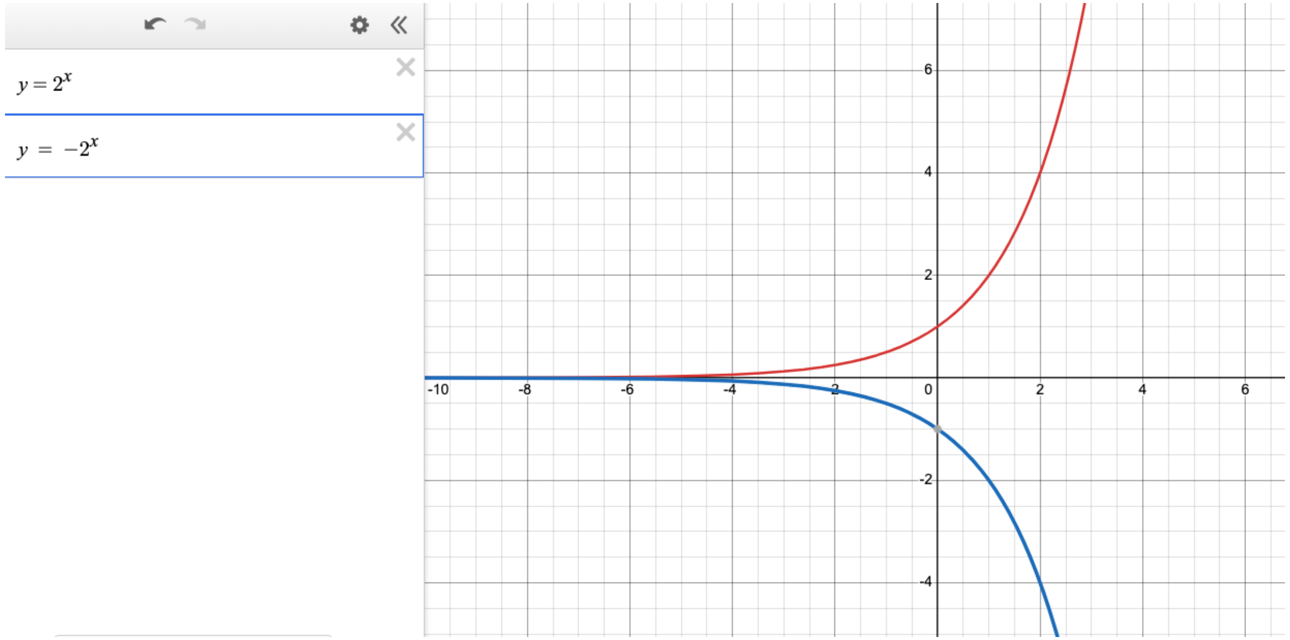
TRUE

True or False: Shift 5 units up parallel to y axis



TRUE

True or False: Reflection in the y axis



False

Transforming a Function

For each of the following functions, a. and b., sketch a graph by using a sequence of transformations of a well-known function.

a. $f(x) = -|x+2| - 3$
b. $f(x) = 3\sqrt{-x} + 1$

Transforming a Function

For each of the following functions, a. and b., sketch a graph by using a sequence of transformations of a well-known function.

