

Turn and Talk

A line passes through the point $(4, -3)$ and is perpendicular to the line

$$3x + 2y = 12.$$

Find the equation of the line.

Turn and Talk

$$3x + 2y = 12 \Rightarrow y = -\frac{3}{2}x + 6$$

$$\text{Slope} = -\frac{3}{2}$$

$$\text{Perpendicular slope} = \frac{2}{3}$$

Using point-slope form:

$$y + 3 = \frac{2}{3}(x - 4)$$

$$y = \frac{2}{3}x - \frac{17}{3}$$

Trigonometric Functions

trigonometric function — 三角函数 (*sān jiǎo hán shù*)

angle — 角 (*jiǎo*)

degree — 度 (*dù*)

radian — 弧度 (*hú dù*)

sine — 正弦 (*zhèng xián*)

cosine — 余弦 (*yú xián*)

tangent — 正切 (*zhèng qiē*)

right triangle — 直角三角形 (*zhí jiǎo sān jiǎo xíng*)

hypotenuse — 斜边 (*xié biān*)

opposite side — 对边 (*duì biān*)

adjacent side — 邻边 (*lín biān*)

unit circle — 单位圆 (*dān wèi yuán*)

period — 周期 (*zhōu qī*)

amplitude — 振幅 (*zhèn fú*)

Radians - we do

$$360 \text{ degrees} = \text{----} \text{ Rad}$$

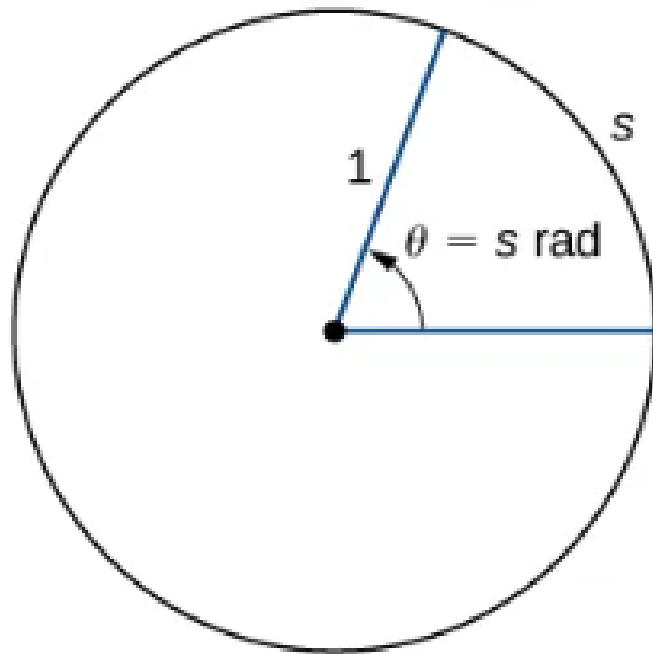
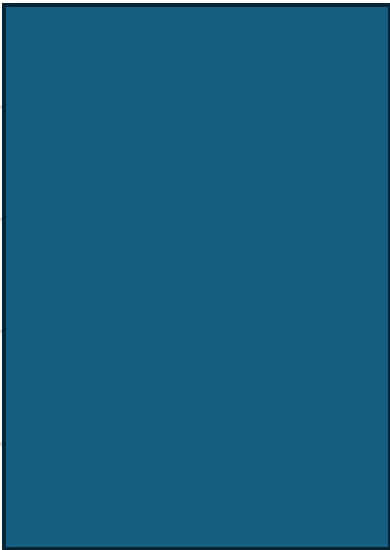
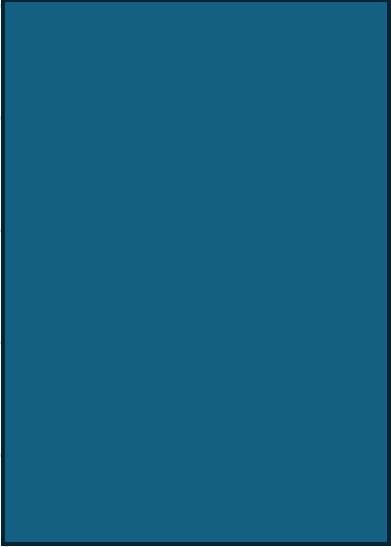


Figure 1.30 The radian measure of an angle θ is the arc length s of the associated arc on the unit circle.

$$\text{---} \text{ degrees} = 1 \text{ Rad}$$

Radians - you do

Degrees	Radians	Degrees	Radians
0		120	
30		135	
45		150	
60		180	
90			

Converting between Radians and Degrees

- a. Express 225° using radians.
- b. Express $5\pi/3$ rad using degrees.

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Solution

Use the fact that 180° is equivalent to π radians as a conversion factor: $1 = \frac{\pi \text{ rad}}{180^\circ} = \frac{180^\circ}{\pi \text{ rad}}.$

$$\text{a. } 225^\circ = 225^\circ \cdot \frac{\pi}{180^\circ} = \frac{5\pi}{4} \text{ rad}$$

$$\text{b. } \frac{5\pi}{3} \text{ rad} = \frac{5\pi}{3} \cdot \frac{180^\circ}{\pi} = 300^\circ$$

Six Basic Trigonometric Functions

DEFINITION

Let $P = (x, y)$ be a point on the unit circle centered at the origin O . Let θ be an angle with an initial side along the positive x -axis and a terminal side given by the line segment OP . The **trigonometric functions** are then defined as

$$\begin{aligned}\sin \theta &= y & \csc \theta &= \frac{1}{y} \\ \cos \theta &= x & \sec \theta &= \frac{1}{x} \\ \tan \theta &= \frac{y}{x} & \cot \theta &= \frac{x}{y}\end{aligned}$$

(1.9)

If $x = 0$, $\sec \theta$ and $\tan \theta$ are undefined. If $y = 0$, then $\cot \theta$ and $\csc \theta$ are undefined.

