

TURN and TALK

A quadratic function $f(x) = ax^2 + bx + c$ satisfies the following conditions:

- The roots are real and differ by 4
- The sum of the roots is 6
- The value of the function at $x = 1$ is 5

Find the exact values of a , b , and c .

TURN and TALK

Let roots be r_1, r_2 .

Given:

$$r_1 + r_2 = 6, |r_1 - r_2| = 4$$

Solve:

$$r_1 = 5, r_2 = 1$$

$$\text{So } f(x) = a(x - 5)(x - 1)$$

Use $f(1) = 5$:

$$a(0) = 5 \rightarrow \text{contradiction } \times$$

Conclusion:

No such quadratic exists.

1. Straight line — 直线
2. Gradient (slope) — 斜率
3. Intercept — 截距
4. y-intercept — y截距
5. x-intercept — x截距
6. Equation of a line — 直线方程
7. Slope-intercept form — 斜截式 ($y = mx + b$)
8. Gradient formula — 斜率公式
9. Parallel lines — 平行线
10. Perpendicular lines — 垂直线

Examples

This means we can **substitute** the values of a coordinate into our equation whenever we know the point lies on the line.

The point $(5, a)$ lies on the line with equation $y = 3x + 2$. Determine the value of a .

?

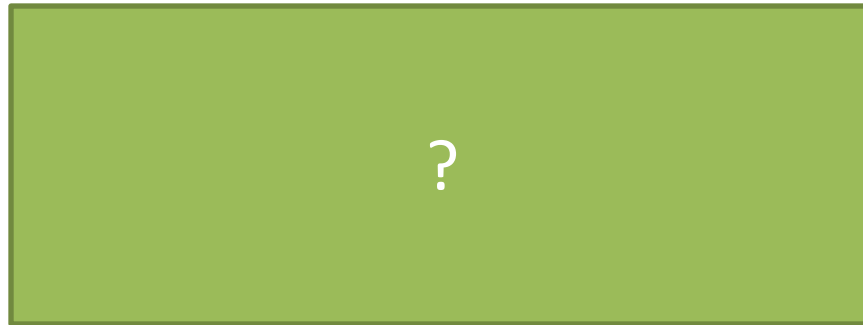
Find the coordinate of the point where the line $2x + y = 5$ cuts the x -axis.

?

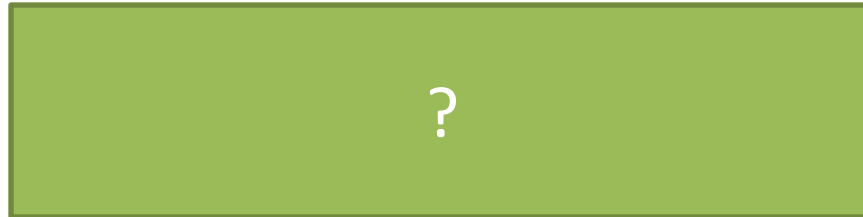
Another intercept example

Determine where the line $x + 2y = 3$ crosses the:

a) y -axis:

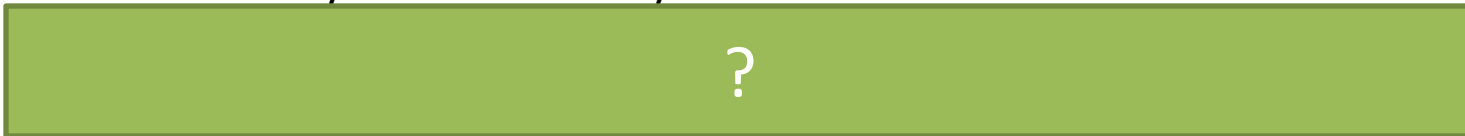


b) x -axis:

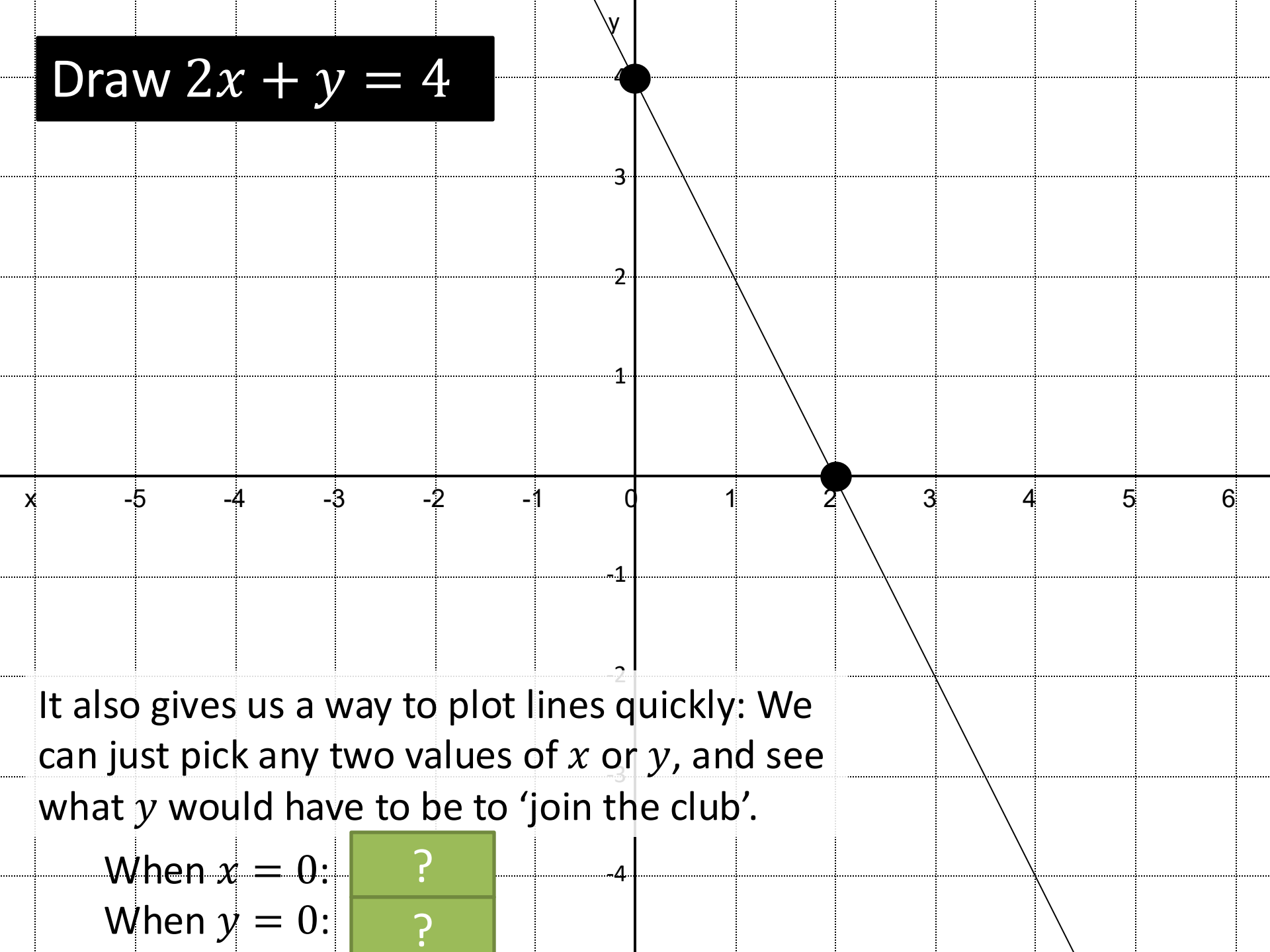


What mistakes do you think it's easy to make?

-
-



Draw $2x + y = 4$



It also gives us a way to plot lines quickly: We can just pick any two values of x or y , and see what y would have to be to 'join the club'.

When $x = 0$:

?

When $y = 0$:

?

Test Your Understanding

- 1 A point lies on the line with equation $y = 5x - 2$. The x -value of the point is 6. What would the y value have to be?

?

- 2 The point $(k, 8)$ lies on the line with equation $y = 20 - 4x$. Determine the value of k .

?

