Turn and Talk

1. Obtuse Angle in the Second Quadrant

- Find the values of $\sin 120°$, $\cos 120°$, and $\tan 120°$.

2. Obtuse Angle in the Second Quadrant

- Find the values of $\sin 135°$, $\cos 135°$, and $\tan 135°$.

3. Reflex Angle in the Third Quadrant

- Find the values of $\sin 210°$, $\cos 210°$, and $\tan 210°$.

1. 120°
   - $\sin 120^\circ = \frac{\sqrt{3}}{2}$
   - $\cos 120^\circ = -\frac{1}{2}$
   - $\tan 120^\circ = -\sqrt{3}$
2. 135°
   - $\sin 135^\circ = \frac{\sqrt{2}}{2}$
   - $\cos 135^\circ = -\frac{\sqrt{2}}{2}$
   - $\tan 135^\circ = -1$
3. 210°
   - $\sin 210^\circ = -\frac{1}{2}$
   - $\cos 210^\circ = -\frac{\sqrt{3}}{2}$
   - $\tan 210^\circ = \frac{\sqrt{3}}{3}$

Inverse Function

- Inverse function — 反函数
- One-to-one function — 一一对应函数 / 单射函数
- Horizontal line test — 水平线测试
- Inverse notation — 反函数表示法
- Domain — 定义域
- Range — 值域
- Domain–range switch — 定义域和值域互换
- Reflection over $y = x$ — 关于直线 $y = x$ 的对称
- Inverse exists — 反函数存在
- Restrict the domain — 限制定义域

Reminders:

- Q2 Gradebook is closed
- Desmos Project is now due 3rd March

Inverse Functions

DEFINITION

Given a function f with domain D and range R, its **inverse function** (if it exists) is the function f^{-1} with domain R and range D such that $f^{-1}(y) = x$ if $f(x) = y$. In other words, for a function f and its inverse f^{-1},

$$f^{-1}(f(x)) = x \text{ for all } x \text{ in } D, \text{and } f\left(f^{-1}(y)\right) = y \text{ for all } y \text{ in } R. \tag{1.11}$$

https://openstax.org/books/calculus-volume-1/pages/1-1-review-of-functions

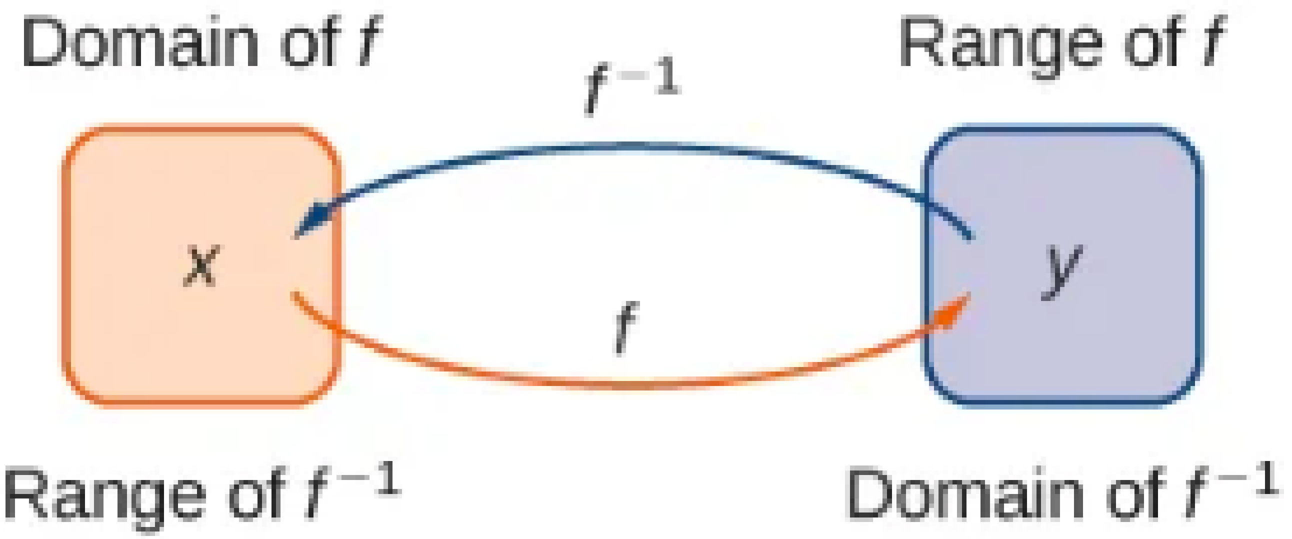


Figure 1.37 Given a function f and its inverse f^{-1}, $f^{-1}(y) = x$ if and only if $f(x) = y$. The range of f becomes the domain of f^{-1} and the domain of f becomes the range of f^{-1}.

DEFINITION

We say a f is a **one-to-one function** if $f\left(x_1\right) \neq f\left(x_2\right)$ when $x_1 \neq x_2$.

RULE: HORIZONTAL LINE TEST

A function f is one-to-one if and only if every horizontal line intersects the graph of f no more than once.

