UNIT 1 KNOWLEDGE – CALCULUS 12 – LIMITS AND CONTINUITY

- 1.1 Introducing Calculus: Can Change Occur at an Instant? ✓
- 1.2 Defining Limits and Using Limit Notation ✓
- 1.3 Estimating Limit Values from Graphs ✓
- 1.4 Estimating Limit Values from Tables
- 1.5 Determining Limits Using Algebraic Properties of Limits
- 1.6 Determining Limits Using Algebraic Manipulation
- 1.7 Selecting Procedures for Determining Limits
- 1.8 Determining Limits Using the Squeeze Theorem
- 1.9 Connecting Multiple Representations of Limits
- 1.10 Exploring Types of Discontinuities
- 1.11 Defining Continuity at a Point
- 1.12 Confirming Continuity over an Interval
- 1.13 Removing Discontinuities
- 1.14 Connecting Infinite Limits and Vertical Asymptotes
- 1.15 Connecting Limits at Infinity and Horizontal Asymptotes
- 1.16 Working with the Intermediate Value Theorem (IVT)

Turn and Talk

$$\lim_{x \to \infty} \frac{3x^2 + 5x}{2x^2 - x}$$

$$\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{x+1} \right)$$

$$\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4}$$

Turn and Talk

$$\lim_{x\to\infty} \frac{3x^2+5x}{2x^2-x} = \frac{3}{2}$$

$$\lim_{x\to 0} \left(\frac{1}{x} - \frac{1}{x+1}\right)$$ → does not exist

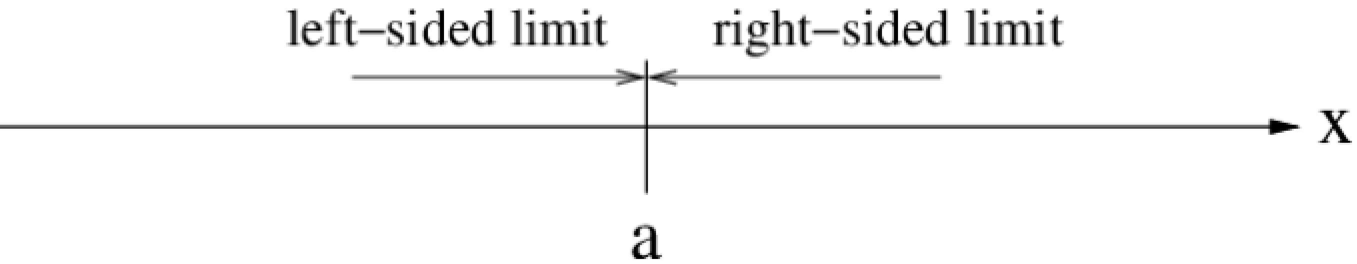
$$\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4} = \frac{1}{4}$$

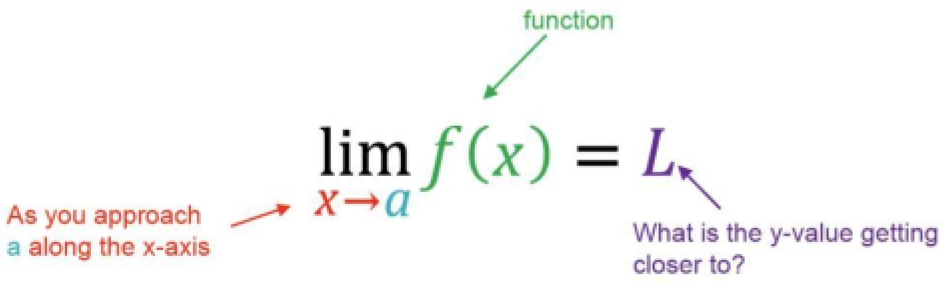
- **Discrete（离散的）** – individual, separate values
- **Continuous（连续的）** – unbroken, values can take any number in a range
- **Linear（线性的）** – forms a straight line; proportional relationship
- **Unbounded（无界的）** – not limited, extends infinitely
- **Limit（极限）** – the value a function or sequence approaches
- **Approaches to（趋向于）** – getting closer to a particular value

What Will We Learn?

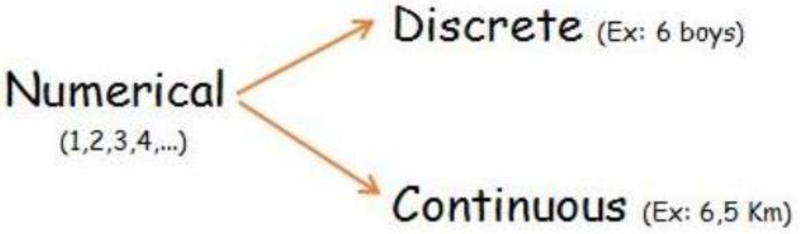
- Numerical data (tables) can often provide insight into likely behavior of a function

- Tables do not give a COMPLETE picture and can lead to false assumptions and errors.