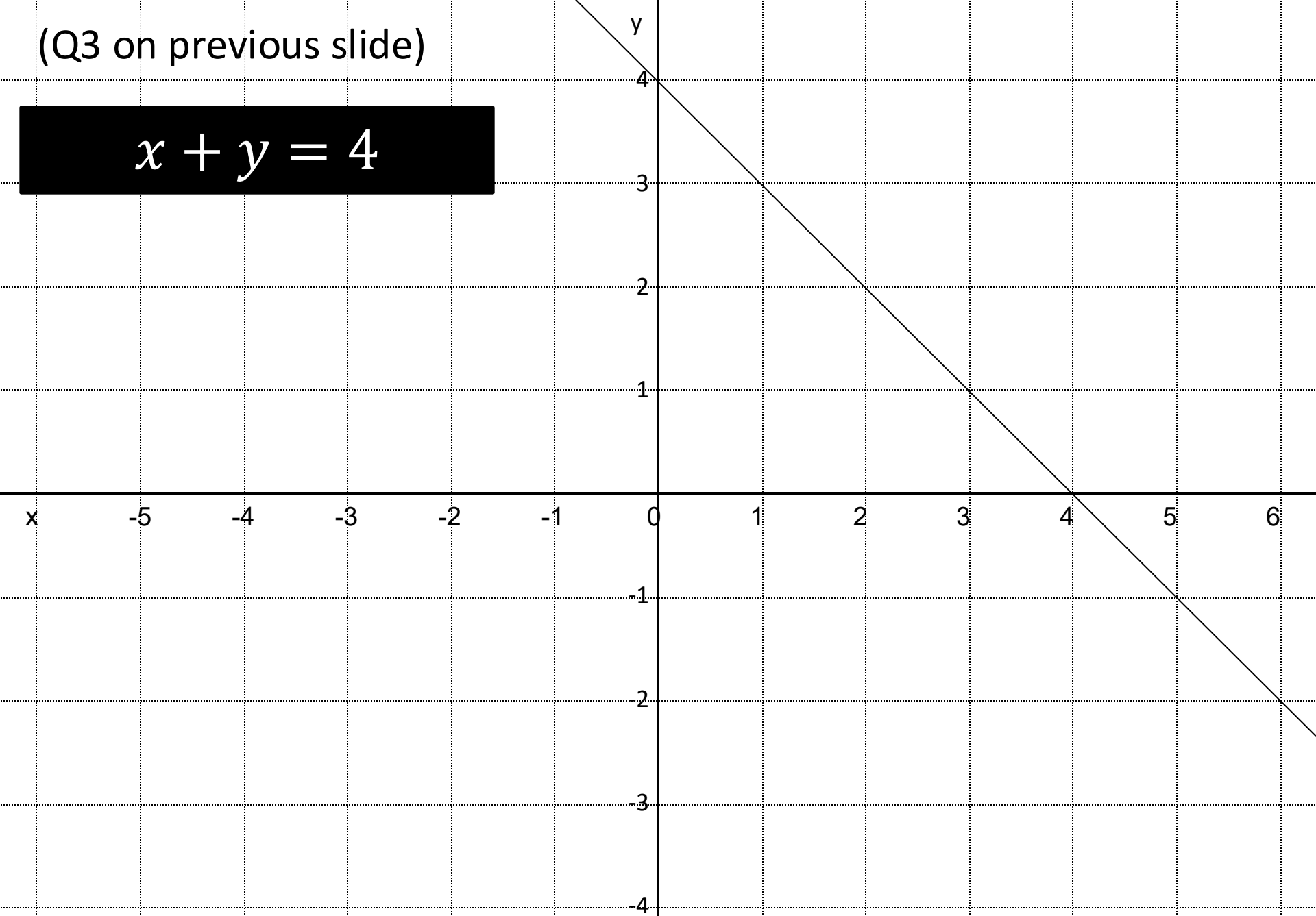
- 3 Draw coordinate axis in your book with x going from 0 to 6 and y going from 0 to 6. Draw the line with equation $x + y = 4$.

- 4 A line has equation $3x - 2y + 4 = 0$. Determine the coordinate of the point it intercepts the x -axis.

?

(Q3 on previous slide)