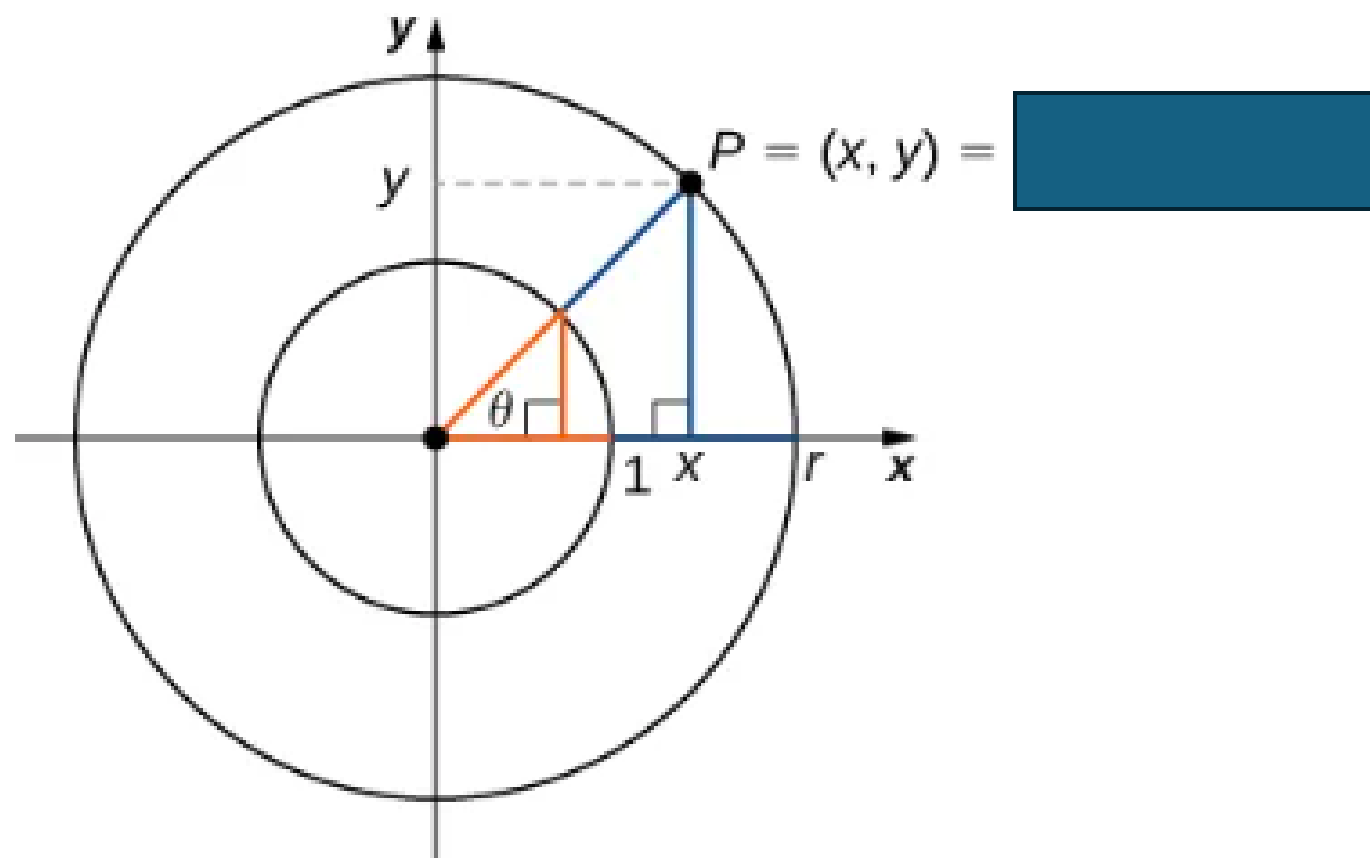
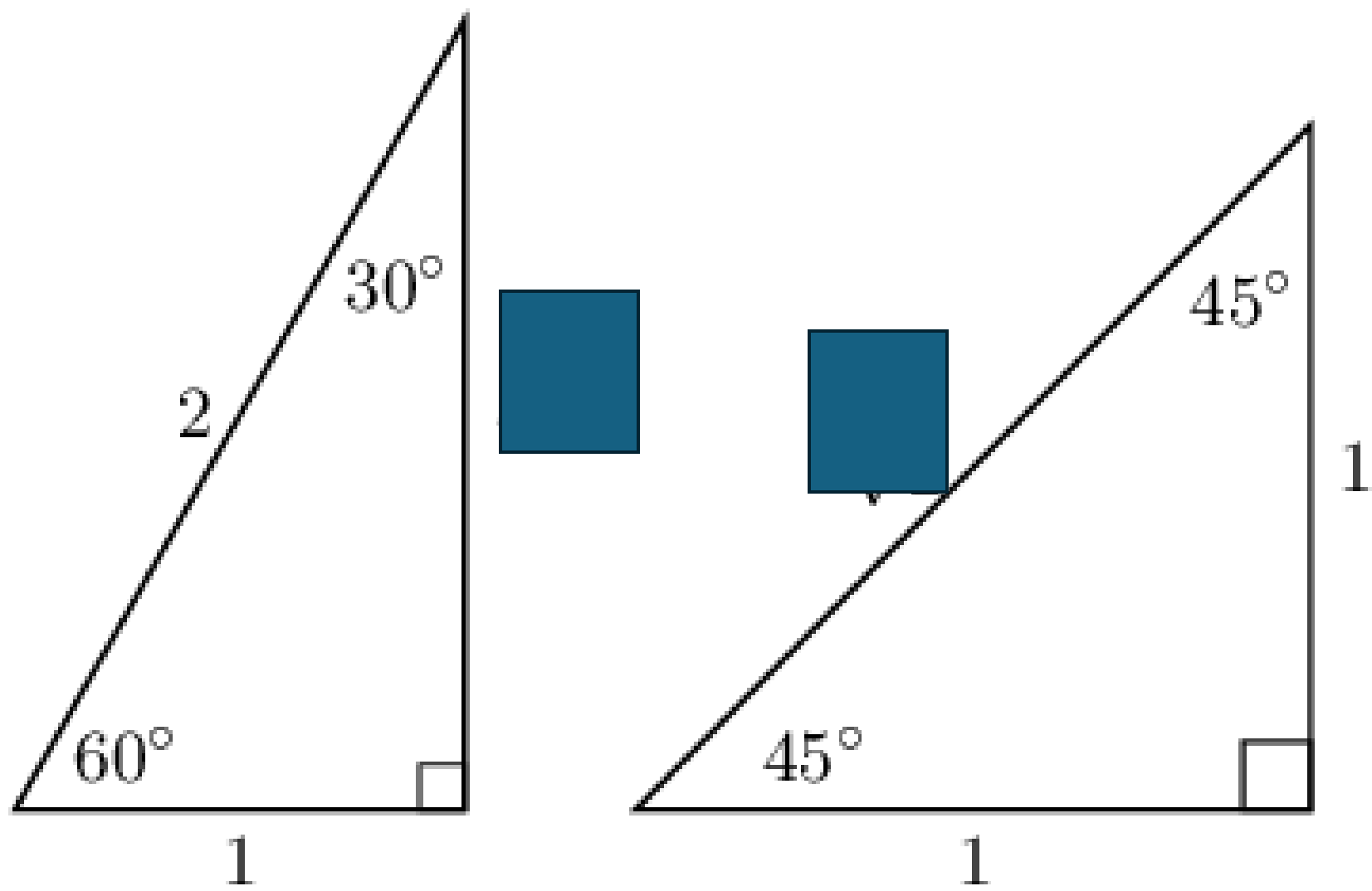


