F1

Annie Bai
Franklin Chen
Charlie Cheng
Zoey Cheng
Harry Cui
King Deng
Aisling Fu
Ken He
Ivy Jing
Lee Li
Ziheng Li
Lewis Liu
Sam Liu
Ava Peng
Peter Shen
Sylvia Song
Coco Wang
Eileen Wang
Jason Wang
Luna Wang
Cathy Yan
Melody You
Allen Zhang
Lucas Zhang
Lynette Zhang
Ryder Zhang
Flora Zhou
Cyntina Zuo

F2

Angelina Wang	Azura Cheng	Bonnie Zhao	Cecilia Jia	Diana Li	Oscar Liu	Zavier Liu	Tom Ou	Mia Peng
Joshua Hui	Julius Lv	Justin Jia	Lucas Li	Lyra Zhang	Iris Shao	Dylan Suo	PeytonAson Wang	Struck Wang
Selene Hou	Serena Feng	Silas Lv	Simon Wang	Stella Sun	Yola Wang	Mark Xiao	Elsa Ye	Emily Zhang
Zinnia Dong	Flora Zhang							

F6

Lucas Chang	Chloe Fang	Bruce Gao	Wyatt Hou	Jackson Jiang	Ivan Lei	Eric Li	Keira Li	Vivian Li
Jeremy Lin	Orange Liu	Serika Ren	Yumiko Shi	Hannah Si	Eric Song	Eric Tan	Silas Wang	Niki Wei
Kiki Wen	Miyu Wu	Tia Wu	Kumi Yuan	Estelle Zhang	Soren Zhang	Viakey Zhang	Jack Zhao	

F4

Aris Cheng	Tracy Dang	Jason Du	Alanna Fan	Iris Gao	Eason Gong	Vardy Hai	Edmund He	Liora Li
Miranda Li	Leo Liu	Francis Lv	Dorothy Qian	Dary Song	Jerry Tu	Belinda Wang	Winnie Wang	Elena Wei
James Xu	Mike Yan	Simon Yang	Coco Zhang	Patrick Zhang	Sophia Zhang	Molly Zheng	Carrie Zhou	Rudy Zhu

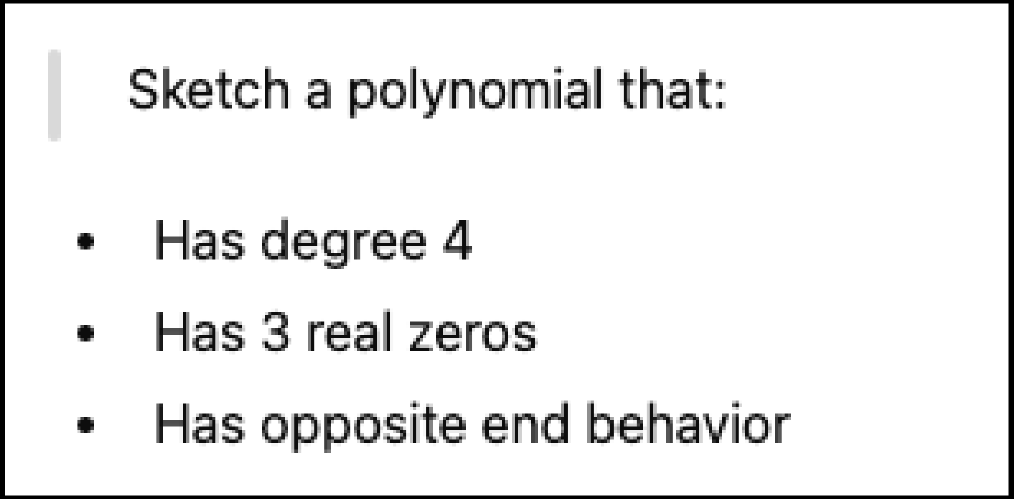
F3

Allen Peng	Aurora Yuan	Brittney Wei	Cynthia Liu	Erya Hu	Eva Gai	Felicity Pan	Fiona Ding	Freya Fan
Gordon Yao	Helios Luo	Honey Ruan	Iris Xie	Jasper Wu	Kaven Zhang	Kevin Gao	Leo Yang	Linger Li
Lydia Wei	Maggie Gao	Micheal Zhao	Ray Meng	Rose Jiang	Ross Ma	Roy Liu	Ryan Wang	Sky Bai
Star Su	Stella Xi							