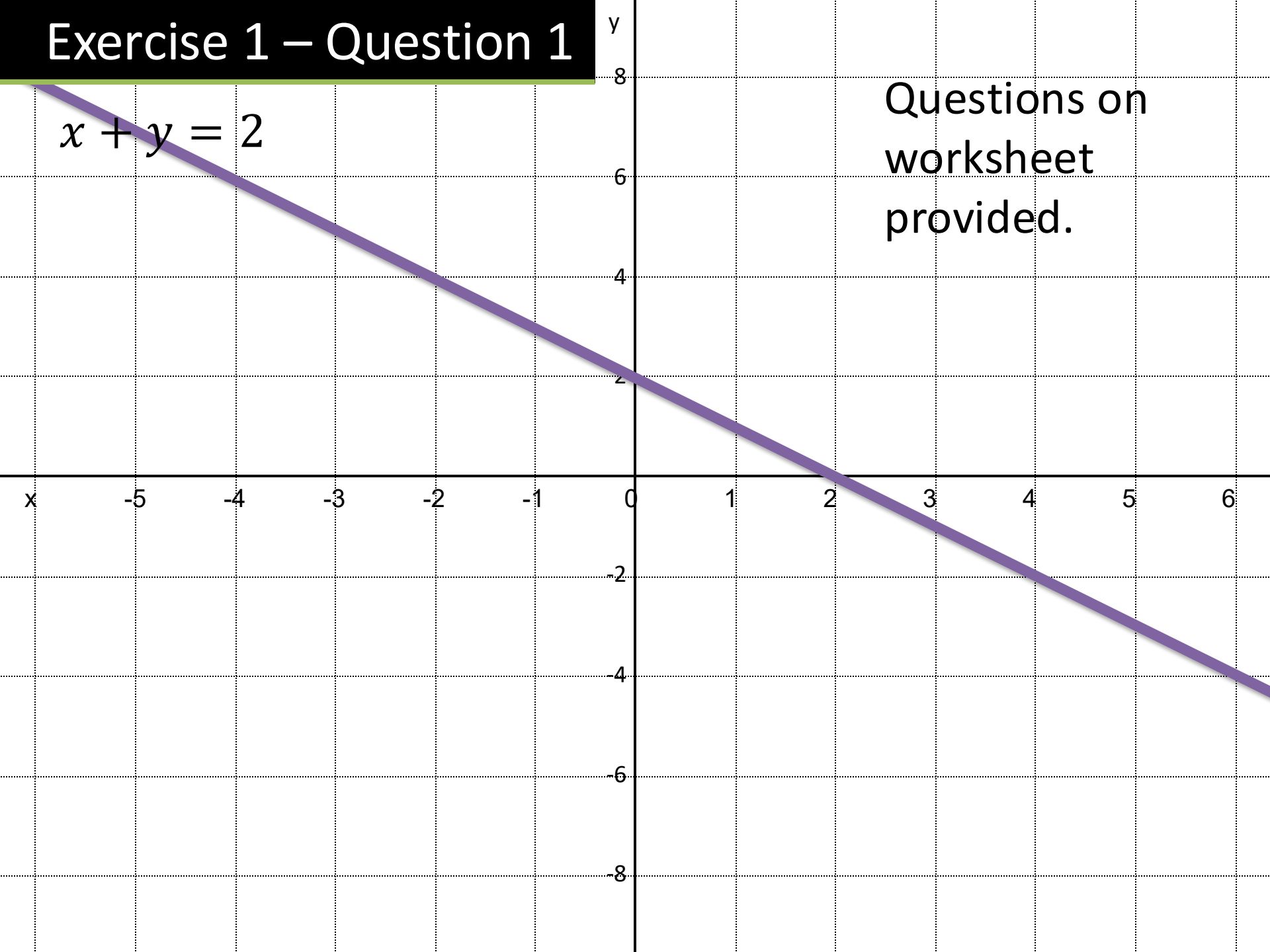
$$x + y = 4$$



Exercise 1 – Question 1

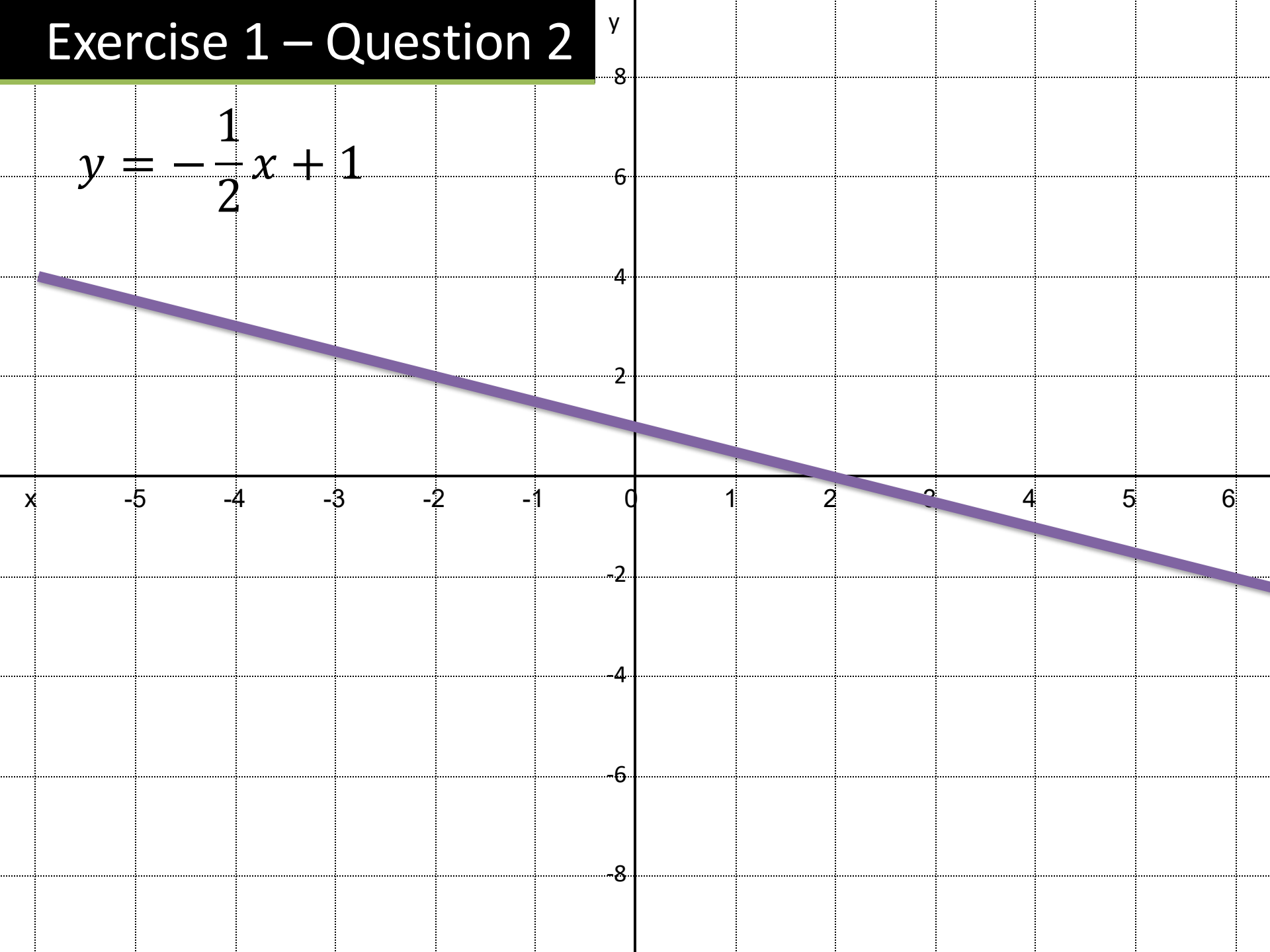
$$x + y = 2$$

Questions on
worksheet
provided.



Exercise 1 – Question 2

$$y = -\frac{1}{2}x + 1$$



Exercise 1 – Question 3

When the point $(3, k)$ lies on each of these lines, find the value of k .

a	$y = 3x + 2$?
b	$y = 4x - 2$?
c	$y = 3 - 2x$?
d	$x + y = 7$?
e	$x - 2y = 1$?

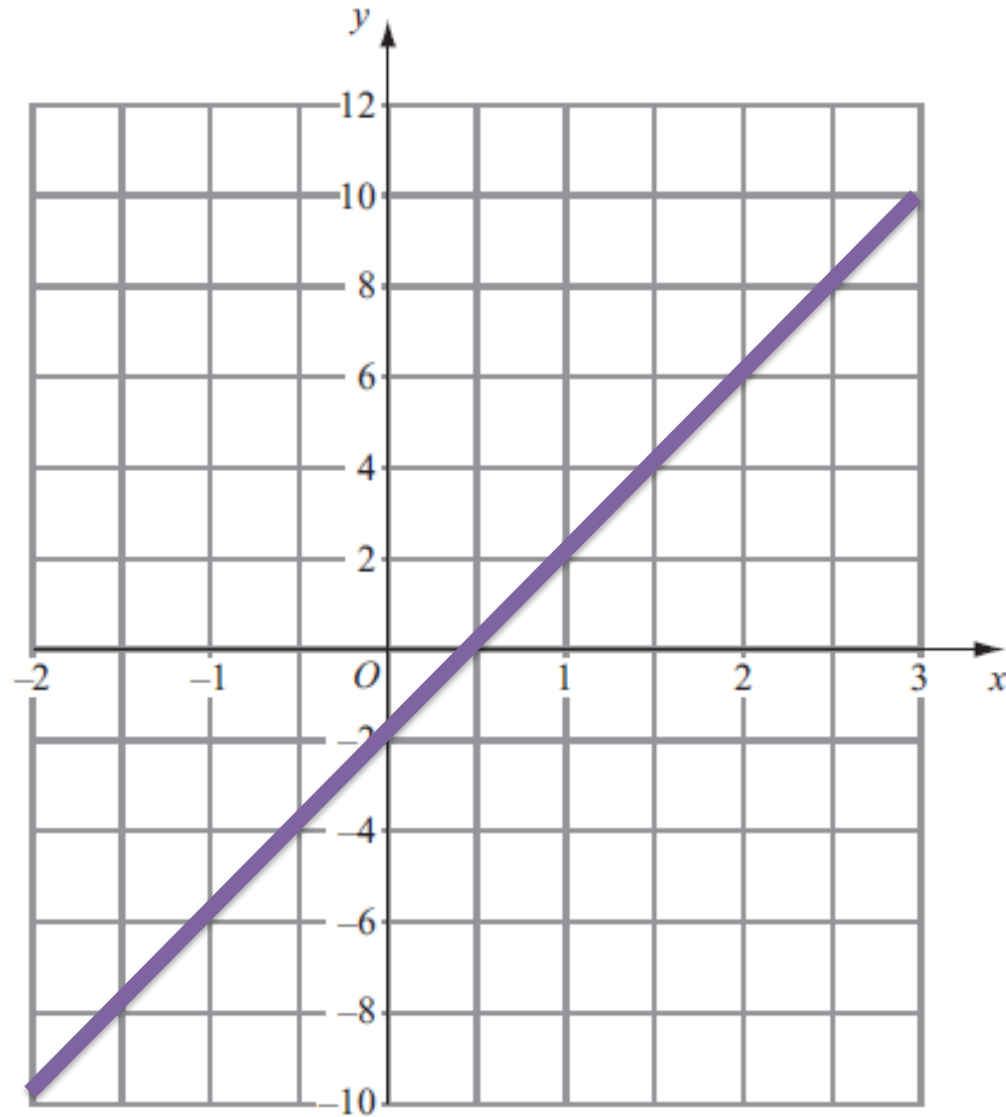
Exercise 1 – Question 4

Copy and complete this table.

Equation	The point where the line crosses the:	
	y-axis	x-axis
$y = 3x + 1$?	?
$y = 4x - 2$?	?
$y = \frac{1}{2}x - 1$?	?
$2x + 3y = 4$?	?

Exercise 1 – Question 5

$$y = 4x - 2$$



Exercise 1 – Question 6

When the point $(k, 3)$ lies on each of these lines, find the value of k .

$$y = 2x + 1$$

$$y = 2x - 1$$

$$y = 8 - 2x$$

$$2x + 3y = 4$$

?

?

?

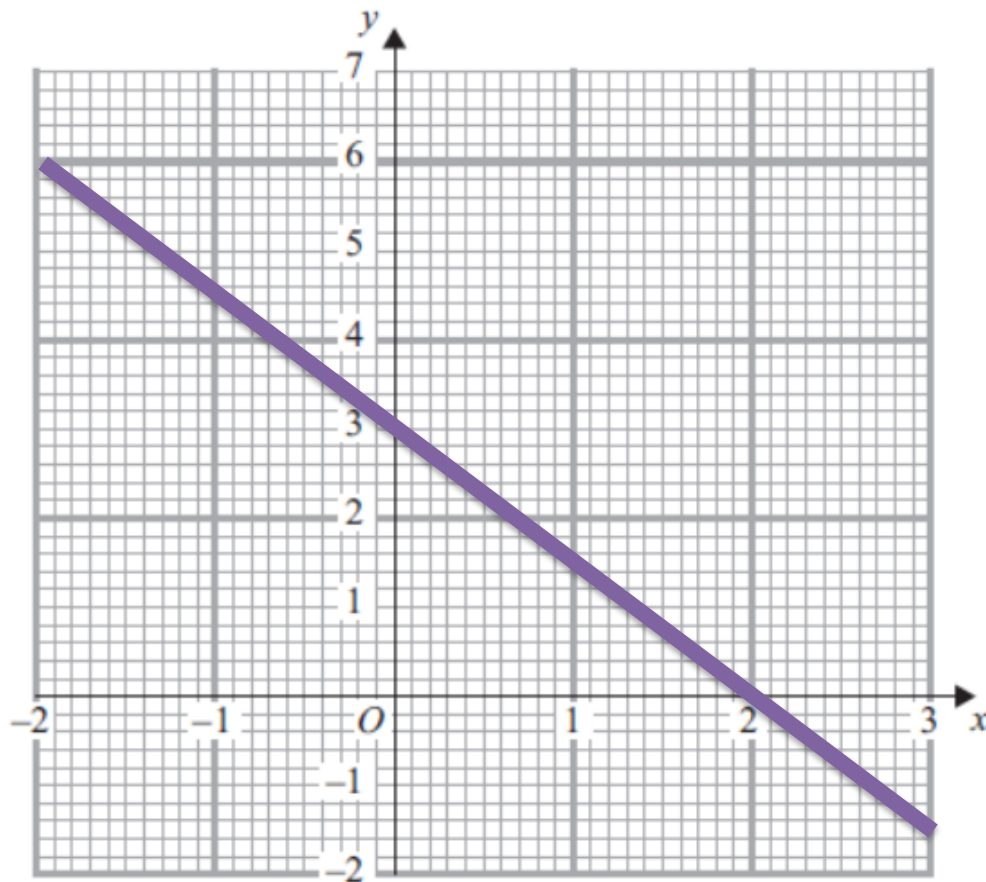
?

Exercise 1 – Question 7

Complete the table of values for $3x + 2y = 6$

x	-2	-1	0	1	2	3
y	?	4.5	3	?	?	-1.5

(b) On the grid, draw the graph of $3x + 2y = 6$



Click to Reveal

Exercise 1 – Question 8

Complete the table of values for $x + 2y = 1$.

x	-2	-1	0	1	2
y	?	1	?	?	?