Figure 1.32 For a point $P = (x, y)$ on a circle of radius r , the coordinates x and y satisfy $x = r \cos \theta$ and $y = r \sin \theta$.

SPECIAL ANGLES



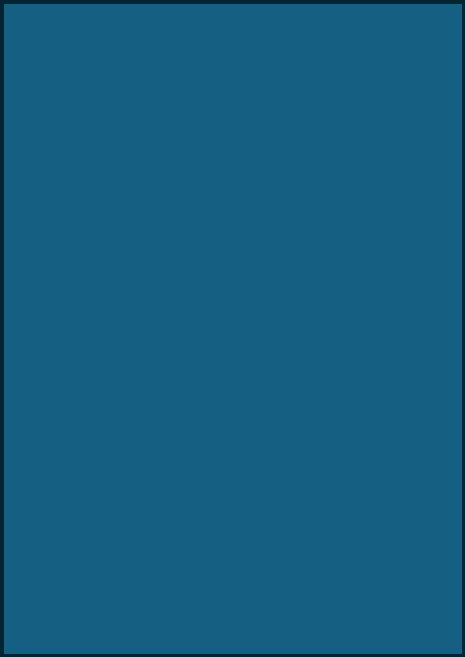
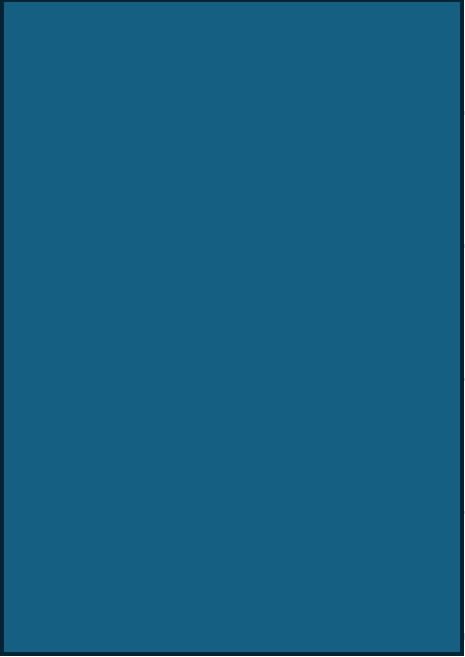
θ	$\sin \theta$	$\cos \theta$
0		
$\frac{\pi}{6}$		
$\frac{\pi}{4}$		
$\frac{\pi}{3}$		
$\frac{\pi}{2}$		

Table 1.9 Values of $\sin \theta$ and $\cos \theta$ at Major Angles θ in the First Quadrant

Evaluating Trigonometric Functions

Evaluate each of the following expressions.

a. $\sin\left(\frac{2\pi}{3}\right)$

b. $\cos\left(-\frac{5\pi}{6}\right)$

c. $\tan\left(\frac{15\pi}{4}\right)$

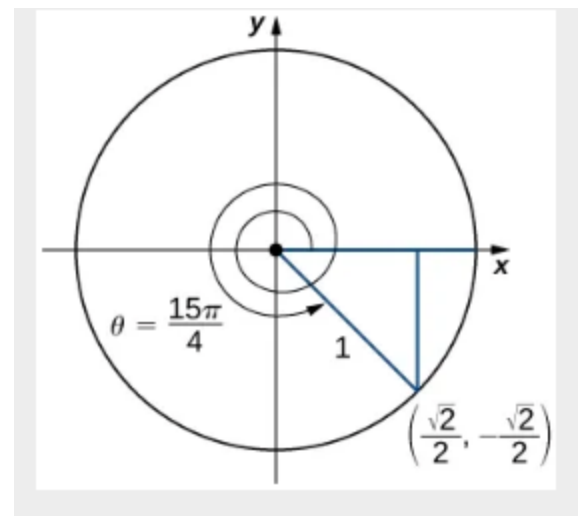
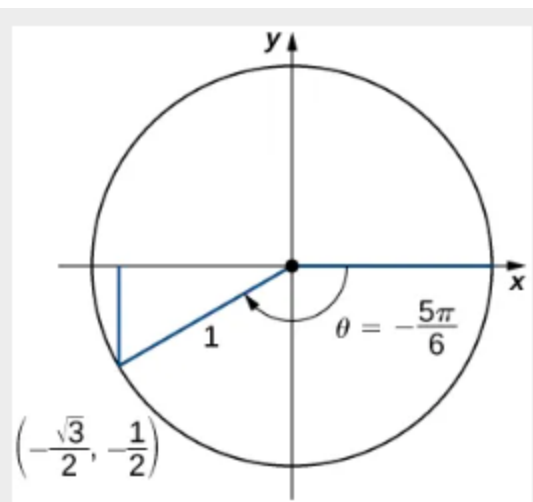
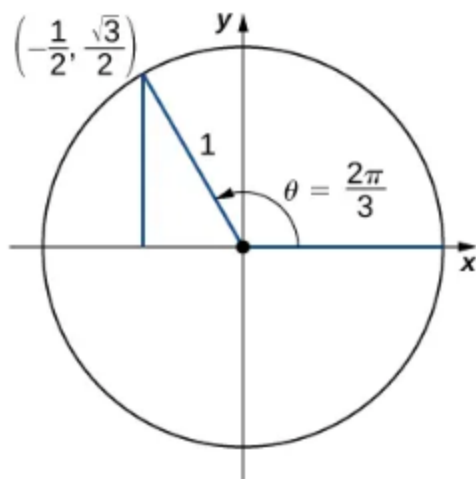
Evaluating Trigonometric Functions