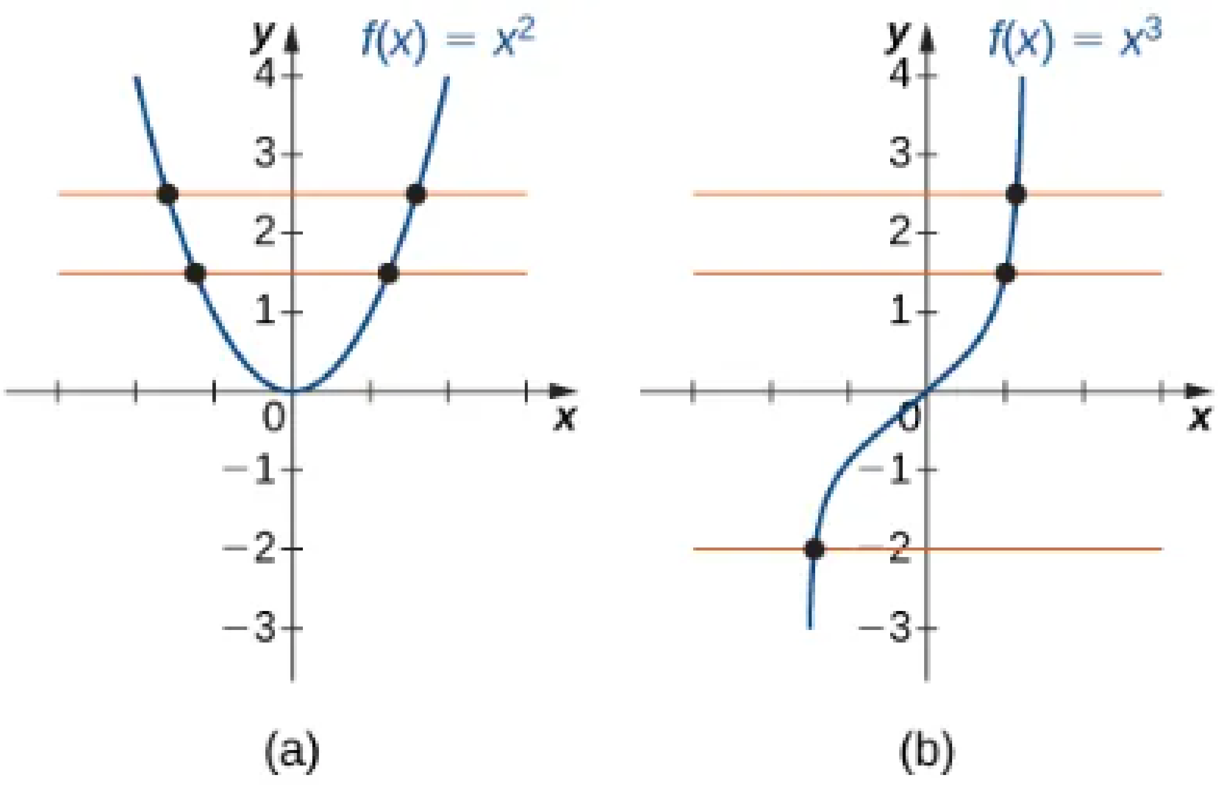
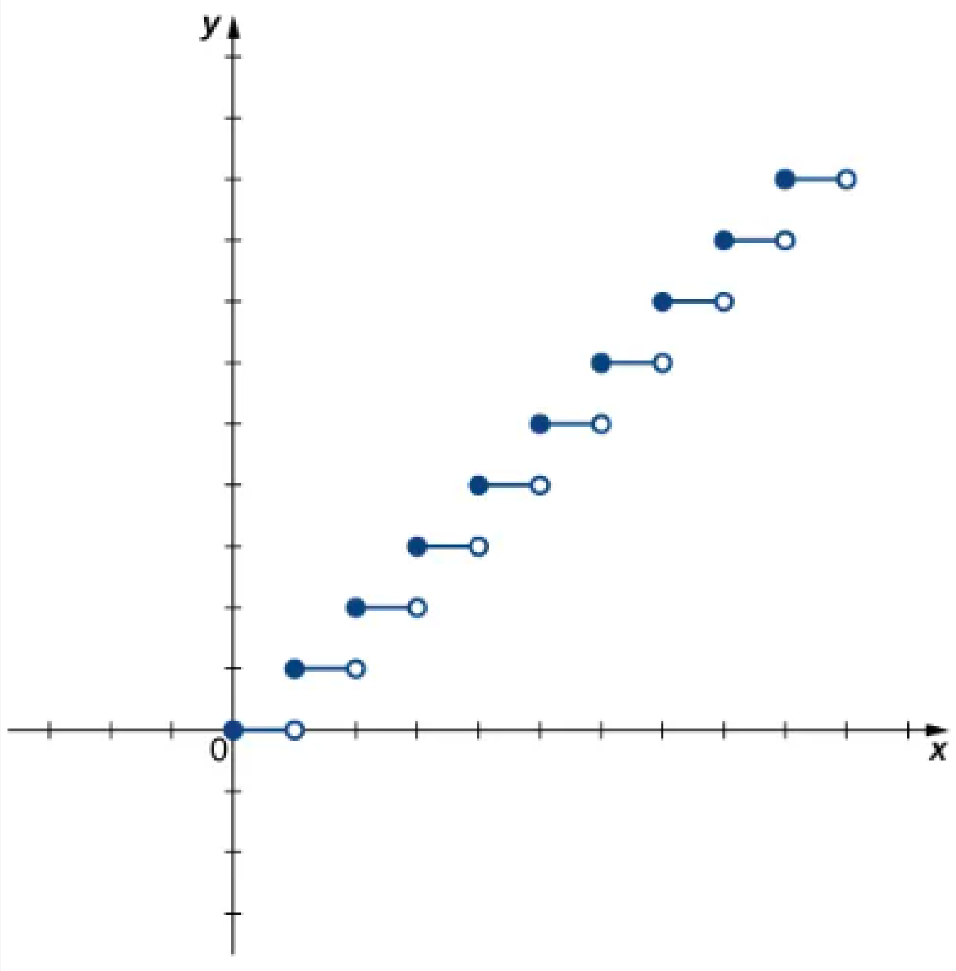