F2

Angelina Wang	Azura Cheng	Bonnie Zhao	Cecilia Jia	Diana Li	Oscar Liu	Zavier Liu	Tom Ou	Mia Peng
Joshua Hui	Julius Lv	Justin Jia	Lucas Li	Lyra Zhang	Iris Shao	Dylan Suo	PeytonAson Wang	Struck Wang
Selene Hou	Serena Feng	Silas Lv	Simon Wang	Stella Sun	Yola Wang	Mark Xiao	Elsa Ye	Emily Zhang
Zinnia Dong	Flora Zhang							

TURN AND TALK



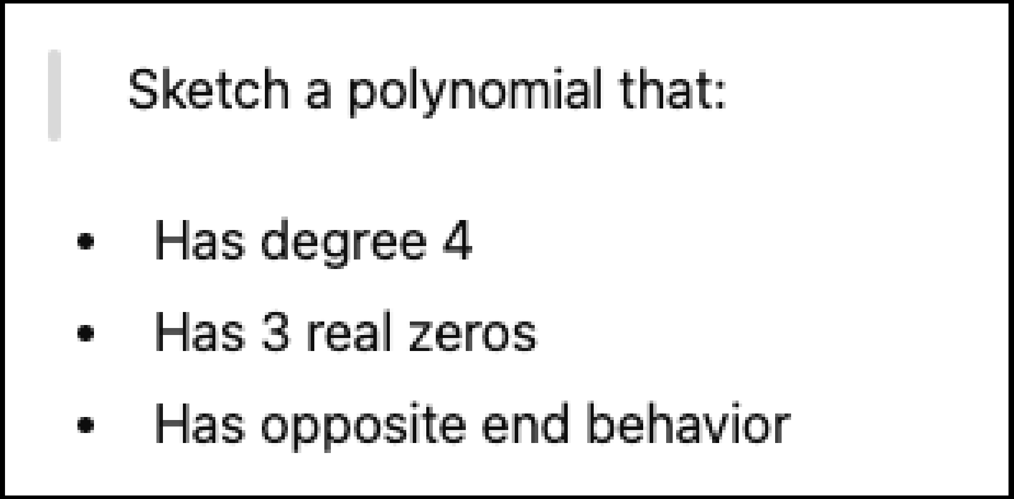
Sketch a polynomial that:

- Has degree 4
- Has 3 real zeros
- Has opposite end behavior

Try to satisfy as many of the conditions as possible.

Is it possible to have a polynomial that meets all three conditions?

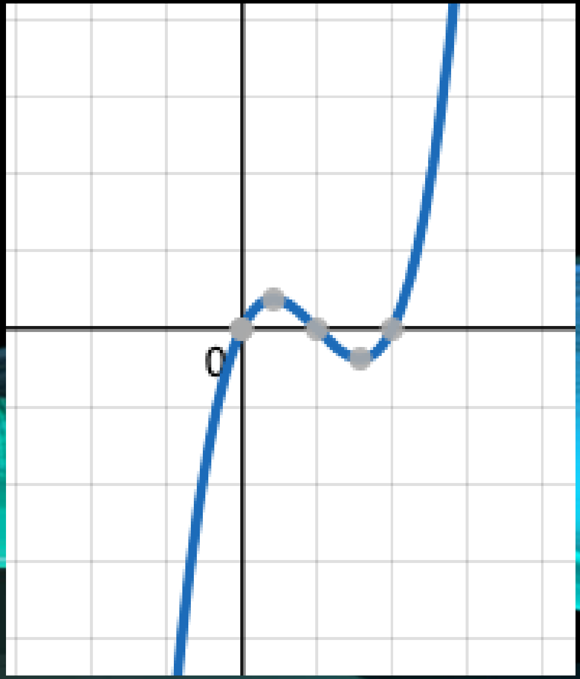
TURN AND TALK



Sketch a polynomial that:

- Has degree 4
- Has 3 real zeros
- Has opposite end behavior

Try to satisfy as many of the conditions as possible.



Is it possible to have a polynomial that meets all three conditions?