If $x = -2$ just sub it
into the equation:

$$-2 + 2y = 1$$

$$2y = 3$$

$$y = \frac{3}{2}$$

Exercise 1 – Question 9

Put a tick or cross to determine whether each of the following points are on the line with the given equation.

	$y = 1 - x$	$x + 2y = 3$
$(3, -2)$?	?
$(1, 2)$?	?
$\left(2, \frac{1}{2}\right)$?	?
$(-1, 2)$?	?

Exercise 1 – Question 10

For the given equation of a line and point, indicate whether the point is above the line, on the line or below the line. (Hint: Find out what y is on the line for the given x)

		Below the line	On the line	Above the line
$y = 3x + 4$	$(3, 11)$?		
$x + y = 5$	$(7, -2)$?		
$y = 3 - 2x$	$(-3, 10)$?		
$2x + 3y = 4$	$\begin{pmatrix} 3 & 4 \\ - & - \\ 4 & 5 \end{pmatrix}$?		

Exercise 1 – Question _1

The equation of a line is $ax + by = c$. If the x value of some point on the line is d , what is the full coordinate of the point, in terms of a, b, c, d ?

?

Exercise 1 – Question 2

What is the area of the region enclosed between the line with equation $2x + 7y = 3$, the x axis, and the y axis?



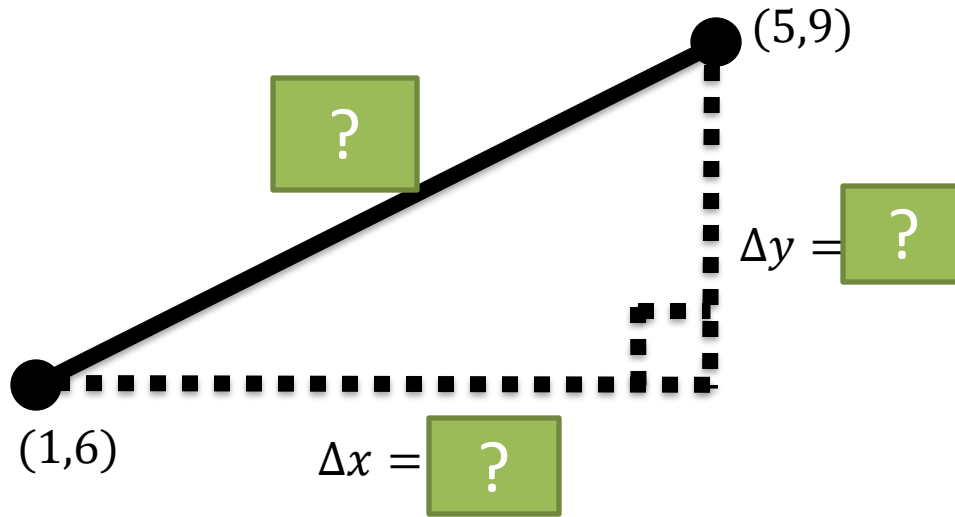
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Part 4

Distances between points
and points of intersection

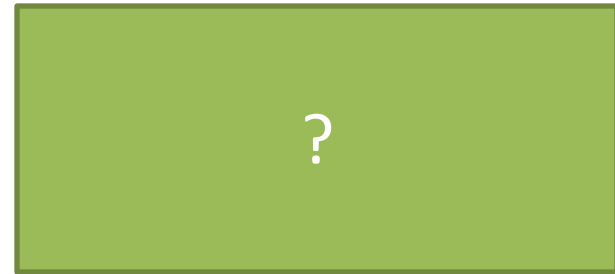
Distances between points

Δ (Greek letter 'delta') means "change in"



How could we find the **distance** between these two points?

Hint: [?]



Distance between two points:

$$\sqrt{(\Delta x)^2 + (\Delta y)^2}$$

Examples

Distance between:

$(3,4)$ and $(5,7)$

$(5,1)$ and $(6,-3)$

$(0,-2)$ and $(-1,3)$



Note: Note that unlike with gradient, we don't care if the difference is positive or negative (it's being squared to make it positive anyway!)

Quickfire Questions:

Distance between:

$(1,10)$ and $(4,14)$

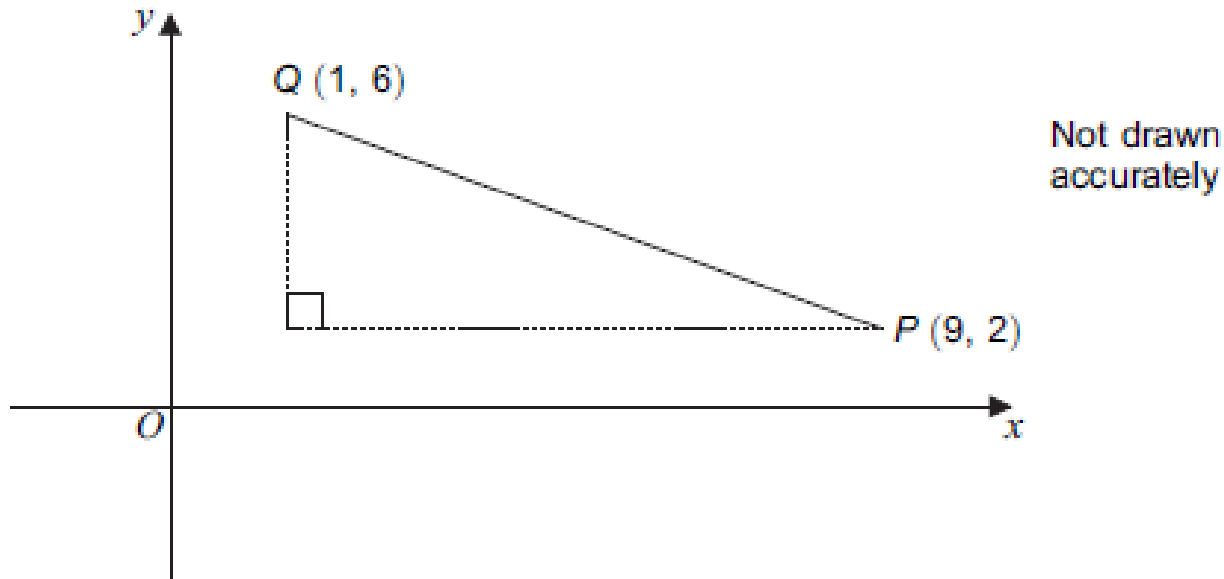
$(3,-1)$ and $(0,1)$

$(-4,-2)$ and $(-12,4)$



Test Your Understanding So Far...

AQA IGCSEFM June 2012 Paper 2 Q3

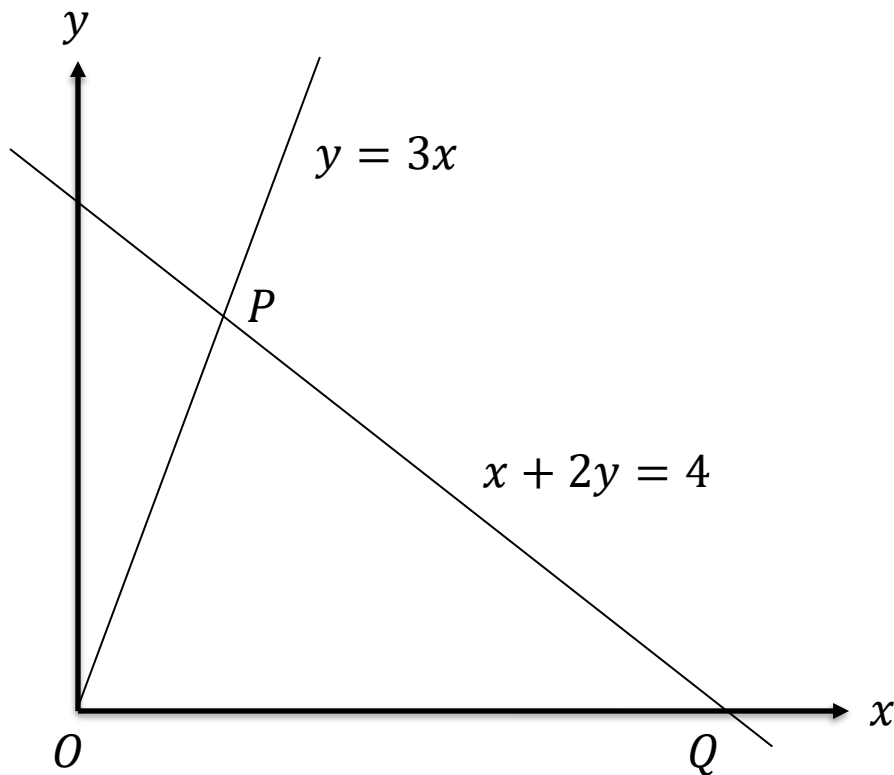


Work out the length of PQ .
Give your answer to 3 significant figures.