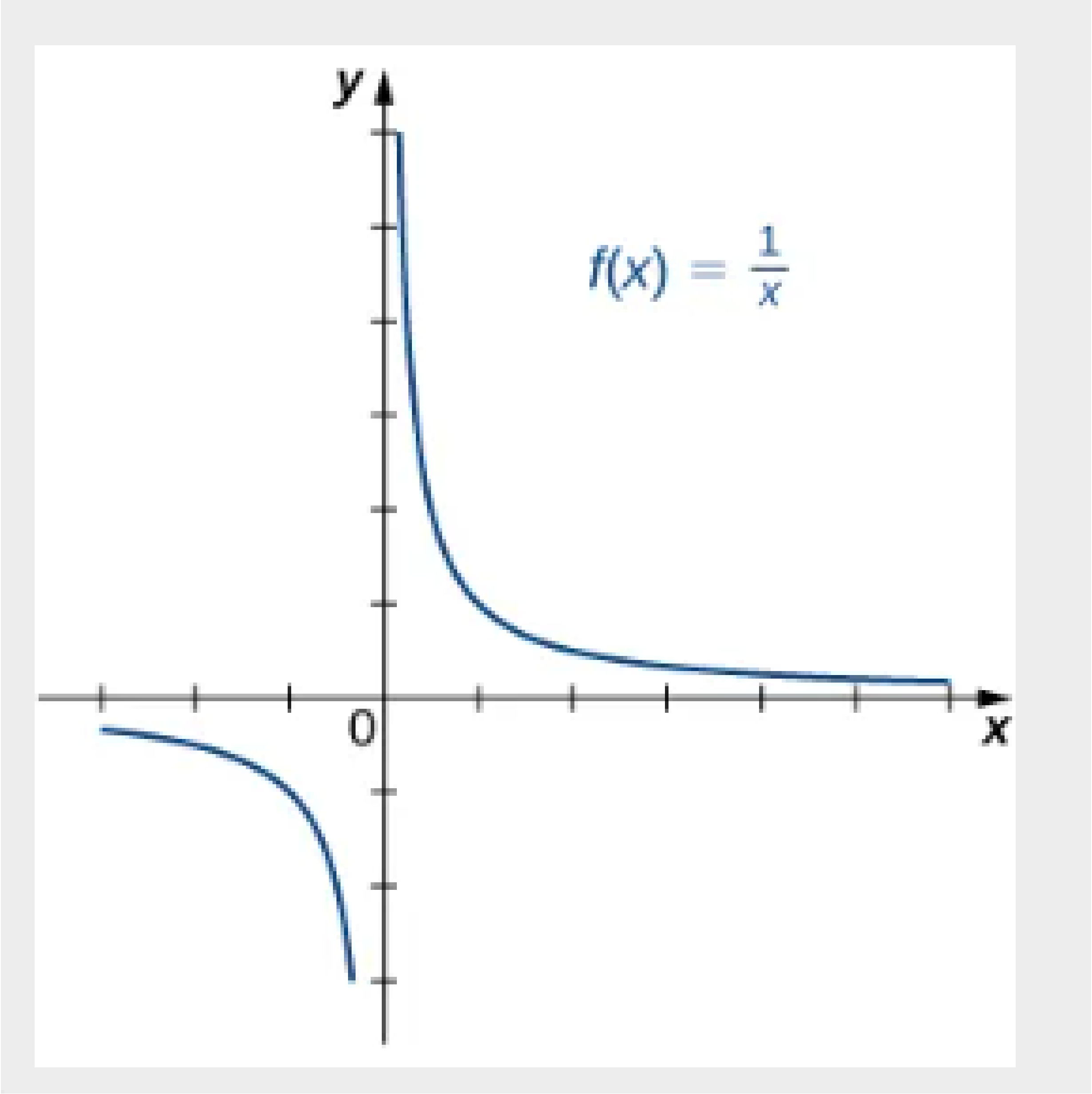
Figure 1.38 (a) The function $f(x) = x^2$ is not one-to-one because it fails the horizontal line test. (b) The function $f(x) = x^3$ is one-to-one because it passes the horizontal line test.

Determining Whether a Function Is One-to-One

For each of the following functions, use the horizontal line test to determine whether it is one-to-one.