DIVIDING POLYNOMIALS

In this section, you will:

- Review synthetic, polynomial division
- Factor Theorem and Remainder Theorem
- Conjugate pairs

- Reminders
- Exam 19th January – polynomials and rational functions
- Presentations start 19th January – opportunity to respond to feedback such as APPLICATION. Everyone needs to speak for approximately 1 minute.

I SAY.   YOU SAY

1. **Polynomial** (多项式 - Duō xiàng shì)
2. **Division** (除法 - Chú fǎ)
3. **Dividend** (被除数 - Bèi chú shù)
4. **Divisor** (除数 - Chú shù)
5. **Quotient** (商 - Shāng)
6. **Remainder** (余数 - Yú shù)
7. **Long division** (长除法 - Cháng chú fǎ)
8. **Synthetic division** (合成除法 - Hé chéng chú fǎ)
9. **Factor** (因子 - Yīn zǐ)
10. **Term** (项 - Xiàng)

POLYNOMIAL DIVISION

$$\frac{x^3+4x^2-3x+10}{x+2}$$

2-04 DIVIDING POLYNOMIALS

- $\frac{x^3+4x^2-3x+10}{x+2}$

- $$\begin{array}{r} x^2+2x-7+\frac{24}{x+2} \\ x+2\overline{)x^3+4x^2-3x+10} \\ \underline{x^3+2x^2} \\ 2x^2-3x \\ \underline{2x^2+4x} \\ -7x+10 \\ \underline{-7x-14} \\ 24 \end{array}$$

Divide the 1st terms

Multiply

Subtract

POLYNOMIAL DIVISION

- $\frac{y^4+2y^2-y+5}{y^2-y+1}$

2-04 DIVIDING POLYNOMIALS

- Long Division
  - Done like long division with numbers
- $\frac{y^4+2y^2-y+5}{y^2-y+1}$

Divide the 1st terms

Multiply

Subtract

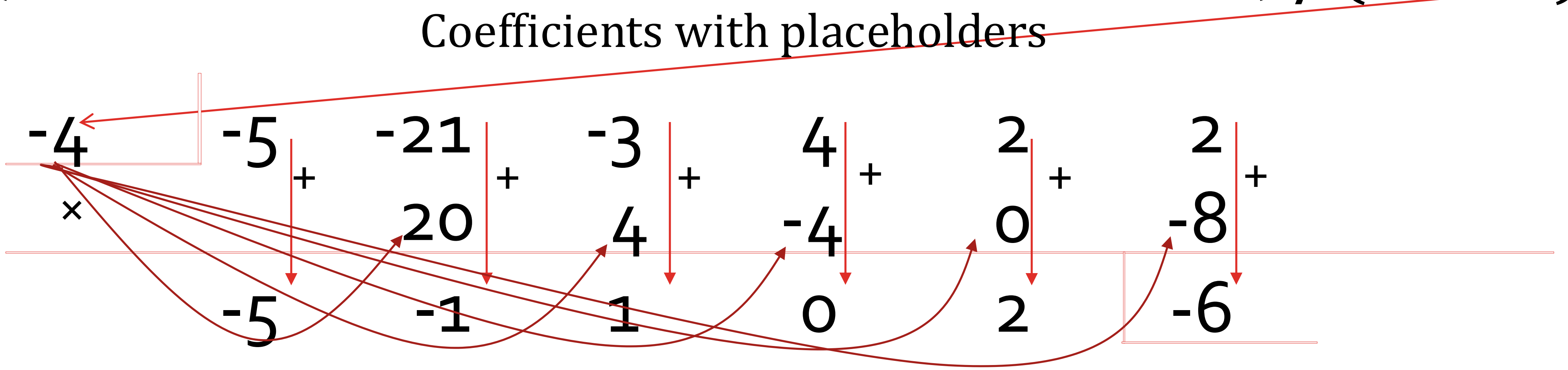
- $$\begin{array}{r}
\boxed{y^2}\boxed{+y}\boxed{+2}+\frac{3}{y^2-y+1} \\
\boxed{y^2-y+1}\overline{)y^4+0y^3+2y^2-y+5} \\
\underline{y^4-y^3+y^2} \\
y^3+y^2-y \\
\underline{y^3-y^2+y} \\
2y^2-2y+5 \\
\underline{2y^2-2y+2} \\
3
\end{array}$$