Key Terms



Categorical Variable

A variable that takes values that are category names or labels.

Quantitative Variable

A variable that takes numerical values for a measured or counted quantity.

A **discrete** variable can take on a countable number of values (with gaps)

Examples: number of siblings e.g, 0,1,2,3,4 nothing in between like 2.5, 2.7.

When you are counting something!

A **continuous** variable can take on infinitely many values, but those values cannot be counted (no gaps)

Examples: height e.g, 61.5 inches, 62 inches, 67.8 inches

When you are measuring something!

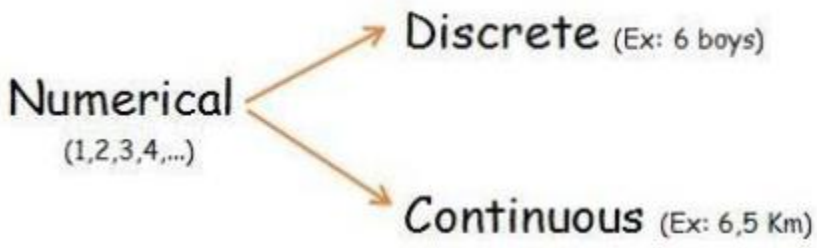
Classify the following variables as either *discrete* or *continuous*:

- The number of students in a classroom.
- The height of a sunflower in cm.
- The number of goals scored in a football match.
- The time (in seconds) it takes to run 100 m.
- The number of cars passing a junction in an hour.
- The weight of apples in kg.
- The number of pages in a book.
- The temperature outside in °C.

1. **Number of students → Discrete** (can only take whole numbers)
2. **Height of a sunflower → Continuous** (can take any value within a range)
3. **Number of goals scored → Discrete** (whole numbers only)
4. **Time to run 100 m → Continuous** (any positive real value)
5. **Number of cars passing a junction → Discrete** (whole numbers)
6. **Weight of apples → Continuous** (can take any value within a range)
7. **Number of pages in a book → Discrete** (whole numbers only)
8. **Temperature outside → Continuous** (any value in a range)

Why should we be careful with some tables of values *WITH DISCRETE DATA*?

Selected values of the function $f(x)$ are given in the table below. Which of the following statements about $f(x)$ must be true?



I. $\lim_{x\to 5} f(x) = 4$

II. $f(6) > f(5)$

III. $f(x) = 2.5$ somewhere on the interval (1,3)

IV. $f(x)$ is linear

V. $f(x)$ is increasing

x	1	3	5	7	9
$f(x)$	2	3	4	5	6

Selected values of the function $f(x)$ are given in the table below. Which of the following statements about $f(x)$ must be true?