FALSE



TRUE

PROBLEM-SOLVING STRATEGY

Finding an Inverse Function

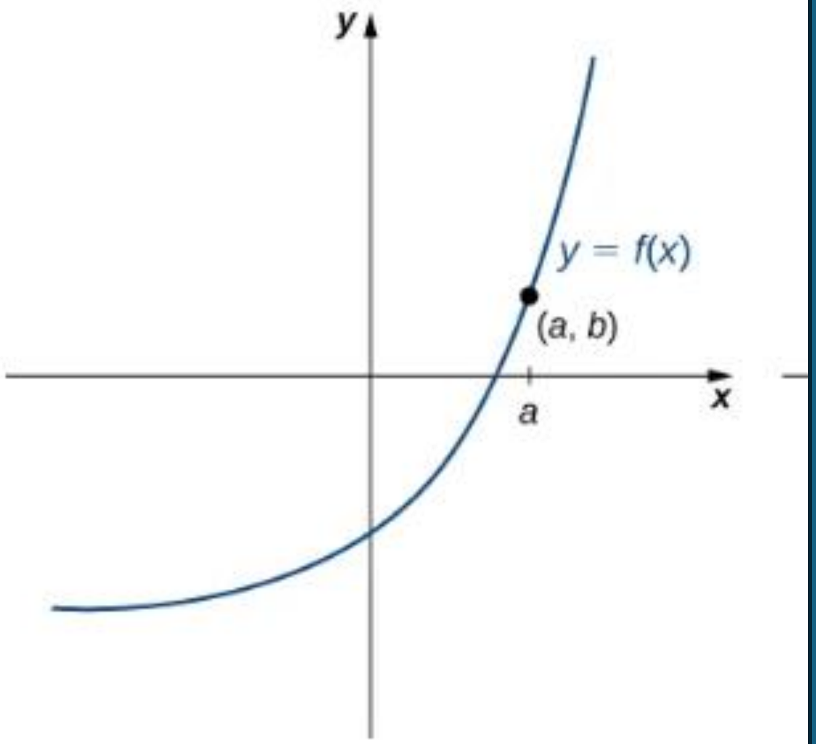
1. Solve the equation $y = f(x)$ for x.
2. Interchange the variables x and y and write $y = f^{-1}(x)$.

Find the inverse for the function $f(x) = 3x - 4$. State the domain and range of the inverse function. Verify that $f^{-1}(f(x)) = x$.

Therefore, $f^{-1}(x) = \frac{1}{3}x + \frac{4}{3}$.

Since the domain of f is $(-\infty, \infty)$, the range of f^{-1} is $(-\infty, \infty)$. Since the range of f is $(-\infty, \infty)$, the domain of f^{-1} is $(-\infty, \infty)$.

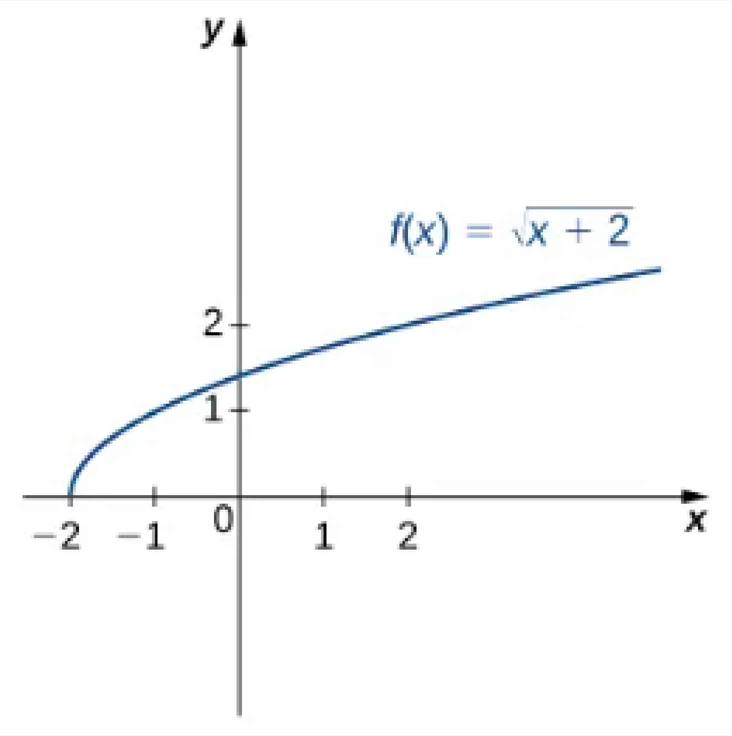
Graphing Inverse Functions



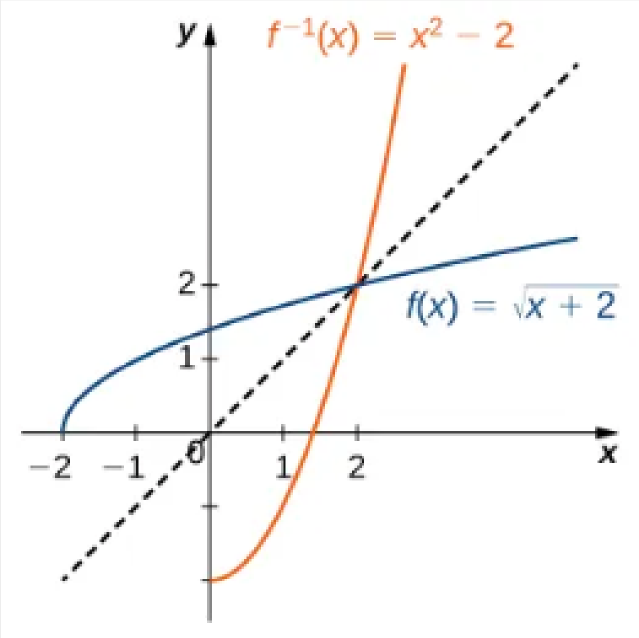
(a)