$PQ =$

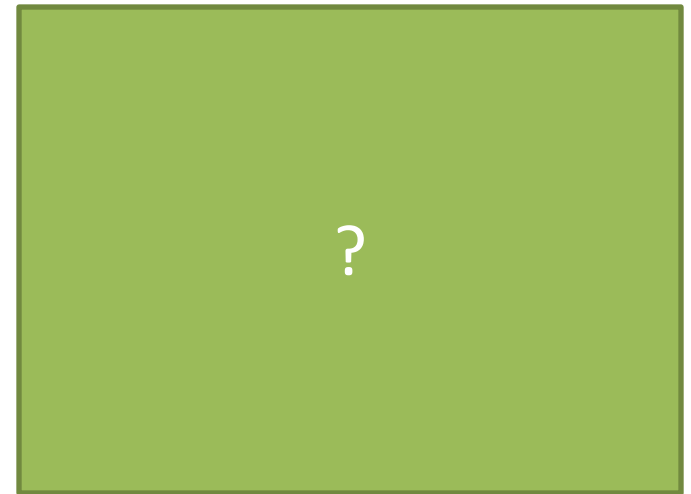
?

Intersection of lines

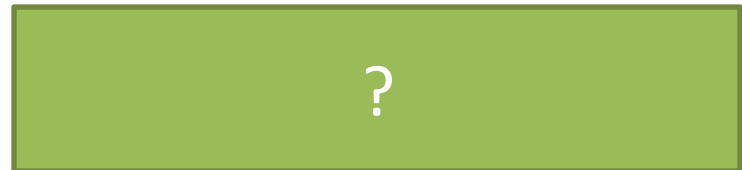


The diagram shows two lines with equations $y = 3x$ and $x + 2y = 4$, which intersect at the point P . The line OP passes through the origin.

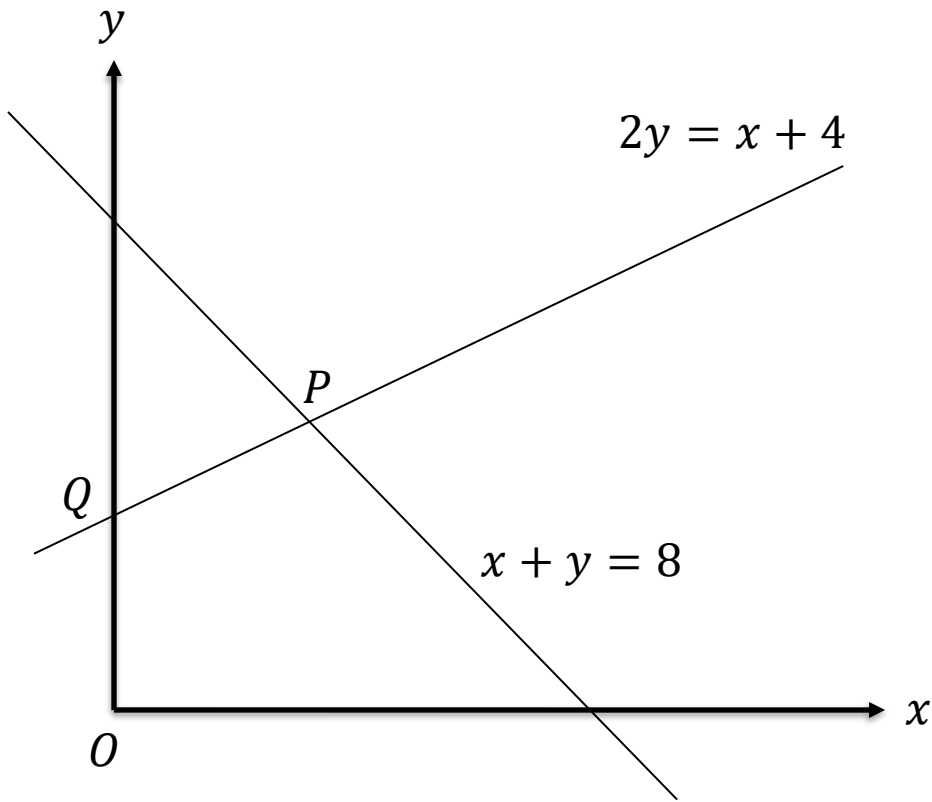
- a) Determine the coordinates of P .



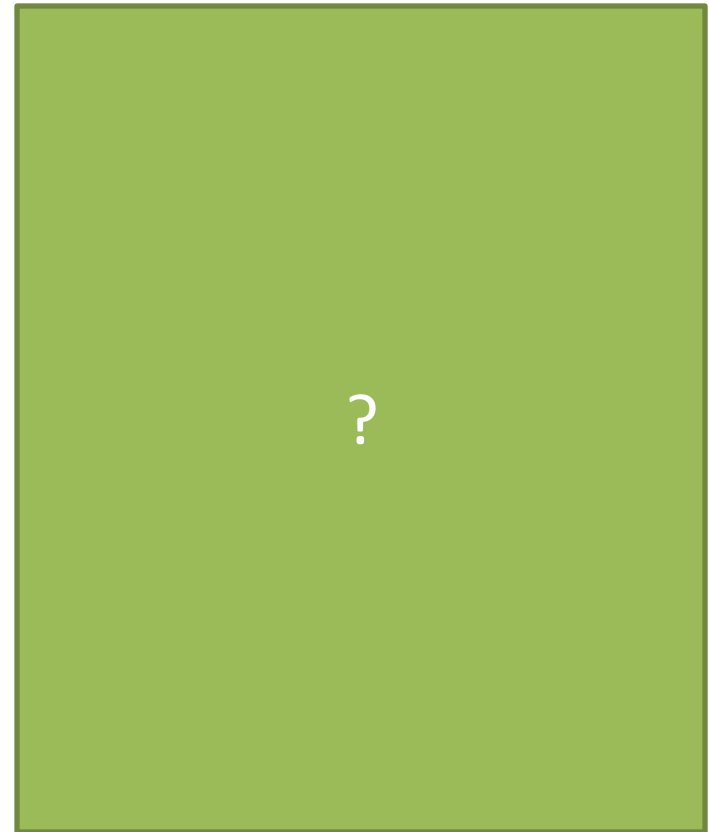
- b) The line $x + 2y = 4$ intersects the x -axis at the point Q . Determine the area of the triangle OPQ .



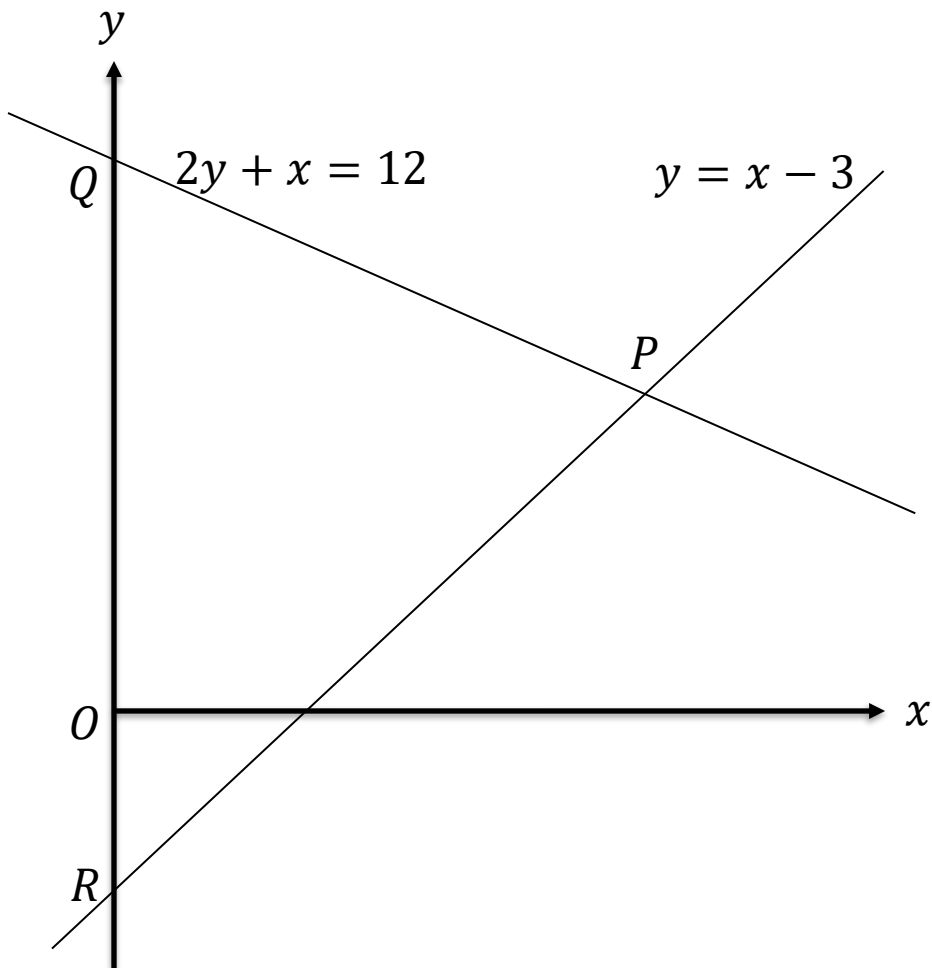
Further Example



Determine the length of PQ .



Test Your Understanding



a) Determine the coordinate of P .

?

b) Determine the area of PQR .

?

c) Determine the length PQ .

?