2-04 DIVIDING POLYNOMIALS

- Synthetic Division
  - Shortened form of long division for dividing by a **binomial**
  - Only when dividing by $(x - r)$

2-04 DIVIDING POLYNOMIALS

- Synthetic Division
  - $(-5x^5 - 21x^4 - 3x^3 + 4x^2 + 2x + 2)/(x + 4)$



Coefficients with placeholders

-4	-5	-21	-3	4	2	2
		20	4	-4	0	-8
	-5	-1	1	0	2	-6

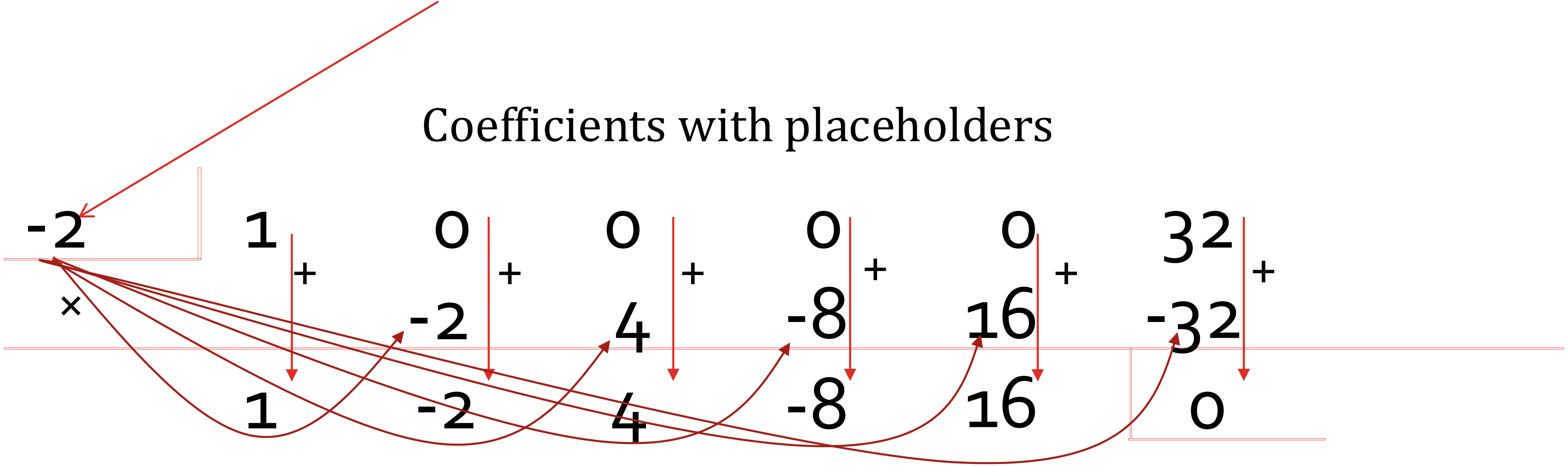
- The bottom row give the coefficients of the answer. This is called the depressed polynomial.
- The number in the box is the remainder.
- Start with an exponent one less than the original expression since you divided by *x*.

$$-5x^4 - x^3 + x^2 + 2 + \frac{-6}{x + 4}$$

2-04 DIVIDING POLYNOMIALS

- $(y^5 + 32)(y + 2)^{-1}$

Coefficients with placeholders



-2	1	0	0	0	0	32
		-2	4	-8	16	-32
	1	-2	4	-8	16	0

- The degree was 5, now the depressed polynomial is one less degree. It’s degree is 4.

$$y^4 - 2y^3 + 4y^2 - 8y + 16$$

2-04 DIVIDING POLYNOMIALS

- Factor Theorem
  - If $f(x)$ is divided by $(x - k)$ and remainder is 0, then $(x - k)$ is a factor of $f(x)$

2-04 DIVIDING POLYNOMIALS

- Show that $(x+3)$ is a factor of $x^3 - 19x - 30$. Then find the remaining factors.

$$(-3)^3 - 19(-3) - 30 = 0$$

$$(x+3)(x^2 - 3x - 10)$$

$$(x+3)(x-5)(x+2)$$

2-05 RATIONAL ZEROS OF POLYNOMIAL FUNCTIONS

In this section, you will:

- Evaluate a polynomial using the Remainder Theorem.
- Use the Factor Theorem to solve a polynomial equation.
- Use the Rational Zero Theorem to find rational zeros.