Sketching Graphs of Inverse Functions

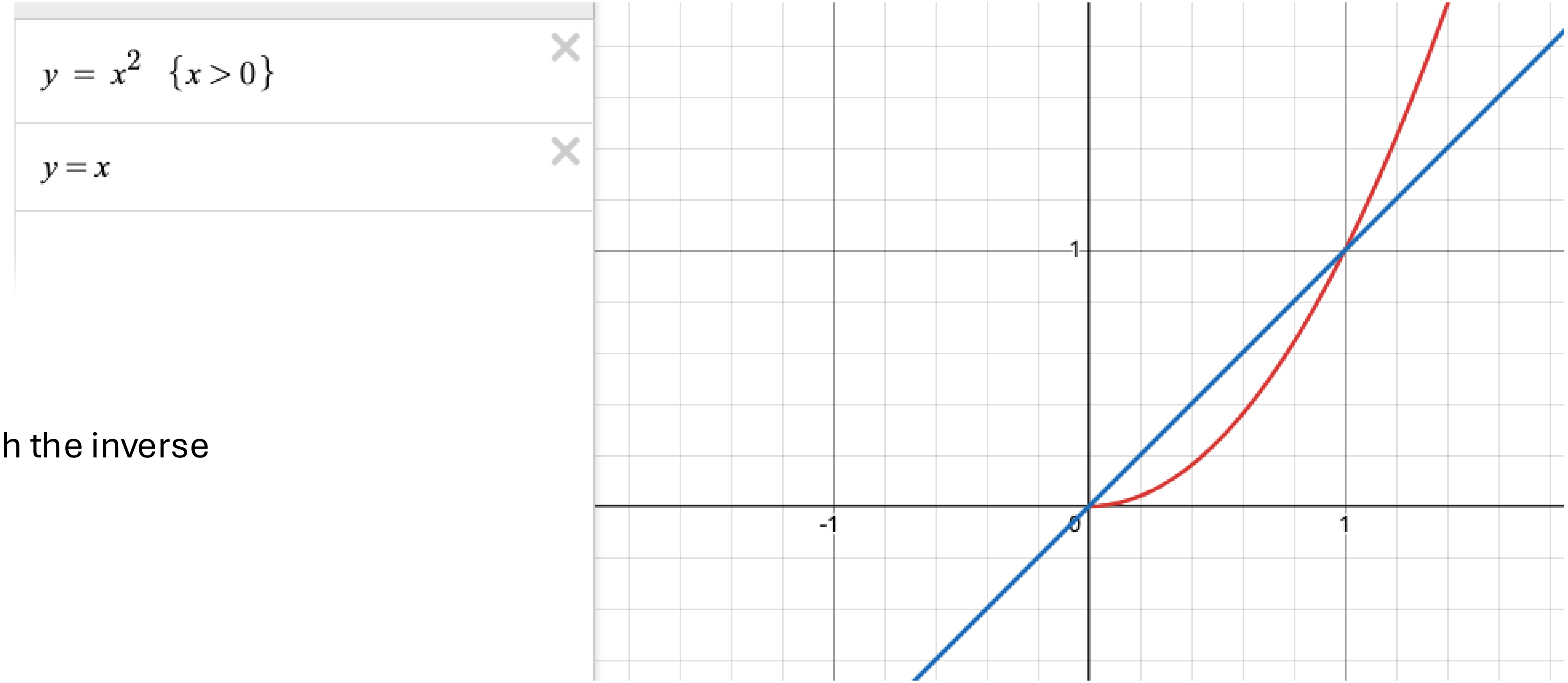
For the graph of f in the following image, sketch a graph of f^{-1} by sketching the line $y = x$ and using symmetry. Identify the domain and range of f^{-1}.



Reflect the graph about the line $y = x$. The domain of f^{-1} is $[0, \infty)$. The range of f^{-1} is $[-2, \infty)$. By using the preceding strategy for finding inverse functions, we can verify that the inverse function is $f^{-1}(x) = x^2 - 2$, as shown in the graph.