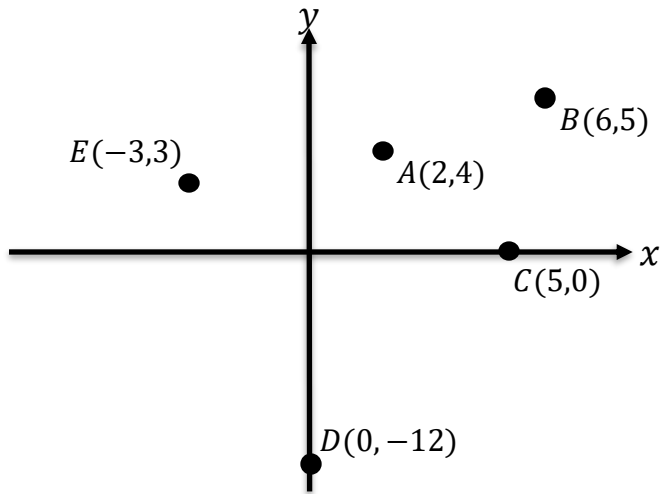
Exercise 4

(On provided sheet)

1 Find the coordinate of the point of intersection between these lines:

- a $y = x + 5, \quad y = 2x \quad \rightarrow$
- b $y = 2x - 5, \quad y = x + 5 \quad \rightarrow$
- c $x + y = 5, \quad y = 2x - 4 \quad \rightarrow$
- d $2x + y = 7, \quad x - 2y = 6 \quad \rightarrow$
- e $4x + 3y = 1, \quad y = 1 - x \quad \rightarrow$

2 Find the distance: (giving exact values)



- a $AB =$
- b $AC =$
- c $CD =$
- d $DE =$
- e $CE =$

3 Find the distance between the two points where $y = 3x + 12$ crosses the coordinate axes.

4 Line l_1 passes through $(-1,1)$ and $(6,15)$. Another line l_2 passes through $(0,-12)$ and $(3,3)$. Determine the coordinate of the point at which they intersect.

5 Line l_1 has the equation $y = x$ and l_2 has the equation $y = -2x + 12$. The two lines intersect at point A and line l_2 intersects the x and y -axis at B and C respectively, as indicated. Find the area of:

- a) OAB (where O is the origin)
- b) OAC

Exercise 4

(On provided sheet)

6 [AQA IGCSEFM Jan 2013 Paper 1 Q16]

A, B and C are points on the line

$$2x + y = 8.$$

DCE is a straight line.

$$AB : BC = 2 : 1$$

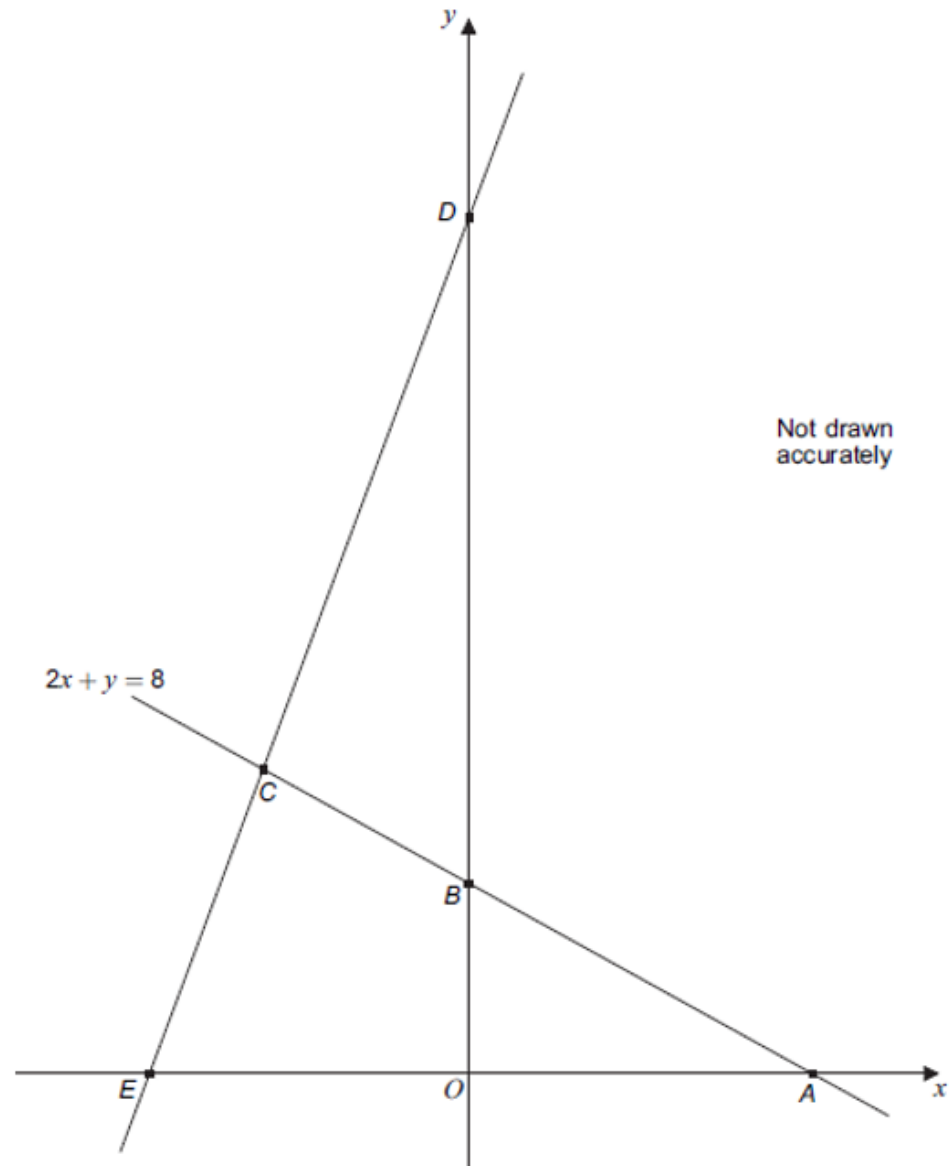
$$EC : CD = 1 : 2$$

Work out the ratio:

Area of triangle AEC

: Area of triangle BCD

Give your answer in its simplest form.



CHALLENGING FURTHER MATHEMATICS QUESTIONS

1. Section Formula + Distance

Points $A(-2, 3)$ and $B(6, -1)$.

Point P divides AB in the ratio $2 : 3$.

Find the exact distance OP , where O is the origin.

2. Perpendicular Condition + Distance

Points $A(1, 2)$ and $B(7, 8)$.

Point P lies on AB such that $OP \perp AB$, where O is the origin.

Find the coordinates of P and its distance from O .

3. Locus + Minimisation (Hard)

Points $A(0, 4)$ and $B(6, 0)$.

Point P moves along segment AB .

Find the minimum possible value of the distance OP , where O is the origin.