a. $f(x) = -|x + 2| - 3$

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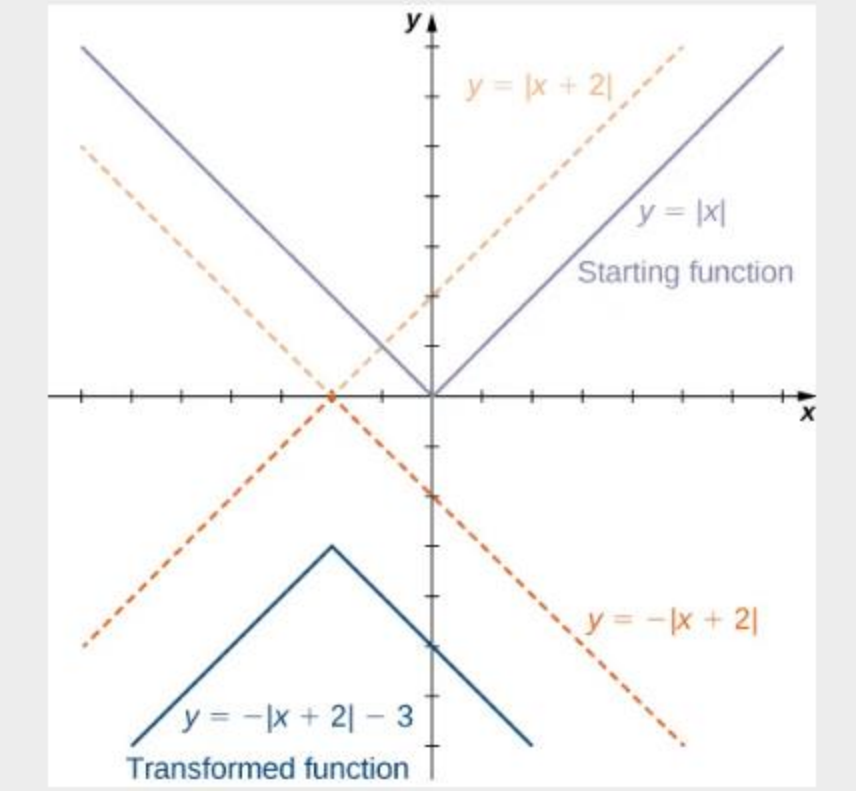