Restricting Domains. - I DO

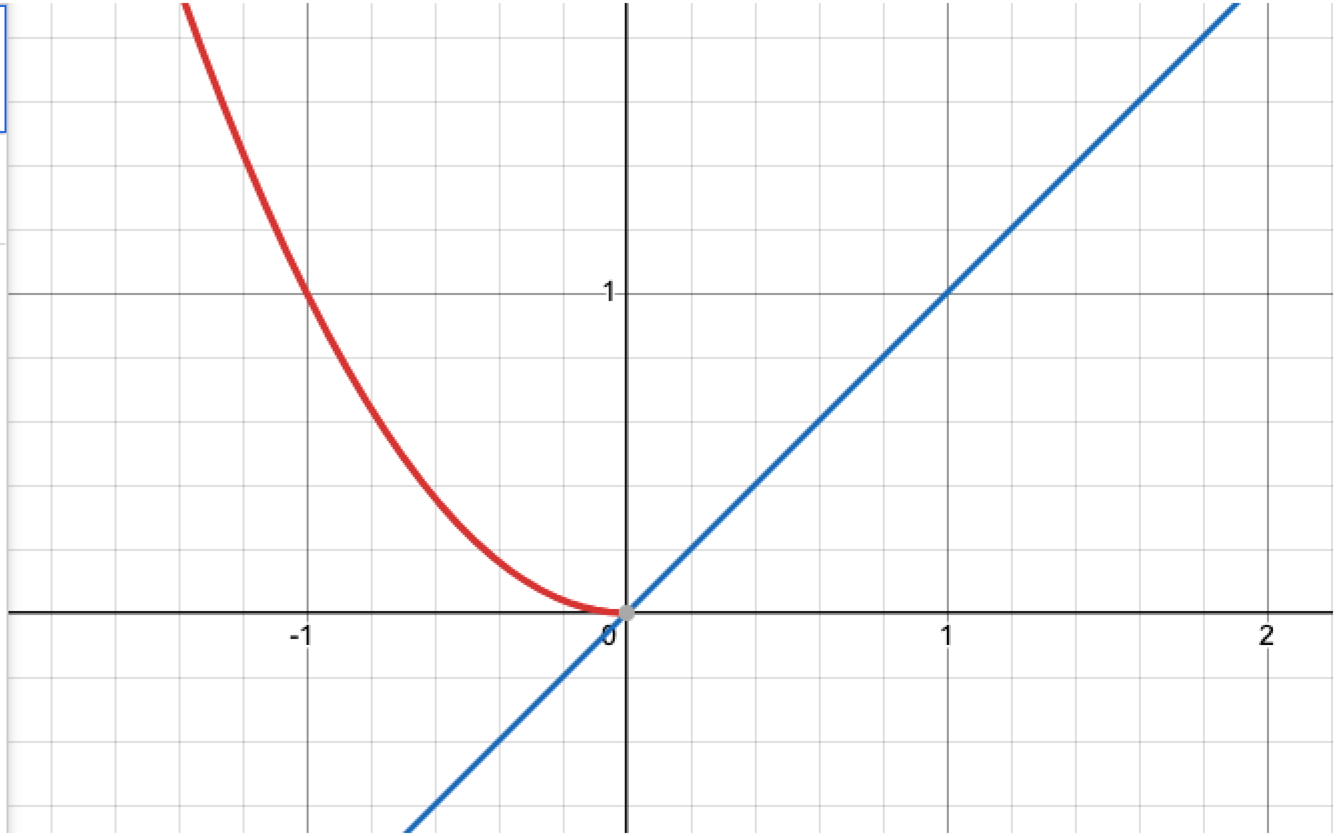


Sketch the inverse

Restricting Domains. - YOU DO

$y = x^2 \quad \{x < 0\}$

$y = x$



Sketch the inverse

Restricting Domains

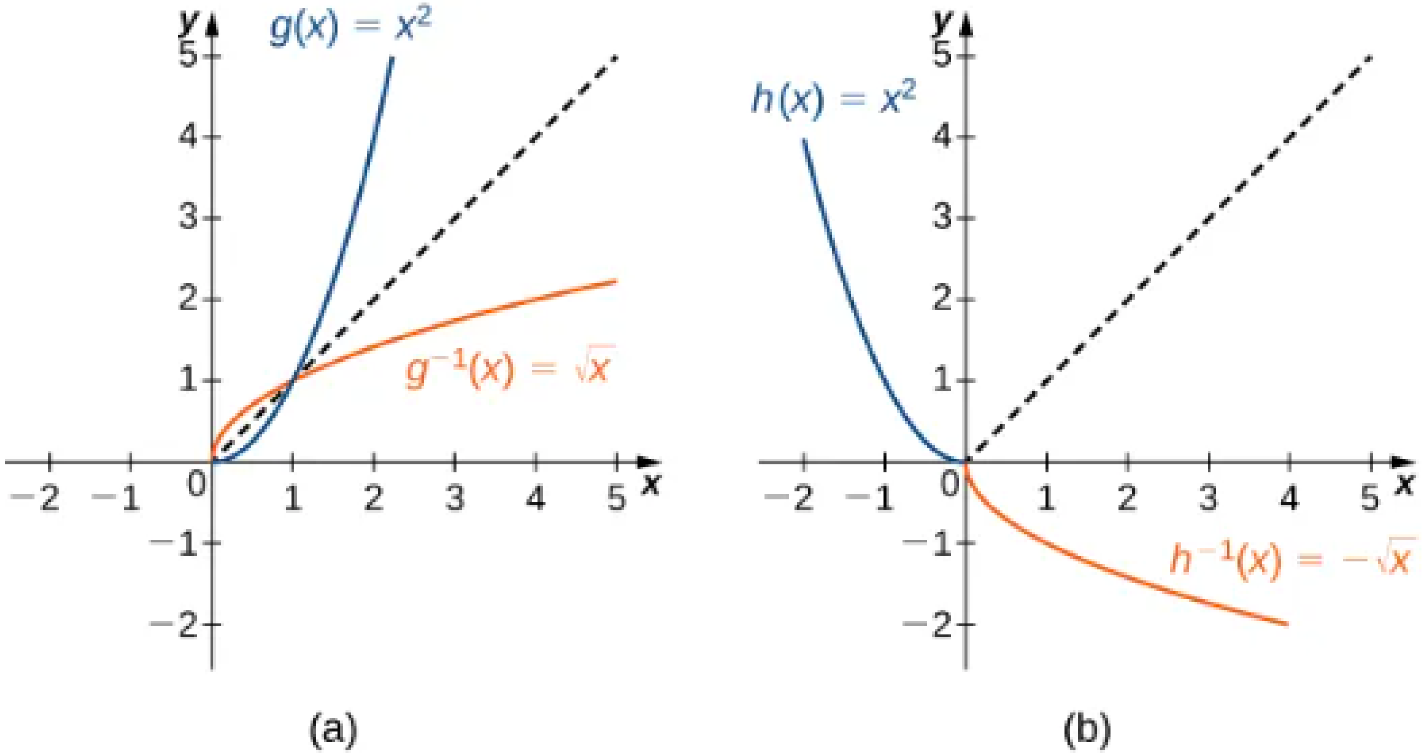
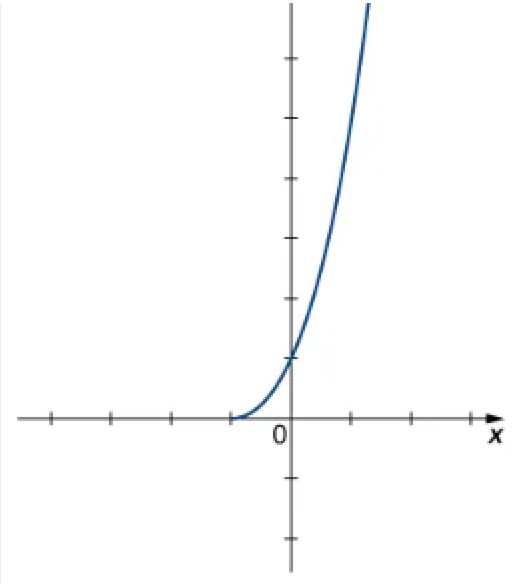


Figure 1.40 (a) For $g(x) = x^2$ restricted to $[0, \infty)$, $g^{-1}(x) = \sqrt{x}$. (b) For $h(x) = x^2$ restricted to $(-\infty, 0]$, $h^{-1}(x) = -\sqrt{x}$.