1

$$P = \left(\frac{6}{5}, \frac{7}{5}\right)$$

$$OP = \frac{\sqrt{85}}{5}$$

2

$$P = \left(-\frac{1}{2}, \frac{1}{2}\right)$$

$$OP = \frac{\sqrt{2}}{2}$$

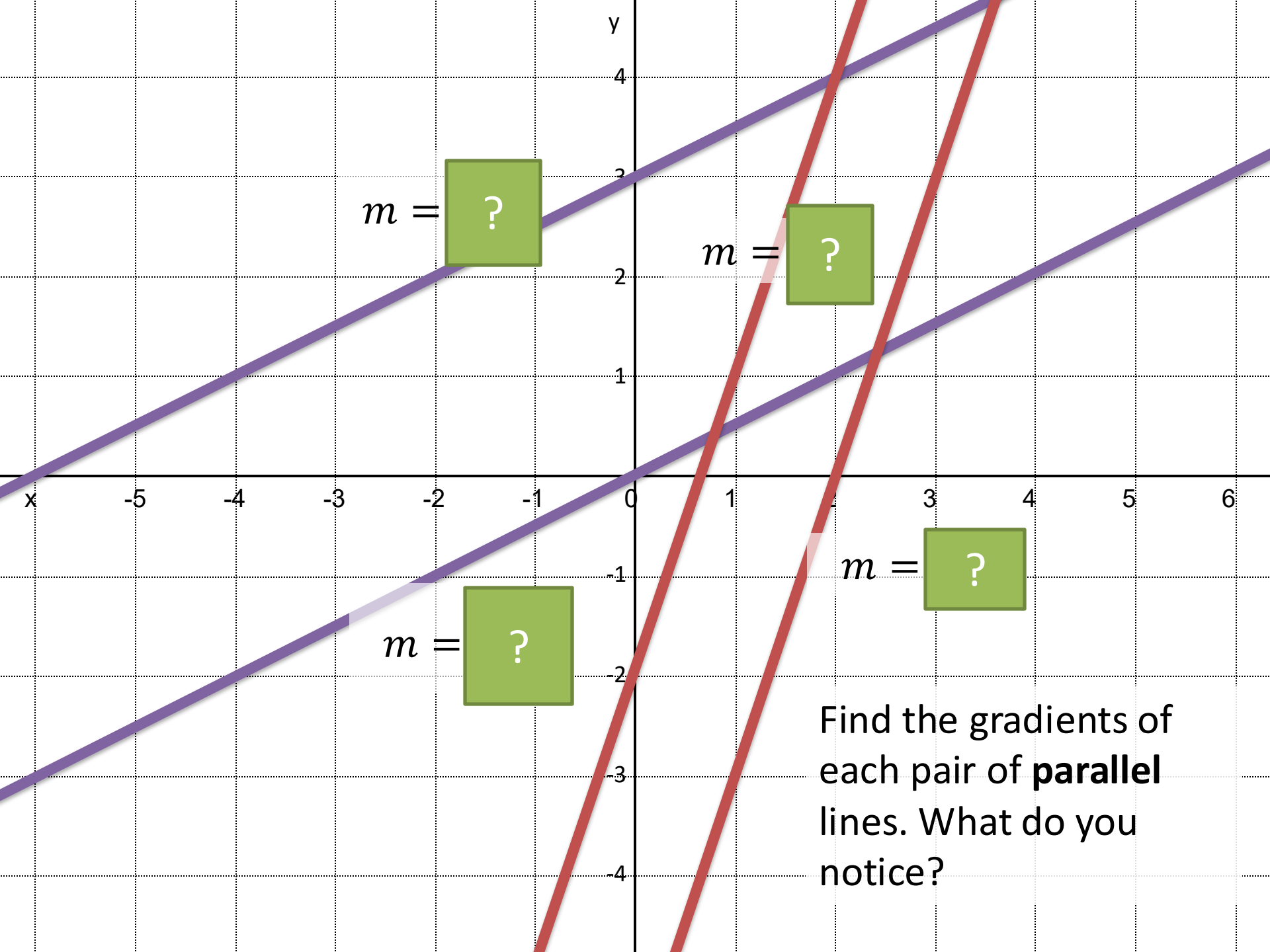
3

Minimum distance:

$$OP_{\min} = \frac{12\sqrt{13}}{13}$$

Part 5

Perpendicular and parallel
lines



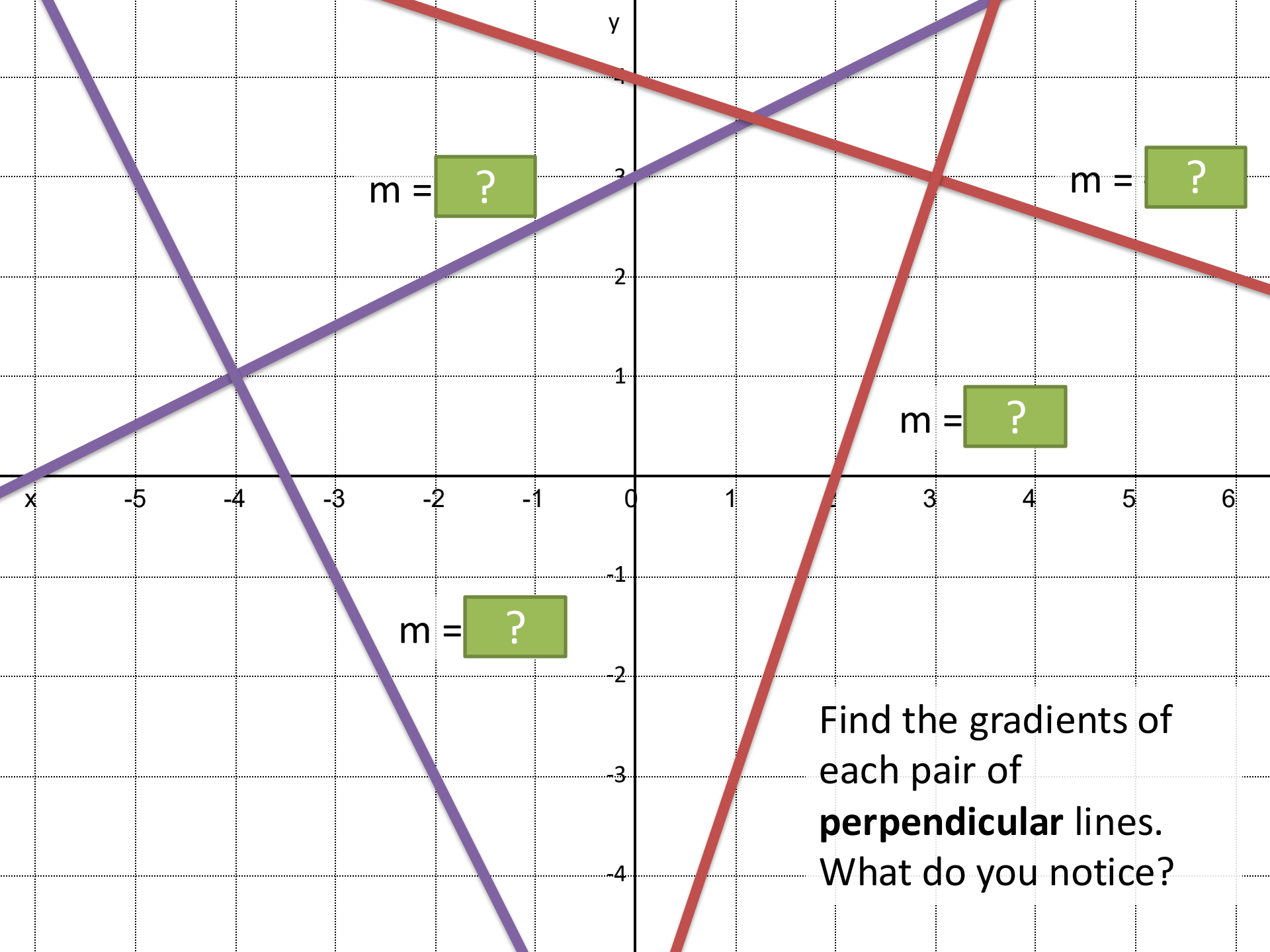
$m = ?$

$m = ?$

$m = ?$

$m = ?$

Find the gradients of each pair of **parallel** lines. What do you notice?



$$m = ?$$

$$m = ?$$

$$m = ?$$

$$m = ?$$

Find the gradients of each pair of **perpendicular** lines. What do you notice?

Perpendicular Lines



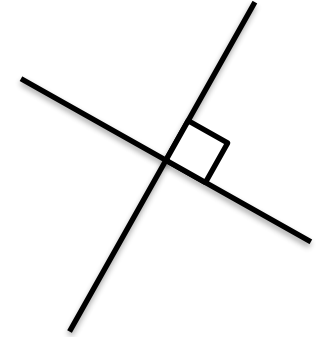
The gradients of parallel lines are equal.

If two lines are perpendicular, then the gradient of one is the **negative reciprocal** of the other.

$$m_1 = -\frac{1}{m_2}$$

To **show** that two lines are perpendicular:

$$m_1 m_2 = -1$$



Gradient	Gradient of Perpendicular Line
2	?
-3	?
$\frac{1}{4}$?
5	?
$-\frac{2}{7}$?
$\frac{7}{5}$?

Example Problems

- 1 A line goes through the point $(9,10)$ and is perpendicular to another line with equation $y = 3x + 2$. What is the equation of the line?

?

- 2 A line L_1 goes through the points $A(1,3)$ and $B(3,-1)$. A second line L_2 is perpendicular to L_1 and passes through point B . Where does L_2 cross the x-axis?

?

- 3 Are the following lines parallel, perpendicular, or neither?

$$y = \frac{1}{2}x$$
$$2x - y + 4 = 0$$

?

Putting in the form $ax + by = c$ or $ax + by + c = 0$

Sometimes a question might ask you to put your equation in a particular form.

...Leave your answer in the form $ax + by = c$, where a, b, c are integers.

$$y = \frac{1}{2}x - 2$$

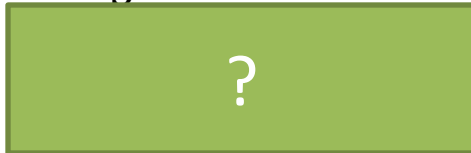


We want whole numbers not fractions, so what should we do to both sides of the equation?

I tend to put everything on the side that makes x positive, but it doesn't hugely matter.

...Leave your answer in the form $ax + by + c = 0$, where a, b, c are integers.

$$y = \frac{2}{3}x + 4$$



Test Your Understanding

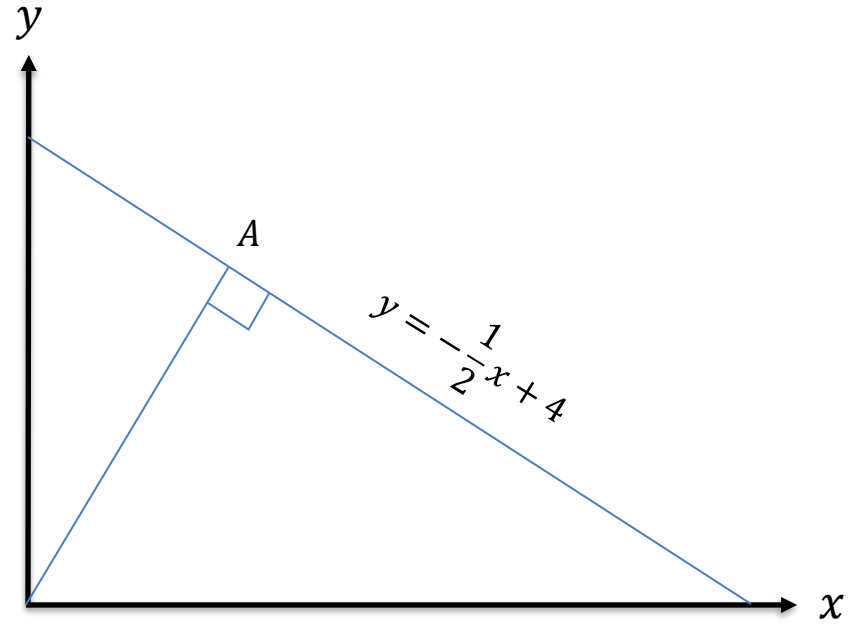
1

A line goes through the point $(4,7)$ and is perpendicular to another line with equation $y = 2x + 2$. What is the equation of the line? Put your answer in the form $ax + by + c = 0$, where a, b, c are integers.