$\lim_{x \to 5} f(x) = 4$
$f(6) > f(5)$
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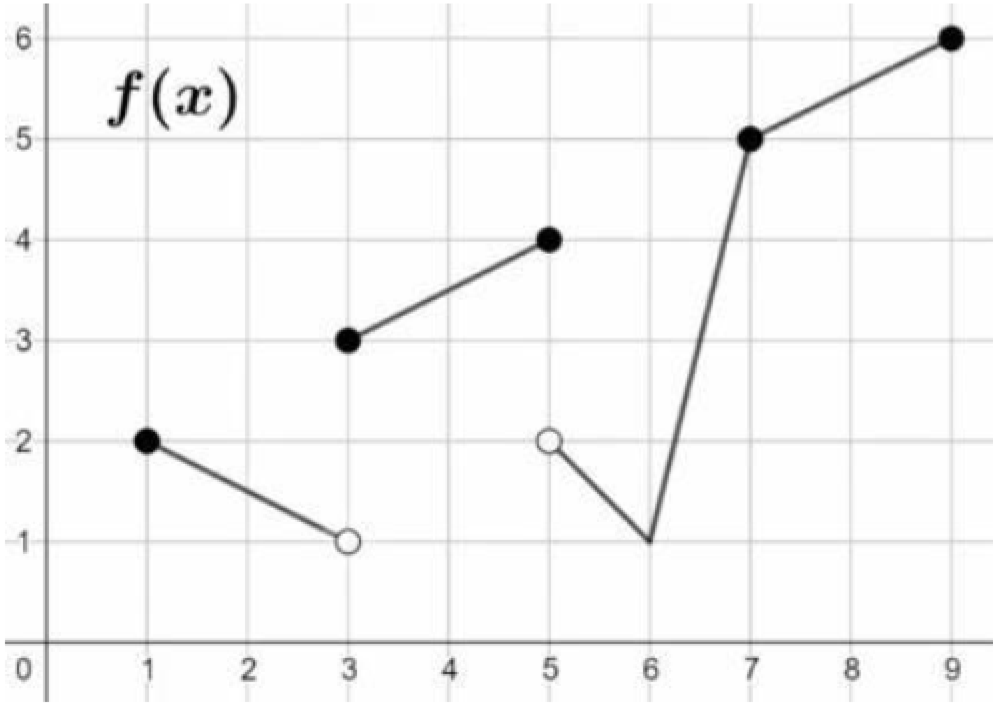
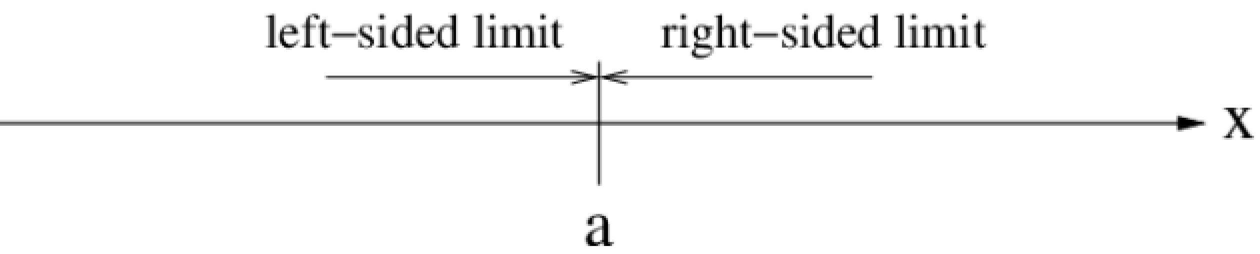
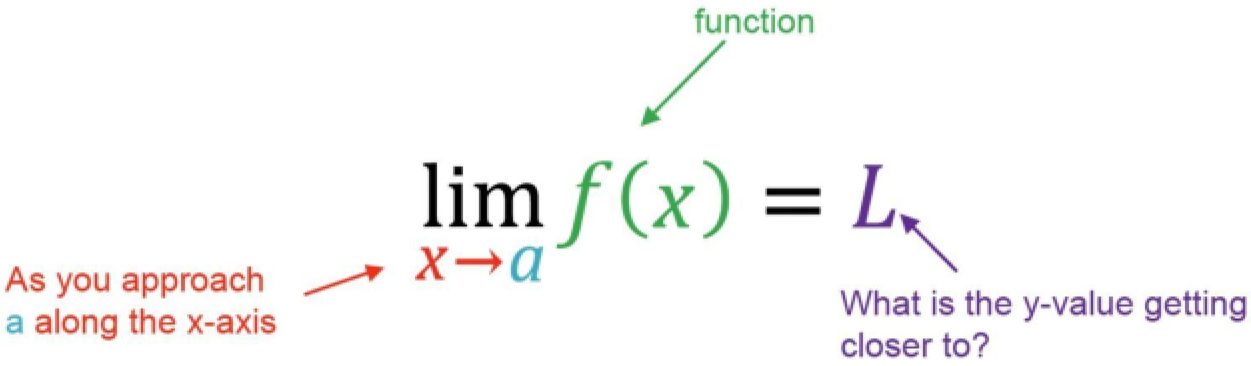


Let’s see an example!





Estimating Limits Using Tables

x	1.923	1.976	1.991	1.997	2.002	2.011	2.035	2.063
$h(x)$	3.89	3.94	3.97	3.98	8.01	8.04	8.07	8.11

Selected values of the function $h(x)$ are given in the table above. Which of the following statements is **supported** by the data in the table?

A. $\lim_{x\to 2} h(x) = 4$

B. $\lim_{x\to 2} h(x) = 8$

C. $h(2)$ is undefined

D. $\lim_{x\to 2} h(x)$ does not exist

Estimating Limits Using Tables

Which of the following tables best **supports** the statement below?

$$\lim_{x \to 3^-} g(x) \neq \lim_{x \to 3^+} g(x)$$

A.

x	2.88	2.93	2.98	3.01	3.05	3.13
$g(x)$	4.22	4.07	4.01	4.02	4.05	4.16

B.

x	2.88	2.93	2.98	3.01	3.05	3.13
$g(x)$	5.1	5.04	5.02	17.99	17.93	17.86

C.

x	5.1	5.04	5.02	17.99	17.93	17.86
$g(x)$	2.88	2.93	2.98	3.01	3.05	3.13

D.

x	2.88	2.93	2.98	3.01	3.05	3.13
$g(x)$	6	51	500	500	51	6

Estimating Limits Using Tables

x	9.89	9.92	9.97	10.01	10.06	10.10
$g(x)$	4	79	?	?	112	8

Selected values of the function g(x) are given in the table above. Which of the following missing values in the table best **support** the statement below?

$$\lim_{x \to 10} g(x) \text{ does not exist due to unbounded behavior}$$

A. $81, 113$

B. $92, 92$

C. $1157, 1773$

D. undefined, undefined

Behavior	Jump	Unbounded (goes all the way to ∞)	Oscillating (bounces up& down & not approaching any specific no.)