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$$\begin{array}{ll} \sin \theta = \frac{O}{H} & \csc \theta = \frac{H}{O} \\ \cos \theta = \frac{A}{H} & \sec \theta = \frac{H}{A} \\ \tan \theta = \frac{O}{A} & \cot \theta = \frac{A}{O} \end{array}$$

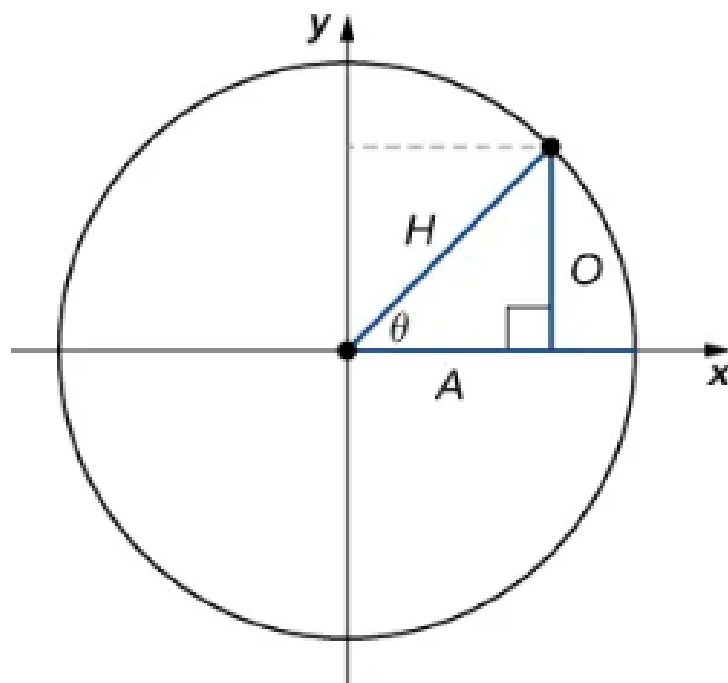


Figure 1.33 By inscribing a right triangle in a circle, we can express the ratios of the side lengths in terms of the trigonometric functions evaluated at θ .

Constructing a Wooden Ramp

A wooden ramp is to be built with one end on the ground and the other end at the top of a short staircase. If the top of the staircase is 4 ft from the ground and the angle between the ground and the ramp is to be 10° , how long does the ramp need to be?

Let x denote the length of the ramp. In the following image, we see that x needs to satisfy the equation $\sin(10^\circ) = 4/x$. Solving this equation for x , we see that $x = 4/\sin(10^\circ) \approx 23.035$ ft.



Trigonometric Identities

Reciprocal identities

$$\begin{aligned}\tan \theta &= \frac{\sin \theta}{\cos \theta} & \cot \theta &= \frac{\cos \theta}{\sin \theta} \\ \csc \theta &= \frac{1}{\sin \theta} & \sec \theta &= \frac{1}{\cos \theta}\end{aligned}$$

Pythagorean identities



A blue rectangular box redacting the Pythagorean identities. To the right of the box, the expression $\sin^2 \theta + \cos^2 \theta = 1$ is partially visible.

Addition and subtraction formulas



A large blue rectangular box redacting the addition and subtraction formulas.

Double-angle formulas



A large blue rectangular box redacting the double-angle formulas.

Solving Trigonometric Equations

For each of the following equations, use a trigonometric identity to find all solutions.

a. $1 + \cos(2\theta) = \cos \theta$

b. $\sin(2\theta) = \tan \theta$

$$\theta = \frac{\pi}{2} + n\pi, \theta = \frac{\pi}{3} + 2n\pi, \text{ and } \theta = -\frac{\pi}{3} + 2n\pi, n = 0, \pm 1, \pm 2, \dots$$

$$\theta = n\pi \quad \text{and} \quad \theta = \frac{\pi}{4} + \frac{n\pi}{2}, n = 0, \pm 1, \pm 2, \dots$$

Prove the trigonometric identity $1 + \tan^2\theta = \sec^2\theta$.

We start with the identity

$$\sin^2\theta + \cos^2\theta = 1.$$

Dividing both sides of this equation by $\cos^2\theta$, we obtain

$$\frac{\sin^2\theta}{\cos^2\theta} + 1 = \frac{1}{\cos^2\theta}.$$

Since $\sin\theta/\cos\theta = \tan\theta$ and $1/\cos\theta = \sec\theta$, we conclude that

$$\tan^2\theta + 1 = \sec^2\theta.$$

Trigonometric Graphs



Angular Speed

Angular speed measures **how fast something rotates**. It tells you the angle covered per unit of time.

- **Formula:**

$$\omega = \frac{\theta}{t}$$

where:

ω = angular speed (radians per second)