2-05 RATIONAL ZEROS OF POLYNOMIAL FUNCTIONS

- Remainder Theorem
  - If $f(x)$ is divided by $(x - k)$, then the remainder is $r = f(k)$

2-05 RATIONAL ZEROS OF POLYNOMIAL FUNCTIONS

- Use the remainder theorem to evaluate $f(x) = 4x^3 + 10x^2 - 3x - 8$ for $f(-1)$

Remainder = 1

2-05 RATIONAL ZEROS OF POLYNOMIAL FUNCTIONS

- Rational Zero Theorem
  - If polynomial $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$ The rational zeros are in the form $\frac{p}{q}$

    where p = factors of a_0

    q = factors of a_n

2-05 RATIONAL ZEROS OF POLYNOMIAL FUNCTIONS

- Find the rational zeros of $f(x) = x^3 - 5x^2 + 2x + 8$ given that $x + 1$ is a factor.

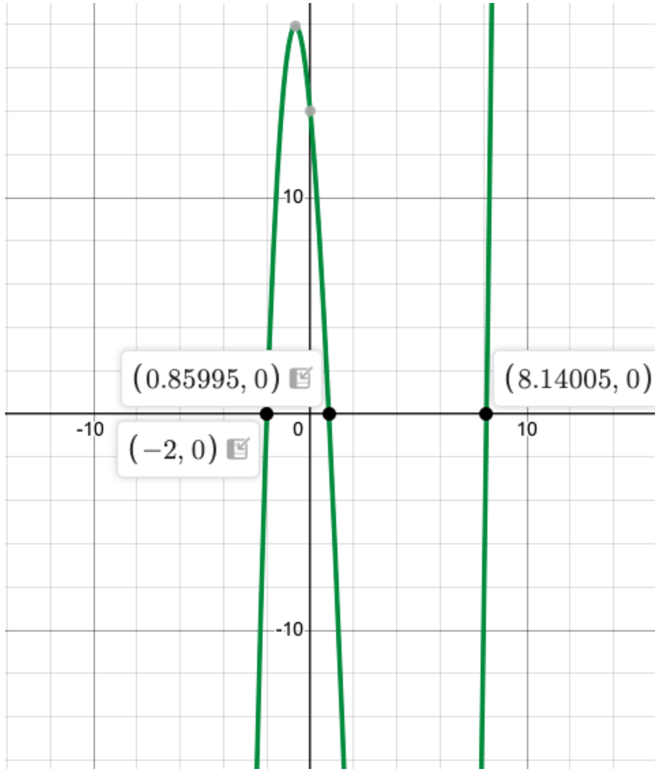
$$(x+1)\left(x^2-6x+8\right) = x^3-5x^2+2x+8$$

$$(x+1)(x-4)(x-2) = x^3-5x^2+2x+8$$

2-05 RATIONAL ZEROS OF POLYNOMIAL FUNCTIONS

- Find the real zeros of $f(x) = x^3 - 7x^2 - 11x + 14$ given that $x + 2$ is a factor.

$$(x+2)\left(x^2 - 9x + 7\right) = x^3 - 7x^2 - 11x + 14$$



2-06 ZEROS OF POLYNOMIAL FUNCTIONS

In this section, you will:

- Find zeros of a polynomial function.
- Use the Linear Factorization Theorem to find polynomials with given zeros.
- Use Descartes' Rule of Signs.