?

2

Determine the point A .



?

Exercise 5

(On provided sheet)

- 1 Are the following lines parallel, perpendicular or neither?

$$y = 2x + 3, \quad y = 2x$$

$$y = 3x - 4, \quad y = -3x + 1$$

$$y = \frac{1}{2}x + 1, \quad y = -2x$$

?

?

?

- 2 A line is parallel to $y = 2x + 3$ and goes through the point $(4,3)$. What is its equation?

?

- 3 A line l_1 goes through the indicated point and is perpendicular to another line l_2 . Determine the equation of l_1 in each case.

$$(2,5) \quad l_2: y = 2x + 1$$

?

$$(-6,3) \quad l_2: y = 3x$$

?

$$(0,6) \quad l_2: y = -\frac{1}{2}x - 1$$

?

$$(-9,0) \quad l_2: y = -\frac{1}{3}x + 1$$

?

$$(10,10) \quad l_2: y = -5x + 5$$

?

- 4 $A(2,5) \quad B(4,9)$

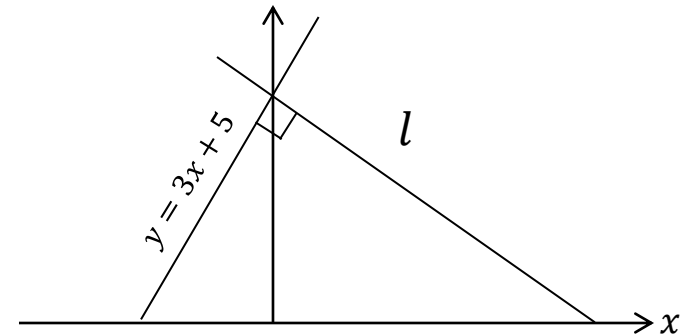
Find the equation of the line which passes through B, and is perpendicular to the line passing through both A and B.

?

- 5 Line l_1 has the equation $2y + 3x = 4$. Line l_2 goes through the points $(2,5)$ and $(5,7)$. Are the lines parallel, perpendicular, or neither?

?

- 6



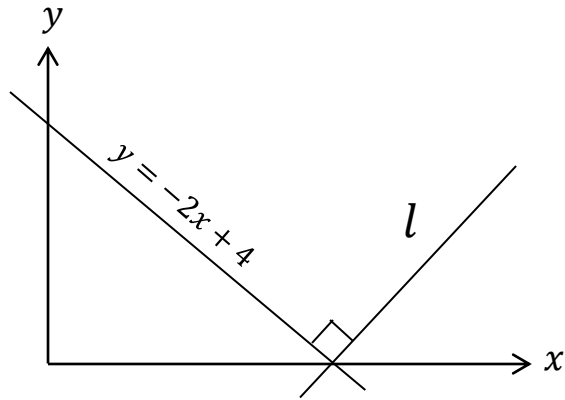
Determine the equation of the line l .

?

Exercise 5

(On provided sheet)

7



Determine the equation of the line l .

?

8

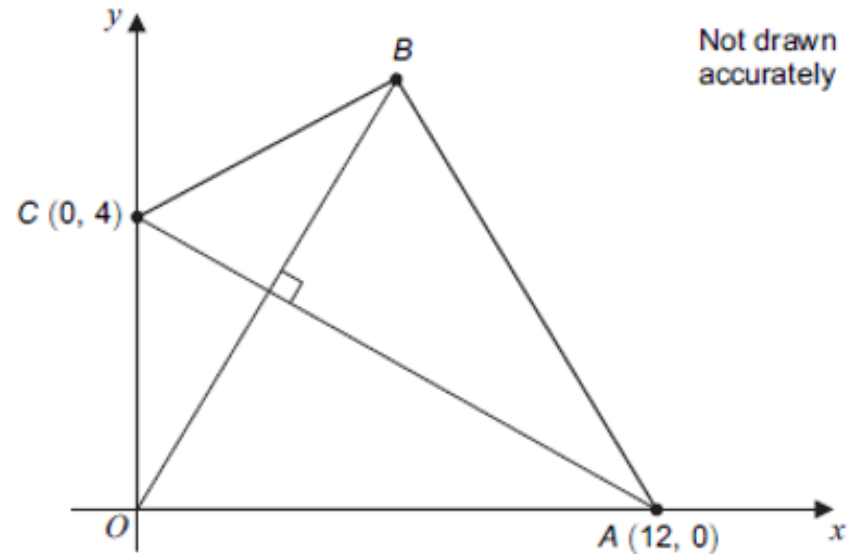
$A(3,7), B(5,13)$

Find the equation of the line passing through B and is perpendicular to the line passing through A and B , giving your answer in the form $ax + by + c$, where a, b, c are integers.

?

9

[AQA IGCSEFM June 2012 Paper 1 Q11]
 $OABC$ is a kite.



a) Work out the equation of AC .

?

b) Work out the coordinates of B .

?

Exercise 5

(On provided sheet)



Suppose O is the origin, and $A(1,2), B(4,2), C(2.2, -0.4)$.

Prove that $OABC$ is a kite.

(Hint: you need to prove **two** things as part of this.)

?

Exercise 6 – Mixed Exercises

(On provided sheet)

- 1 Line l_1 passes through the points (4,5) and (7,11). Line l_2 has the equation $2y = 3x - 1$. Do the lines intersect?

?

- 2 A is the point (4, -1) and B is the point (7,7).

a) Find the coordinates of the midpoint of AB .

?

b) Find the distance AB to 2 dp.

?

- 3 Line l_1 has the equation $y = 2x + 1$ and line l_2 the equation $y = 4x - 3$. Find the coordinates of the point at which they intersect.

?

- 4 a) Find the gradient of the line with equation $3x - 4y = 12$.

?

b) Prove that $3x - 4y = 12$ and $3y = 12 - 4x$ are perpendicular.

?

- 5 A line passes through the points (0,4) and (6,1). Find the equation of the line in the form:

a) $y = mx + c$

?

b) $ax + by = c$ where a, b, c are integers.

?

- 6 Find the coordinates of the points where $2x - 3y = 6$ crosses:

a) The x -axis.

?

b) The y -axis.

?

Exercise 6 – Mixed Exercises

(On provided sheet)

7 [Edexcel]

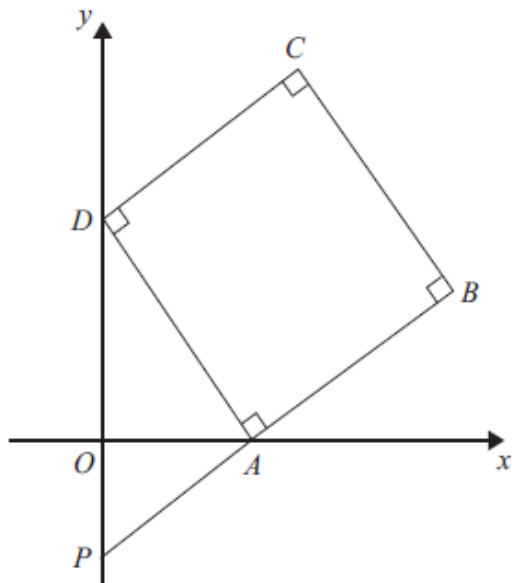
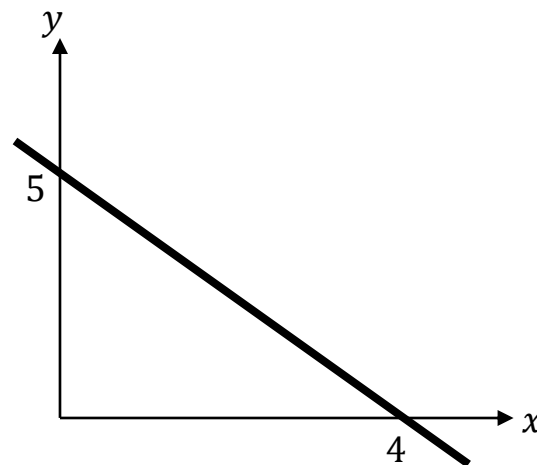


Diagram NOT accurately drawn

$ABCD$ is a square. P and D are points on the y -axis. A is a point on the x -axis. PAB is a straight line. The equation of the line that passes through the points A and D is $y = -2x + 6$. Find the length of PD .

?

8



Determine the equation of this line, putting your answer in the form $ax + by + c = 0$, where a, b, c are integers.

?

Exercise 6 – Mixed Exercises

(On provided sheet)

- 9 A triangle consists of the points $P(3, k)$, $Q(6, 8)$ and $R(10, 10)$. PQR is a right angle. Determine the equation of the line passing through P and R , leaving your answer in the form $ax + by = c$, where a, b, c are integers.