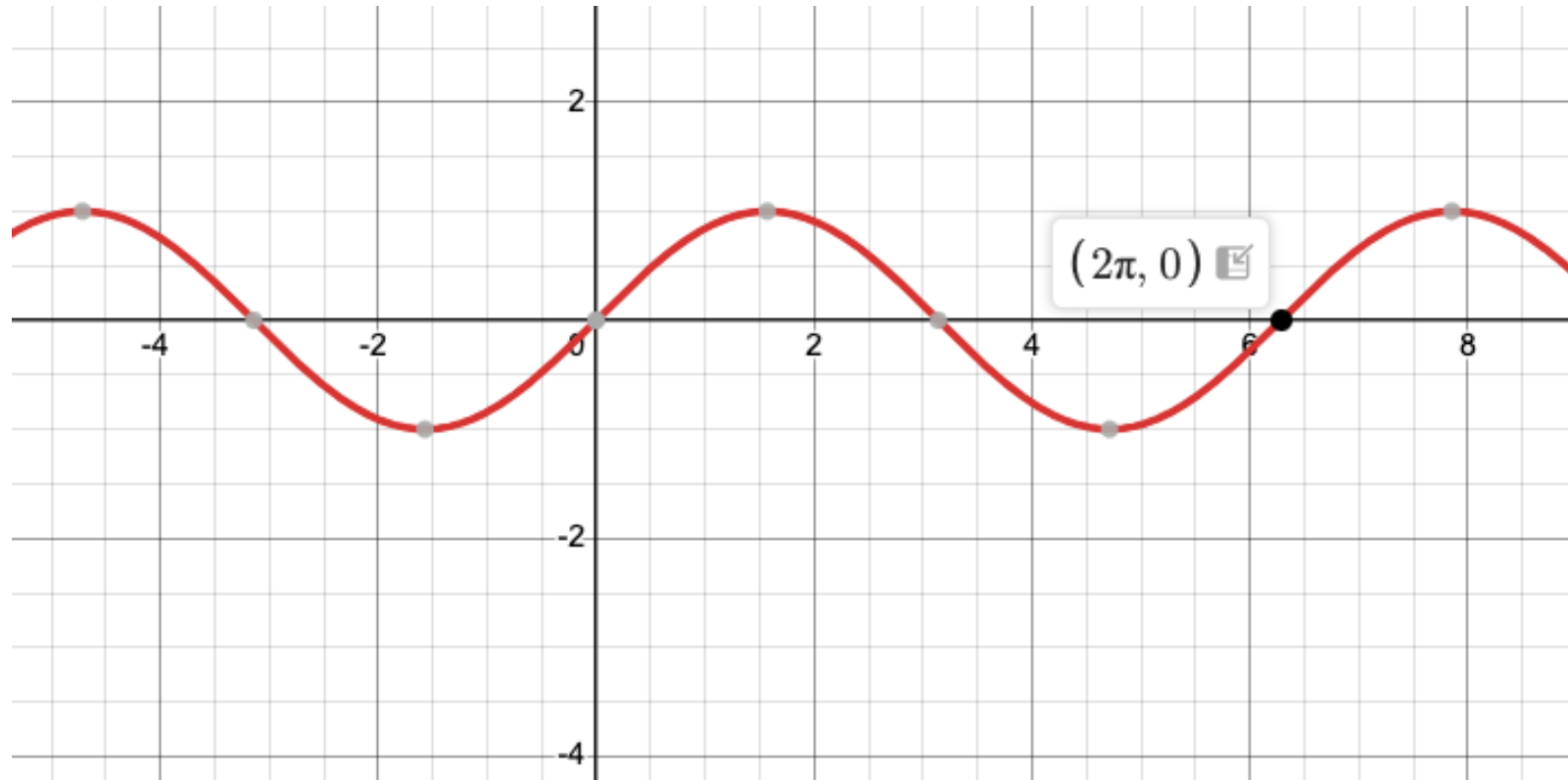
θ = angle rotated (radians)

t = time (seconds)

- If the motion is **circular**, the angular speed is related to **frequency** (f) and **period** (T):

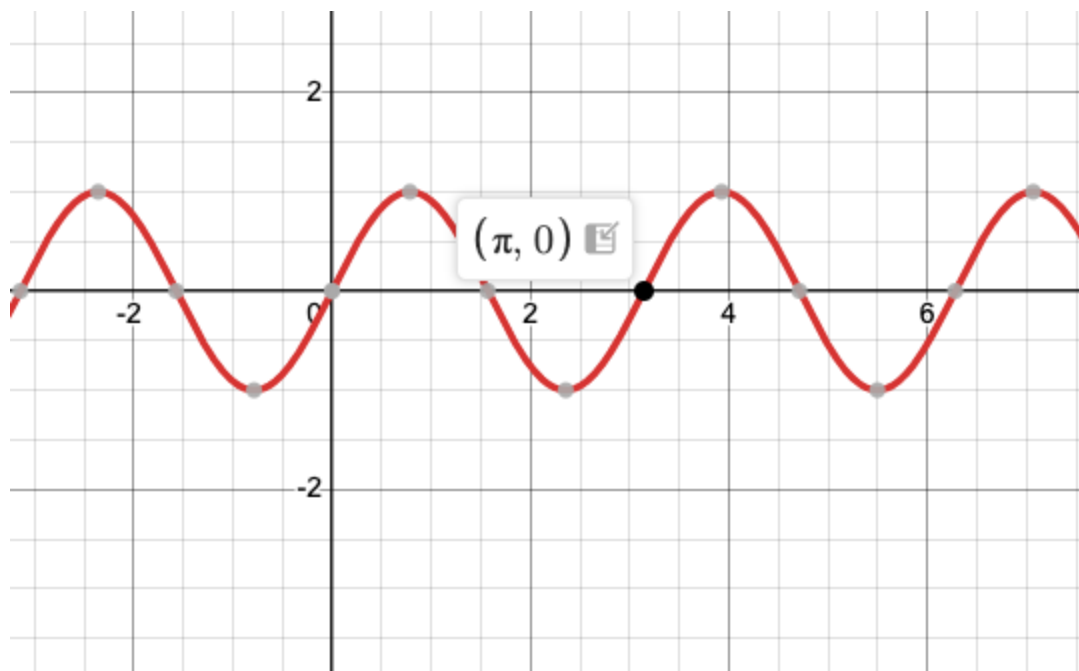
$$\omega = 2\pi f = \frac{2\pi}{T}$$

Sin(t) What is the angular speed?