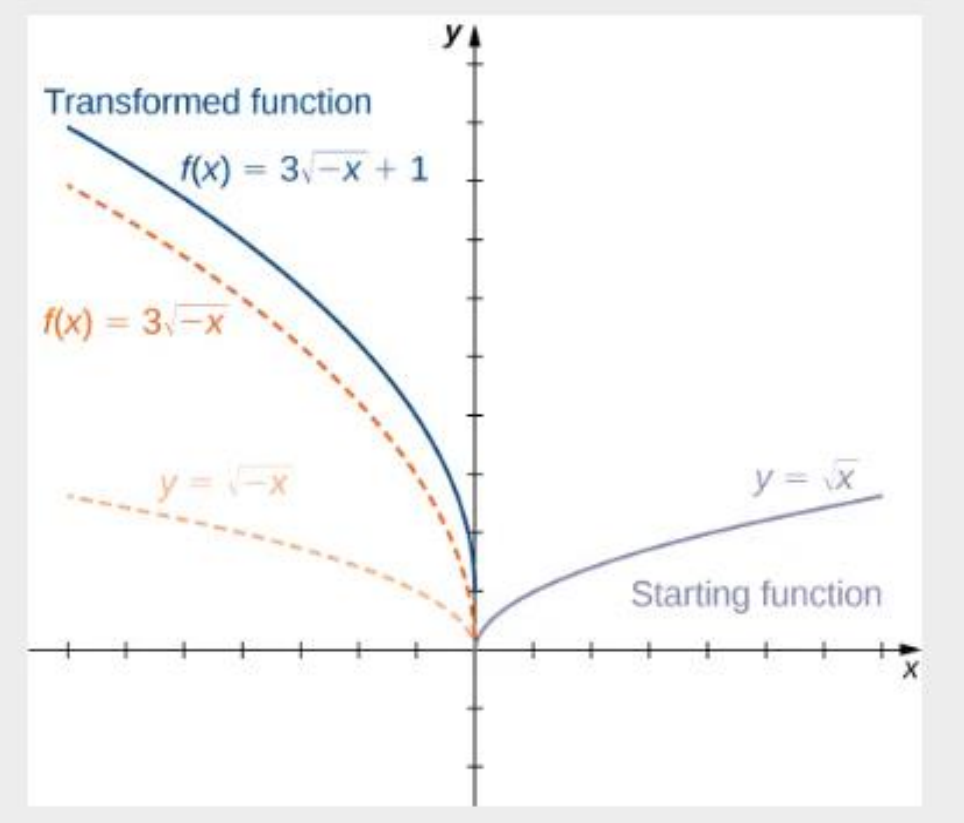
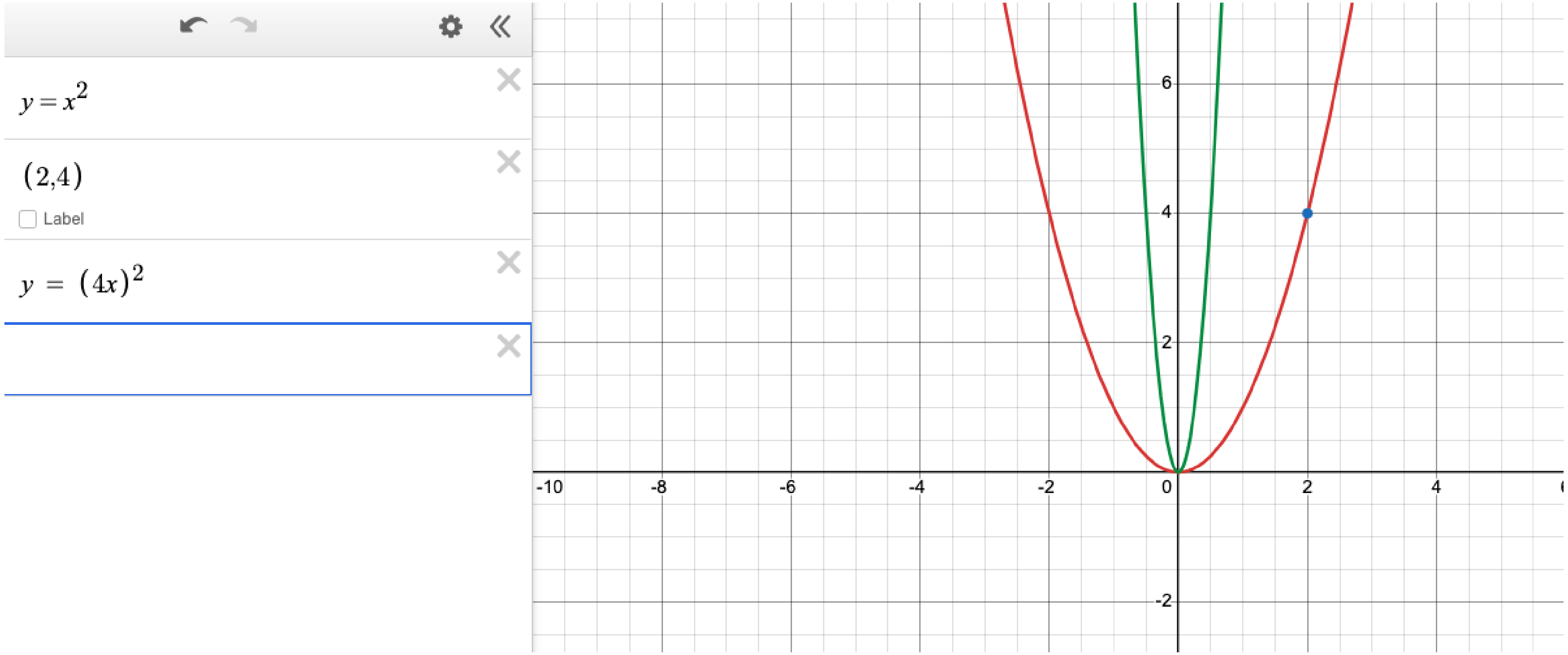