Next-> What Will We Learn?

- We’ll do a short activity.

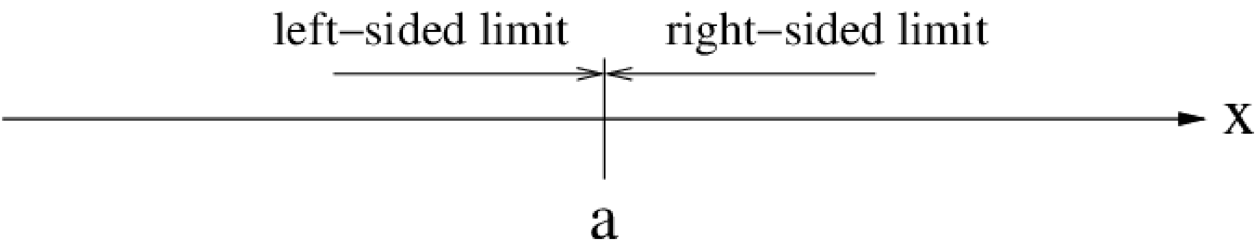
"Limits in 10"

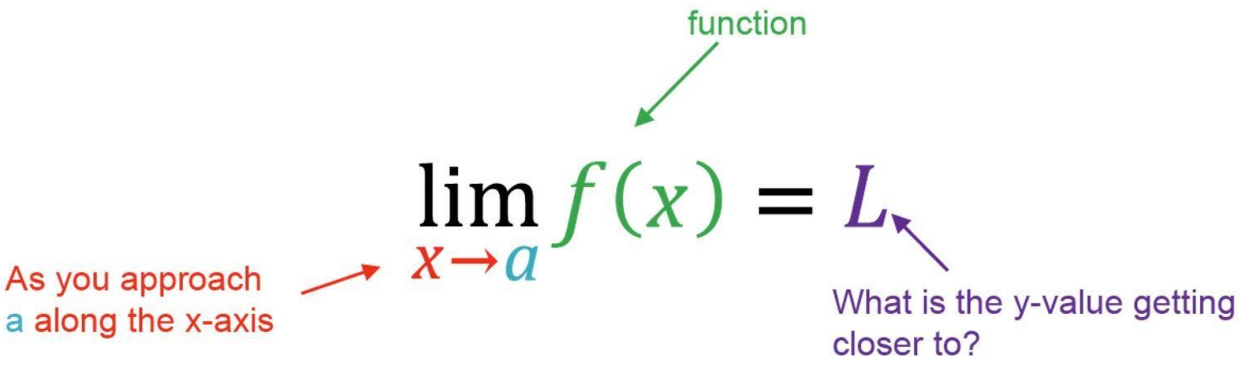
- In this activity, we will work through some problems. The answer to each problem is a whole no. 0-9.

- After answering each problem, cross out the corresponding no. at the top of the handout (The nos. 0-9 are boxed).

- If you answer all problems correctly, you will use each no. exactly ONCE & have each no. crossed off!

Let’s see an example!





"Limits in 10"

0	~~1~~	2	3	4	5	6	7	~~8~~	9

x	1.91	1.96	1.98	2	2.01	2.05	2.11
$f(x)$	7.91	7.96	7.99	?	3.99	3.94	3.89

Selected values of the function $f(x)$ are given in the table above. Use the data from the table for Problems A - B

A. The data in the table best supports that $\lim_{x\to 2^-} f(x) =$

B. The data in the table supports that $\lim f(x)$ ...

1. Does not exist because the limit from the left does not equal the limit from the right
2. does not exist because of unbounded behavior.
3. does not exist because of oscillating behavior.
4. exists

Key Takeaways

Limit Notation	Left: $\lim_{x \to c^-} f(x)$	Right: $\lim_{x \to c^+} f(x)$
A limit exists when...	$\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$	
A limit does NOT exist when...	$\lim_{x \to c^-} f(x) \neq \lim_{x \to c^+} f(x)$	

True or False

Indeterminate forms like 0/0 or ∞/∞ always indicate that the limit does not exist.

False

True or False

A table can sometimes suggest a limit even if the function is undefined at the point of interest.

True

True or False

If the table shows that f(x) grows larger and larger in magnitude as x approaches a point, it can indicate a vertical asymptote.

True