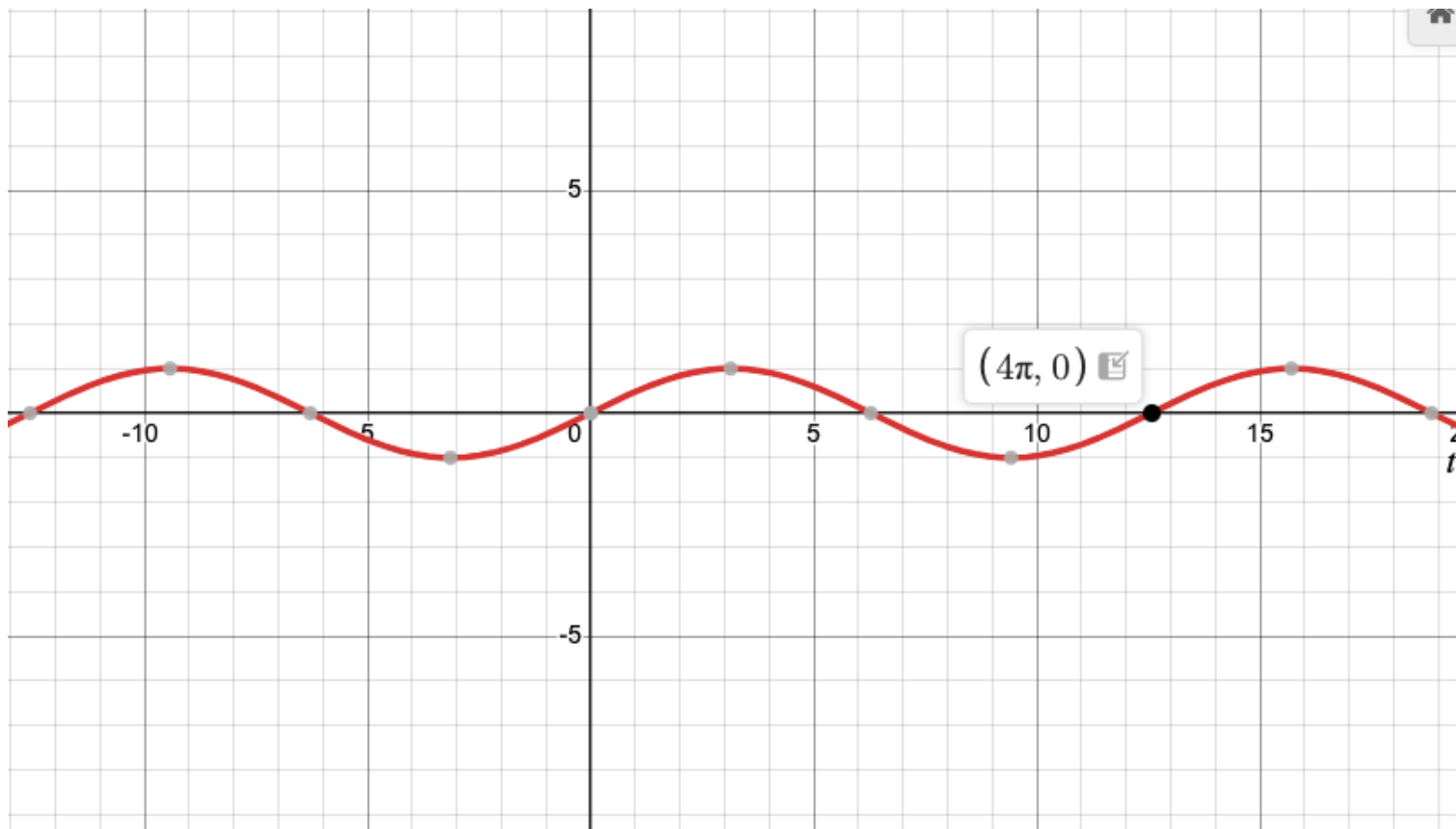
$$W = 1$$

$\sin(2t)$ What is the angular speed?



$$\omega = 2$$

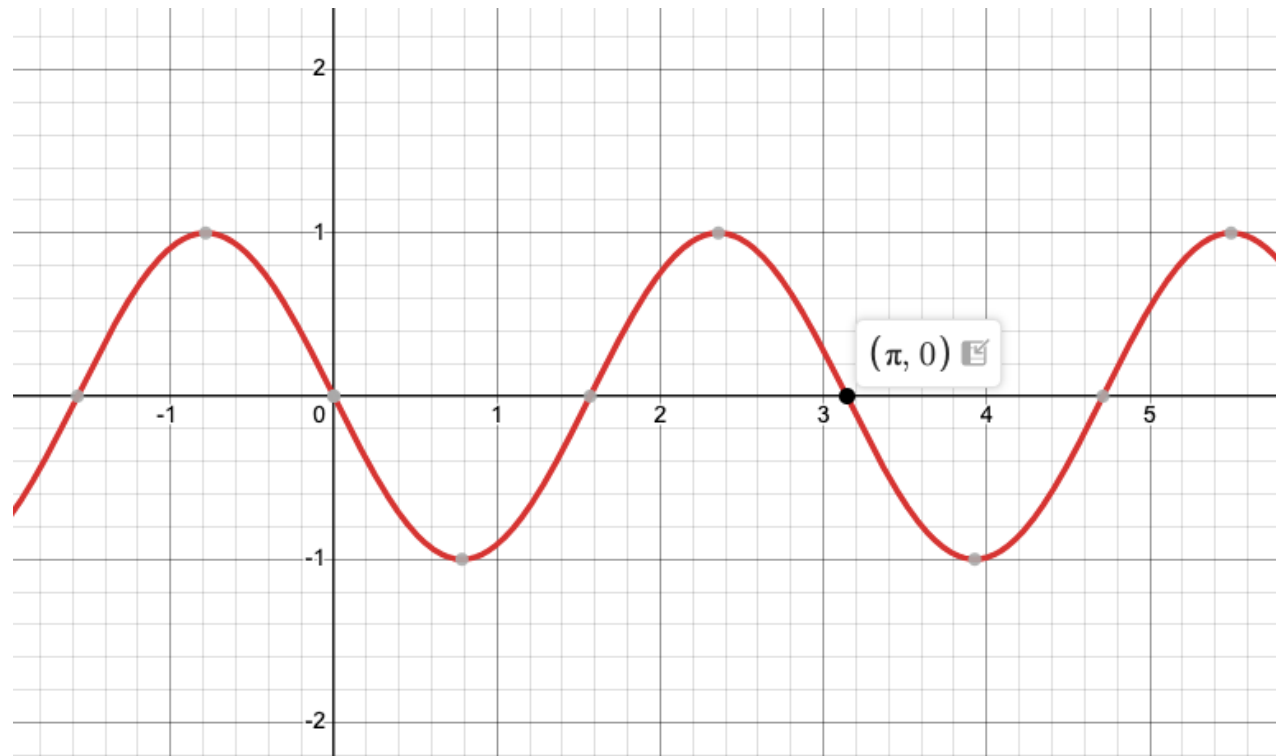
$\sin(t/2)$ What is the angular speed?