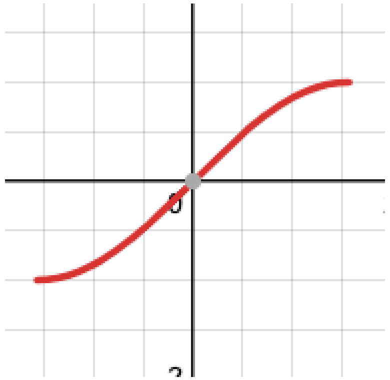
Consider the function $f(x) = (x + 1)^2$.

a. Sketch the graph of f and use the horizontal line test to show that f is not one-to-one.
b. Show that f is one-to-one on the restricted domain $[-1, \infty)$. Determine the domain and range for the inverse of f on this restricted domain and find a formula for f^{-1}.

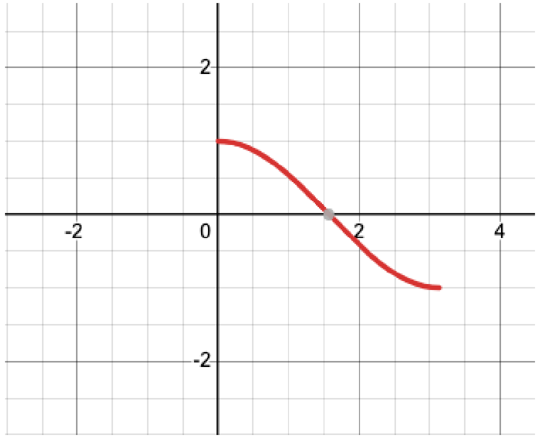


The domain and range of f^{-1} are given by the range and domain of f, respectively. Therefore, the domain of f^{-1} is $[0, \infty)$ and the range of f^{-1} is $[-1, \infty)$. To find a formula for f^{-1}, solve the equation $y = (x + 1)^2$ for x. If $y = (x + 1)^2$, then $x = -1 \pm \sqrt{y}$. Since we are restricting the domain to the interval where $x \geq -1$, we need $\pm\sqrt{y} \geq 0$. Therefore, $x = -1 + \sqrt{y}$. Interchanging x and y, we write $y = -1 + \sqrt{x}$ and conclude that $f^{-1}(x) = -1 + \sqrt{x}$.

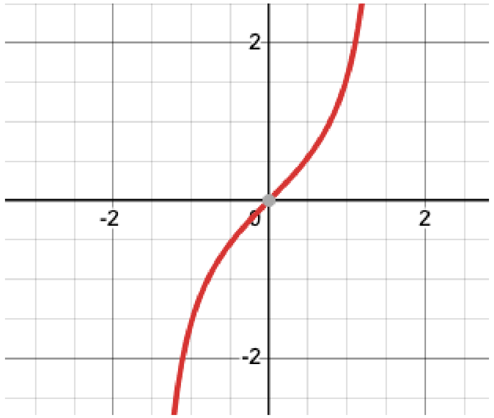
Inverse Trigonometric Functions



sin(x)



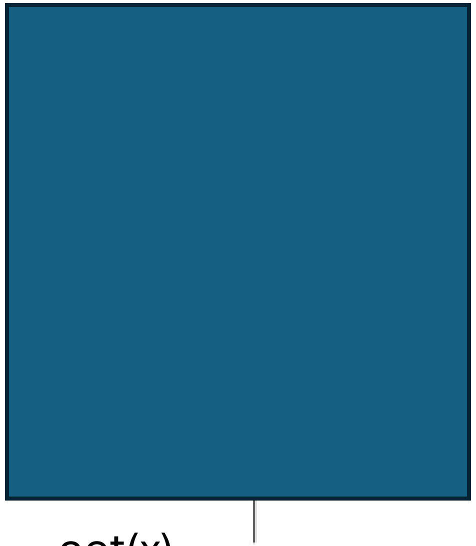
cos(x)



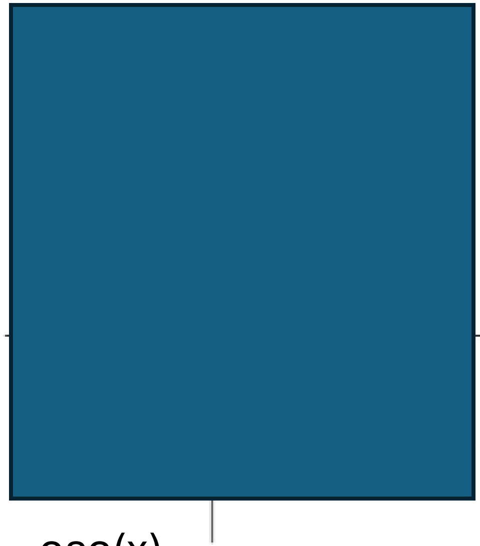
tan(x)