2-06 ZEROS OF POLYNOMIAL FUNCTIONS

- Fundamental Theorem of Algebra
  - If $f(x)$ is polynomial of degree n, then there is at least 1 zero
  - There are exactly n zeros
  - There are n linear factors (Linear Factorization Theorem)

2-06 ZEROS OF POLYNOMIAL FUNCTIONS

- Find all zeros of $f(x) = x^4 - 16$

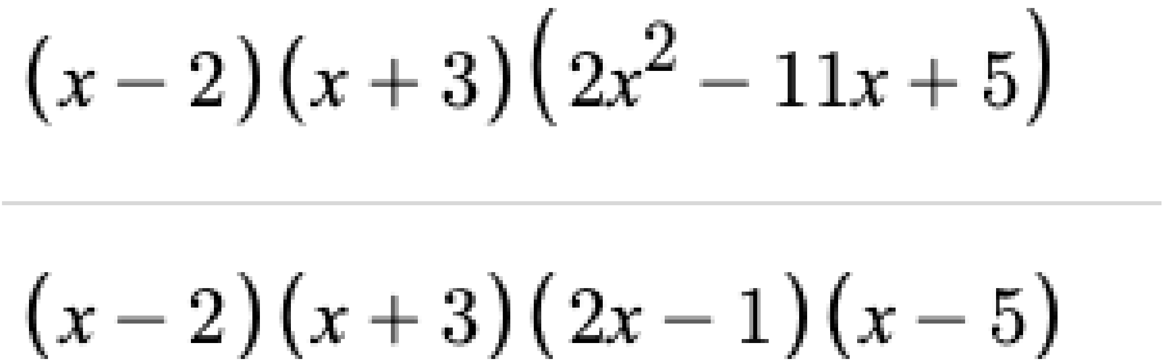
$$(x-2)(x+2)\left(x^2+4\right)$$

$$(x-2)(x+2)(x+2i)(x-2i)$$

2-06 ZEROS OF POLYNOMIAL FUNCTIONS

- Find all the zeros of

$$f(x) = 2x^4 - 9x^3 - 18x^2 + 71x - 30$$



$$(x-2)(x+3)\left(2x^2-11x+5\right)$$

$$(x-2)(x+3)(2x-1)(x-5)$$

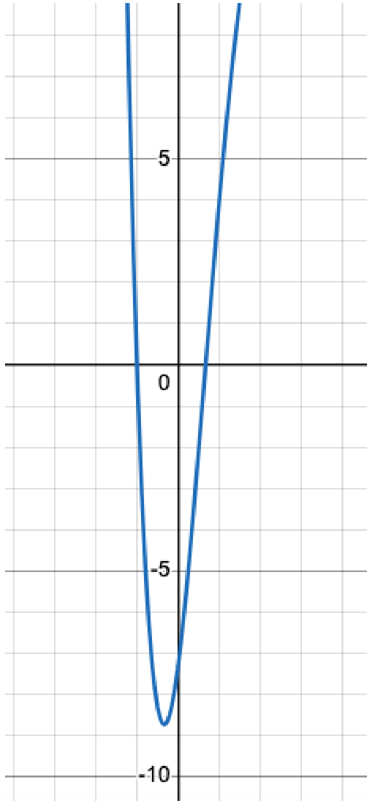
2-06 ZEROS OF POLYNOMIAL FUNCTIONS

- Complex Conjugate Theorem
  - If a complex number $a + bi$ is a zero, then $a - bi$ is also a zero.

2-06 ZEROS OF POLYNOMIAL FUNCTIONS

- Find a polynomial with real coefficients with zeros $\frac{2}{3}, -1, 3+\sqrt{2}i$

$$\left(x-\frac{2}{3}\right)(x+1)\left(x-\left(3+\sqrt{2}\,i\right)\right)\left(x-\left(3-\sqrt{2}\,i\right)\right)$$



TRUE OR FALSE

- If the divisor is a factor of the dividend, the remainder of the division is zero.