$$\omega = 1/2$$

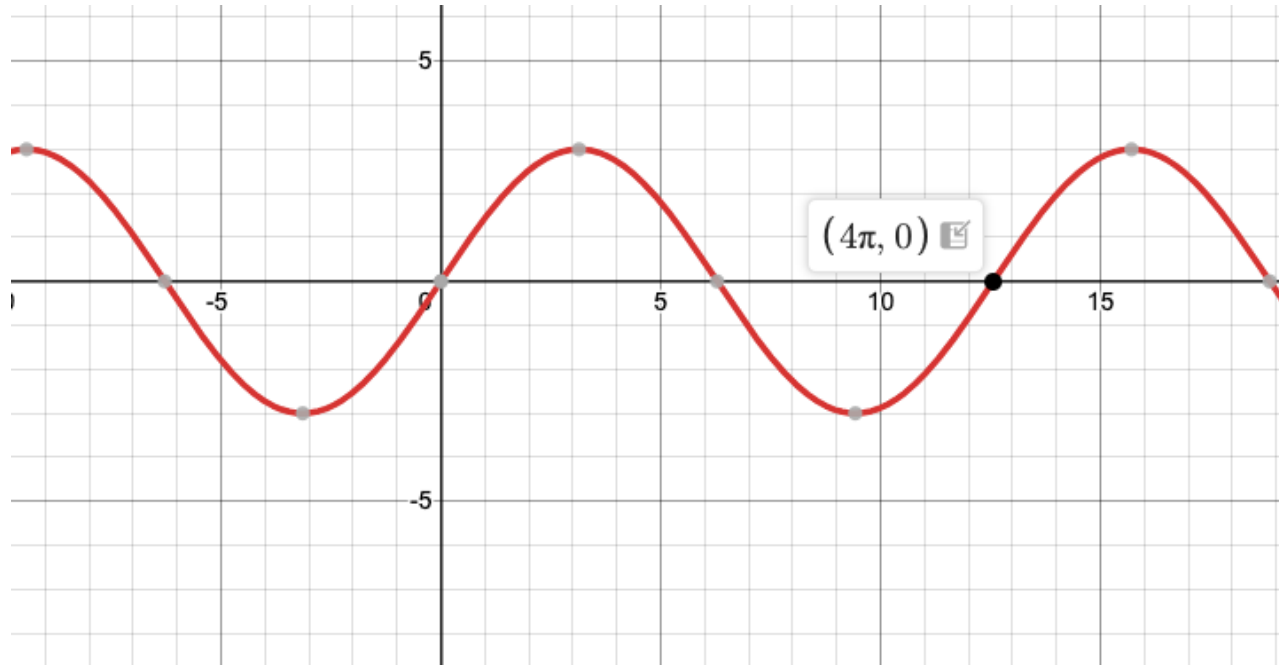
$$\sin(2(t - 0.5\pi))$$

What is the angular speed



$$\omega = 2$$

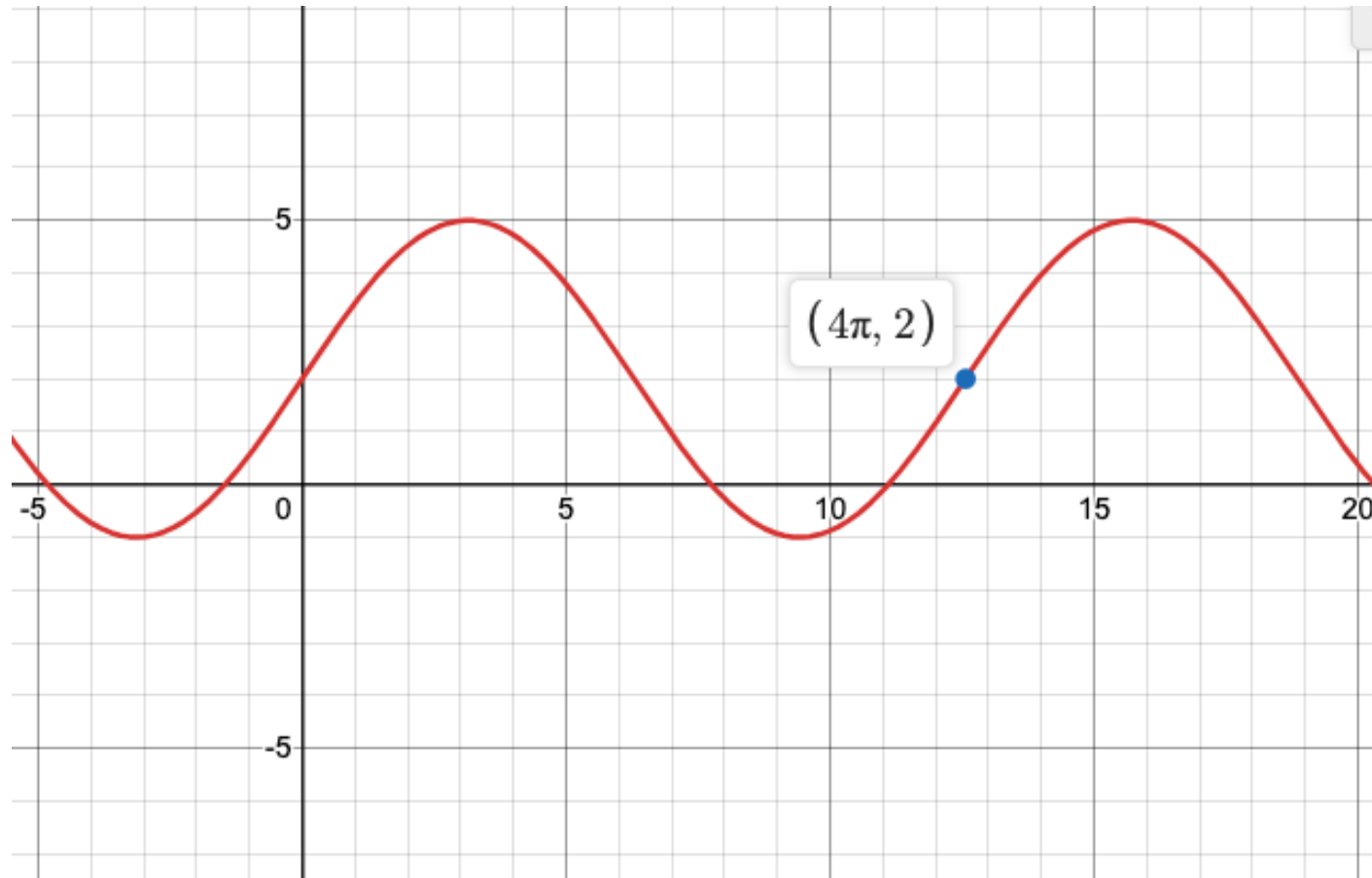
$3\sin(t/2)$ What is the angular speed?