Transfer a point

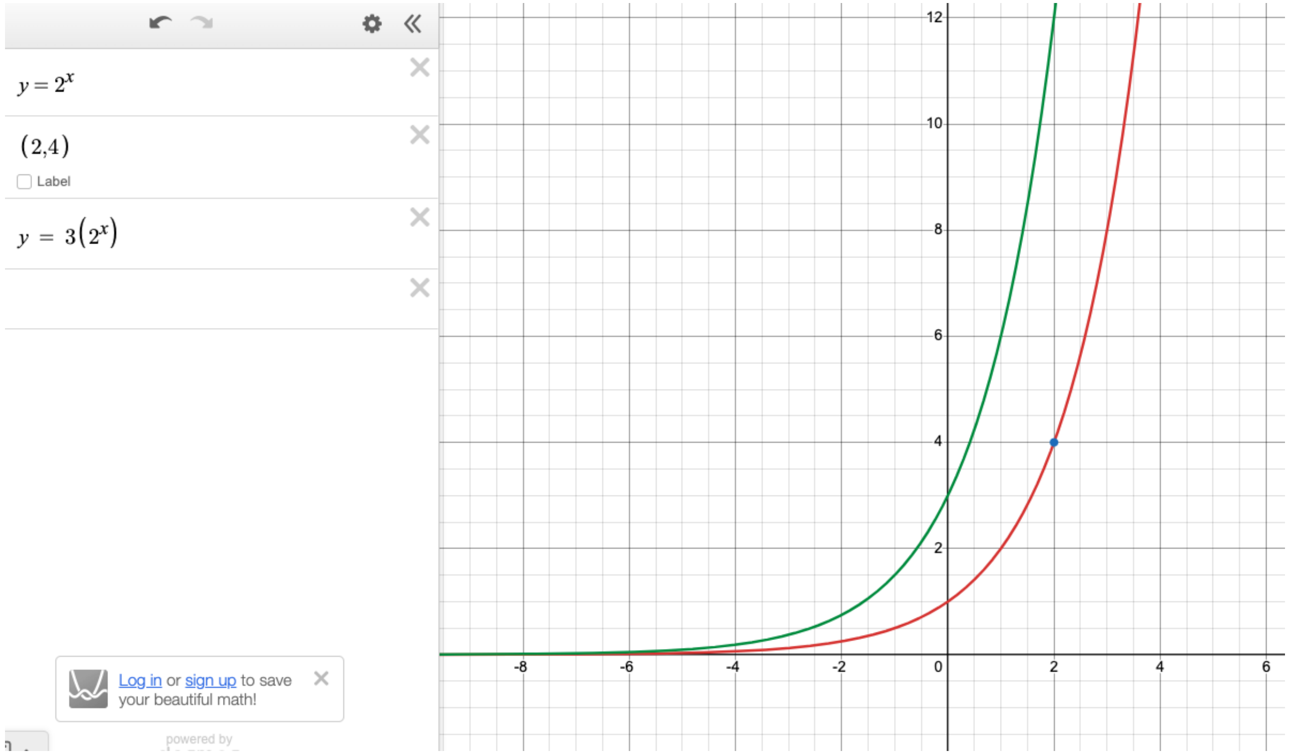
(6,4)

Transfer a point



(0.5,4)