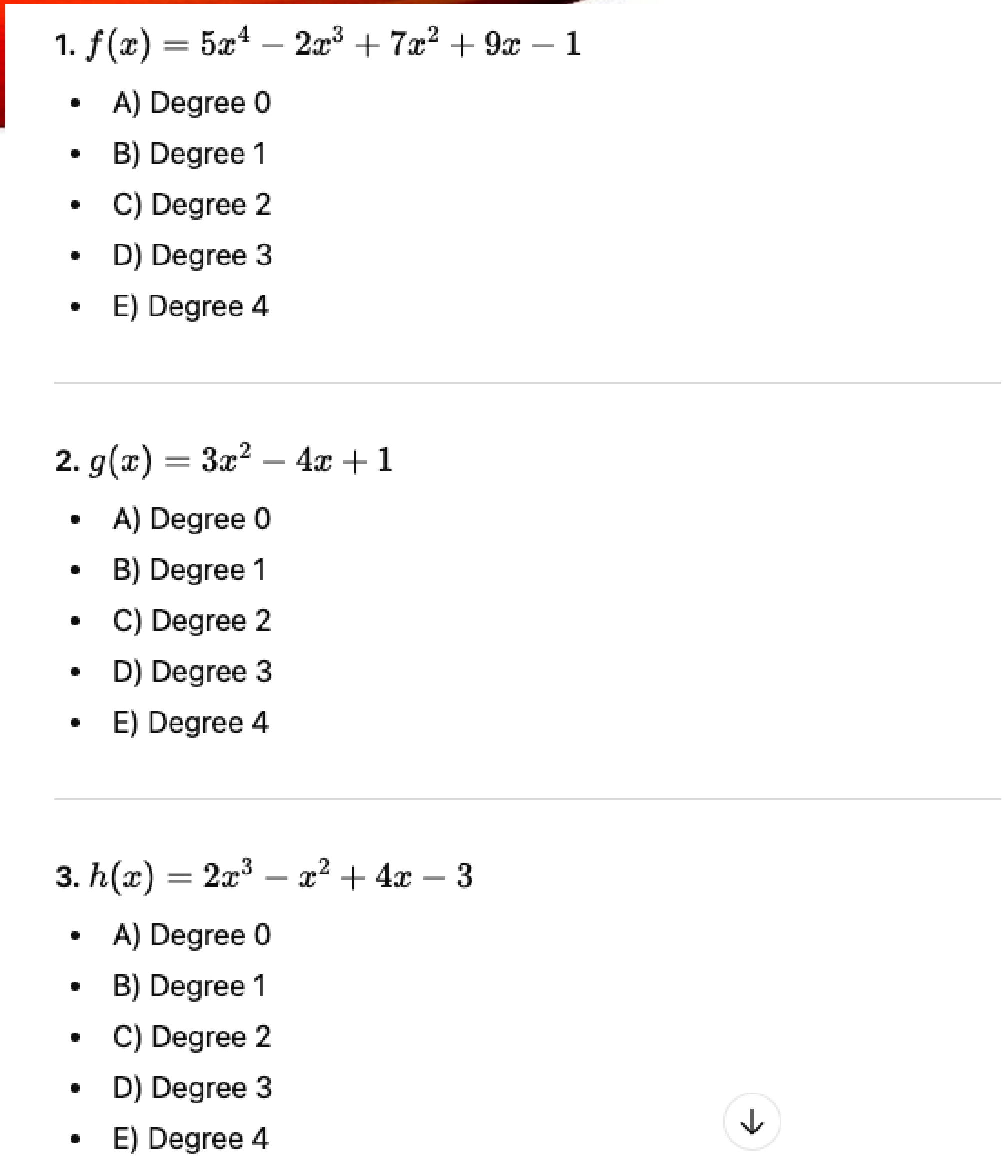
TRUE

TRUE OR FALSE

- If a polynomial has a real coefficient, then its complex roots must occur in conjugate pairs..

TRUE

1. **Linear Polynomial** — [redacted]
   [redacted]
2. **Quadratic Polynomial** — [redacted]
   [redacted]
3. **Cubic Polynomial** — [redacted]
   [redacted]
4. **Quartic Polynomial** — [redacted]
   [redacted]
5. **Quintic Polynomial** — [redacted]
   - [redacted]
6. **Constant Polynomial** — [redacted]
   [redacted]



1. $f(x) = 5x^4 - 2x^3 + 7x^2 + 9x - 1$

- A) Degree 0
- B) Degree 1
- C) Degree 2
- D) Degree 3
- E) Degree 4

2. $g(x) = 3x^2 - 4x + 1$

- A) Degree 0
- B) Degree 1
- C) Degree 2
- D) Degree 3
- E) Degree 4

3. $h(x) = 2x^3 - x^2 + 4x - 3$

- A) Degree 0
- B) Degree 1
- C) Degree 2
- D) Degree 3
- E) Degree 4