$$\omega = 1/2$$

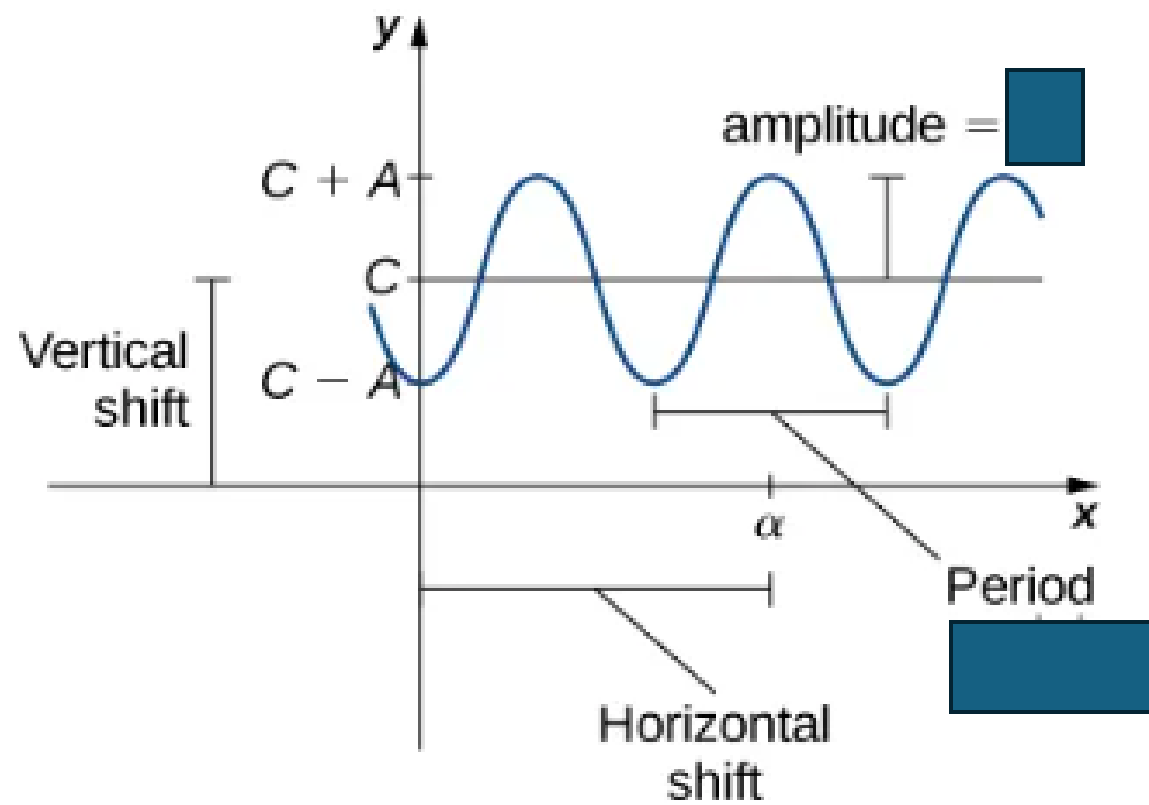
$$3\sin(t/2)+2$$

What is the angular speed?

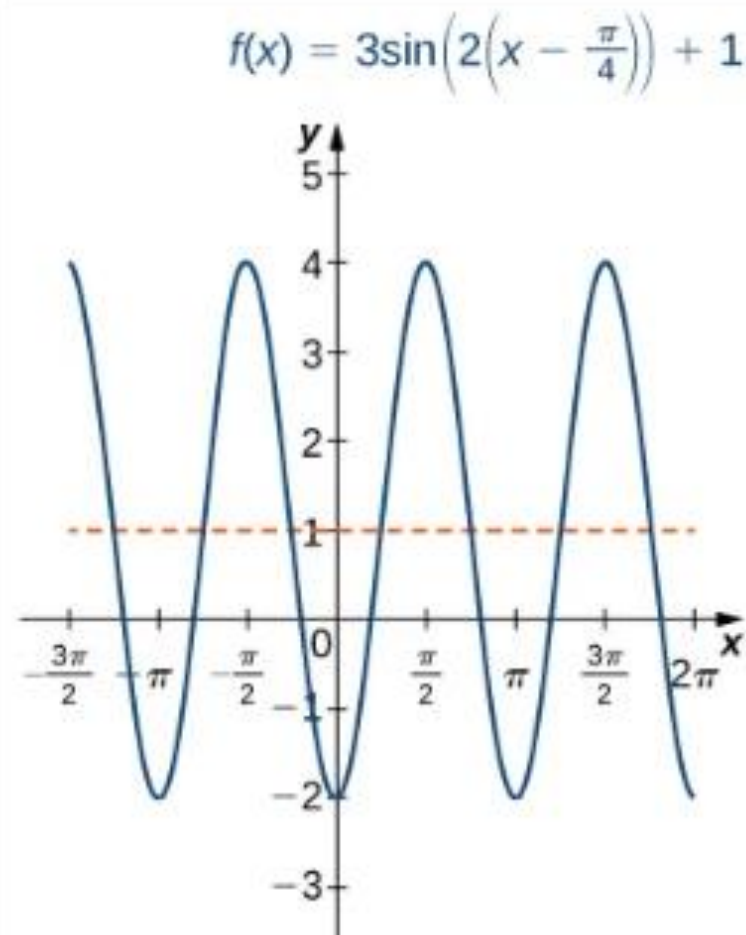


$$\omega = 1/2$$

$$f(x) = A\cos(B(x - \alpha)) + C$$



Sketch a graph of $f(x) = 3\sin\left(2\left(x - \frac{\pi}{4}\right)\right) + 1$.



Sketch a graph of $f(x) = 3\sin\left(2\left(x - \frac{\pi}{4}\right)\right) + 1$.