Transfer a point

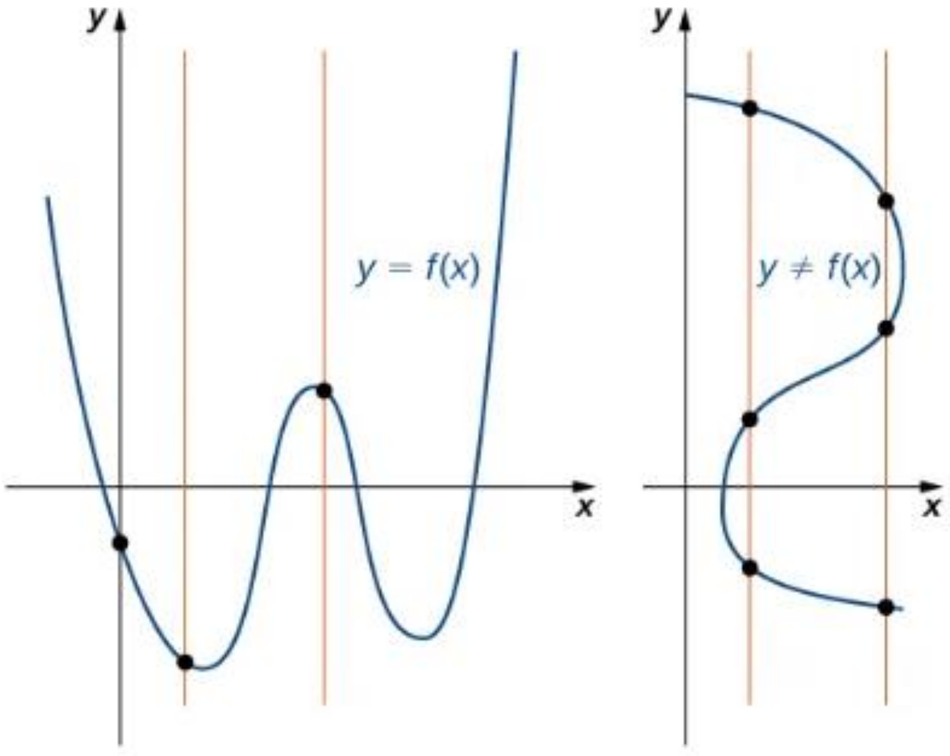


(2,12)

RULE: VERTICAL LINE TEST

Given a function f, every vertical line that may be drawn intersects the graph of f no more than once. If any vertical line intersects a set of points more than once, the set of points does not represent a function.

We can use this test to determine whether a set of plotted points represents the graph of a function (Figure 1.8).



(a) (b)

EXAMPLE 1.3

Finding Zeros and y-Intercepts of a Function

Consider the function $f(x) = -4x + 2$.

a. Find all zeros of f.
b. Find the y-intercept (if any).
c. Sketch a graph of f.

a. To find the zeros, solve $f(x) = -4x + 2 = 0$. We discover that f has one zero at $x = 1/2$.
b. The y-intercept is given by $(0, f(0)) = (0, 2)$.
c. Given that f is a linear function of the form $f(x) = mx + b$ that passes through the points $(1/2, 0)$ and $(0, 2)$, we can sketch the graph of f (Figure 1.9).

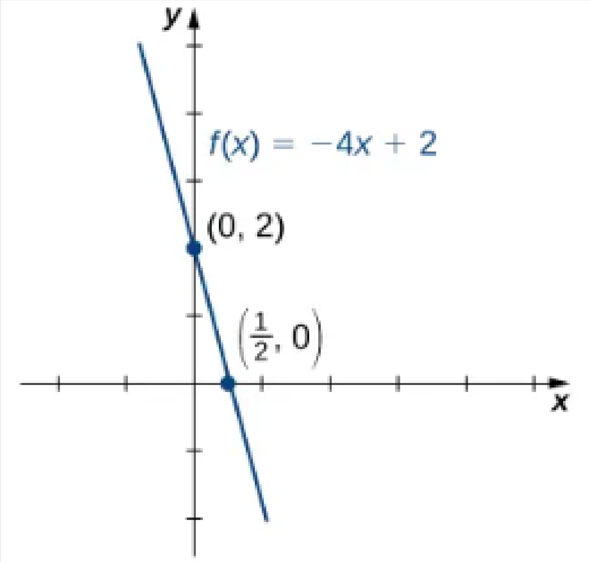


Figure 1.9 The function $f(x) = -4x + 2$ is a line with x-intercept $(1/2, 0)$ and y-intercept $(0, 2)$.