Inverse Trigonometric Functions

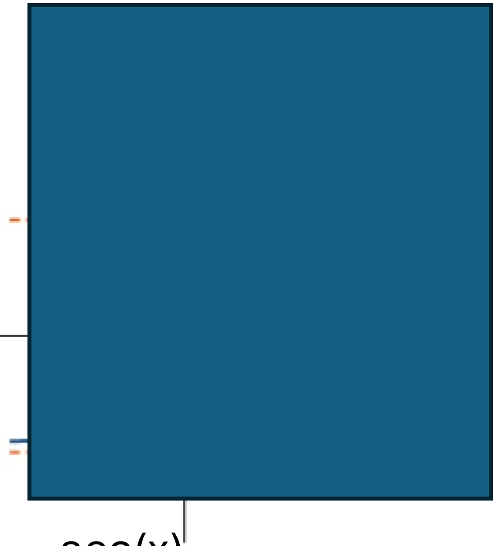
sin(x)



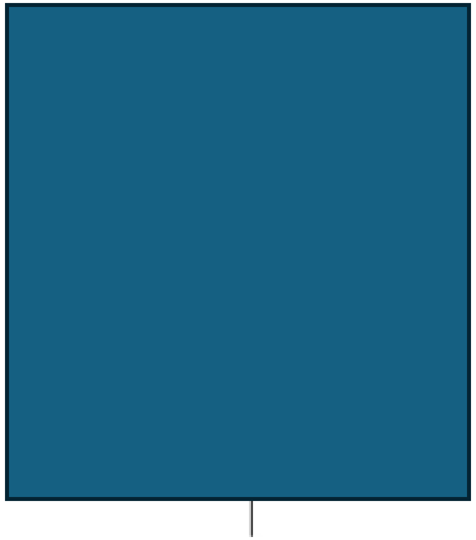
cos(x)



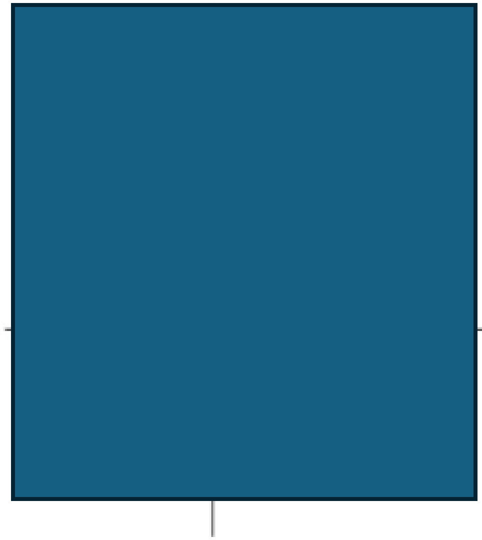
tan(x)



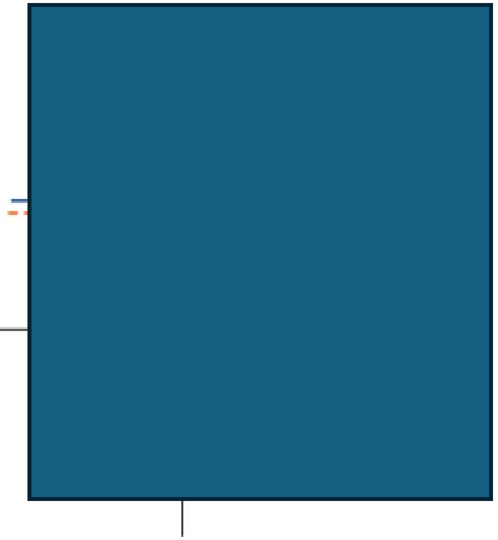
cot(x)



csc(x)



sec(x)



Evaluating Expressions Involving Inverse Trigonometric Functions

Evaluate each of the following expressions.

a. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
b. $\tan\left(\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)\right)$
c. $\cos^{-1}\left(\cos\left(\frac{5\pi}{4}\right)\right)$
d. $\sin^{-1}\left(\cos\left(\frac{2\pi}{3}\right)\right)$

Evaluating $\sin^{-1}\left(-\sqrt{3}/2\right)$ is equivalent to finding the angle θ such that $\sin\theta = -\sqrt{3}/2$ and $-\pi/2 \le \theta \le \pi/2$. The angle $\theta = -\pi/3$ satisfies these two conditions. Therefore, $\sin^{-1}\left(-\sqrt{3}/2\right) = -\pi/3$.

First we use the fact that $\tan^{-1}\left(-1/\sqrt{3}\right) = -\pi/6$. Then $\tan(-\pi/6) = -1/\sqrt{3}$. Therefore, $\tan\left(\tan^{-1}\left(-1/\sqrt{3}\right)\right) = -1/\sqrt{3}$.

To evaluate $\cos^{-1}(\cos(5\pi/4))$, first use the fact that $\cos(5\pi/4) = -\sqrt{2}/2$. Then we need to find the angle θ such that $\cos(\theta) = -\sqrt{2}/2$ and $0 \le \theta \le \pi$. Since $3\pi/4$ satisfies both these conditions, we have $\cos^{-1}(\cos(5\pi/4)) = \cos^{-1}\left(-\sqrt{2}/2\right) = 3\pi/4$.

Since $\cos(2\pi/3) = -1/2$, we need to evaluate $\sin^{-1}(-1/2)$. That is, we need to find the angle θ such that $\sin(\theta) = -1/2$ and $-\pi/2 \le \theta \le \pi/2$. Since $-\pi/6$ satisfies both these conditions, we can conclude that $\sin^{-1}(\cos(2\pi/3)) = \sin^{-1}(-1/2) = -\pi/6$.