Two independent random samples are taken from two populations. Which condition is **necessary** for constructing a confidence interval for $\mu_1 - \mu_2$?

A. The two samples must be the same size

B. The populations must have the same mean

C. The samples must be independent

D. The population standard deviations must be known

C

# Difference in Sample Means

1. **Mean** – 均值 (jūn zhí)
2. **Variance** – 方差 (fāng chā)
3. **Standard error** – 标准误差 (biāo zhǔn wù chā)
4. **Independent samples** – 独立样本 (dú lì yàng běn)
5. **Sampling distribution** – 抽样分布 (chōu yàng fēn bù)

## 🔑 Difference in Sample Means — z vs t (Concise)

**Use z (Normal):**

- Population standard deviations **known** ($\sigma_1, \sigma_2$)
- Normal populations or large samples

**Use t (MOST COMMON):**

- Population standard deviations **unknown** → use $s_1, s_2$
- Normal populations or large samples

---

## ⚡ Exam Shortcut

👉 **Use t unless σ is given**

---

## ⚠️ Small Samples

If $n < 30$:

- No strong skew / no outliers required

---

## 🧠 One-line takeaway

👉 **Unknown σ → t (almost always)**

# Degrees of Freedom

1. **Independent samples, equal variances:**

$$df = n_1 + n_2 - 2$$

- Used with pooled variance; assumes $\sigma_1^2 \approx \sigma_2^2$.

2. **Independent samples, unequal variances (Welch's t-test):**

$$df \approx \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{(s_1^2/n_1)^2}{n_1 - 1} + \frac{(s_2^2/n_2)^2}{n_2 - 1}}$$

- May not be an integer; round down for t-tables.

3. **Paired samples:**

$$df = n_d - 1$$

- n_d = number of pairs; treat differences as a single sample.

1. **Population Mean Difference ($\mu_1 - \mu_2$)** – The parameter of interest.
2. **Sample Mean Difference ($\bar{x}_1 - \bar{x}_2$)** – The statistic estimating the population difference.
3. **Standard Error (SE)** – Measures variability of the difference between sample means:

$$SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

4. **Sampling Distribution Shape** – Approximately normal if populations are normal or samples are large (CLT).
5. **Conditions for Inference** – Independent samples, random sampling, nearly normal populations (or large n), equal variances if pooling.

A nutritionist wants to compare the average daily calorie intake of adults in two cities, City A and City B.

- A random sample of 40 adults from City A has a mean of 2,400 calories with a standard deviation of 300 calories.
- A random sample of 35 adults from City B has a mean of 2,300 calories with a standard deviation of 250 calories.

Assume the conditions for inference are satisfied.

**(a)** Define the parameter of interest.

**(b)** State the null and alternative hypotheses to test whether the mean daily calorie intake differs between the two cities.

**(c)** Identify the appropriate test and justify your choice.

**(d)** Calculate the standard error of the difference in sample means.

**(e)** Write the formula for the test statistic (do not calculate the final t-value).

**(f)** Explain the conditions required for using this test.

**(a) Parameter:** $\mu_1 - \mu_2$, difference in true mean daily calories between City A and City B.

**(b) Hypotheses:**

$$H_0 : \mu_1 - \mu_2 = 0, \quad H_a : \mu_1 - \mu_2 \neq 0$$

**(c) Test:** Two-sample t-test (independent, σ unknown, moderate sample sizes).

**(d) SE:**

$$SE = \sqrt{\frac{300^2}{40} + \frac{250^2}{35}} \approx 63.5$$

**(e) Test statistic:**

$$t = \frac{2400 - 2300}{63.5} \approx 1.57$$

**(f) Conditions:** independent random samples, nearly normal or large n, σ unknown.

**Conclusion:** Do not reject $H_0$; insufficient evidence of a difference.

**Table B** ***t*** **distribution critical values**

| | Tail probability $p$ | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| df | .25 | .20 | .15 | .10 | .05 | .025 | .02 | .01 | .005 | .0025 | .001 | .0005 |
| 1 | 1.000 | 1.376 | 1.963 | 3.078 | 6.314 | 12.71 | 15.89 | 31.82 | 63.66 | 127.3 | 318.3 | 636.6 |
| 2 | .816 | 1.061 | 1.386 | 1.886 | 2.920 | 4.303 | 4.849 | 6.965 | 9.925 | 14.09 | 22.33 | 31.60 |
| 3 | .765 | .978 | 1.250 | 1.638 | 2.353 | 3.182 | 3.482 | 4.541 | 5.841 | 7.453 | 10.21 | 12.92 |
| 4 | .741 | .941 | 1.190 | 1.533 | 2.132 | 2.776 | 2.999 | 3.747 | 4.604 | 5.598 | 7.173 | 8.610 |
| 5 | .727 | .920 | 1.156 | 1.476 | 2.015 | 2.571 | 2.757 | 3.365 | 4.032 | 4.773 | 5.893 | 6.869 |
| 6 | .718 | .906 | 1.134 | 1.440 | 1.943 | 2.447 | 2.612 | 3.143 | 3.707 | 4.317 | 5.208 | 5.959 |
| 7 | .711 | .896 | 1.119 | 1.415 | 1.895 | 2.365 | 2.517 | 2.998 | 3.499 | 4.029 | 4.785 | 5.408 |
| 8 | .706 | .889 | 1.108 | 1.397 | 1.860 | 2.306 | 2.449 | 2.896 | 3.355 | 3.833 | 4.501 | 5.041 |
| 9 | .703 | .883 | 1.100 | 1.383 | 1.833 | 2.262 | 2.398 | 2.821 | 3.250 | 3.690 | 4.297 | 4.781 |
| 10 | .700 | .879 | 1.093 | 1.372 | 1.812 | 2.228 | 2.359 | 2.764 | 3.169 | 3.581 | 4.144 | 4.587 |
| 11 | .697 | .876 | 1.088 | 1.363 | 1.796 | 2.201 | 2.328 | 2.718 | 3.106 | 3.497 | 4.025 | 4.437 |
| 12 | .695 | .873 | 1.083 | 1.356 | 1.782 | 2.179 | 2.303 | 2.681 | 3.055 | 3.428 | 3.930 | 4.318 |
| 13 | .694 | .870 | 1.079 | 1.350 | 1.771 | 2.160 | 2.282 | 2.650 | 3.012 | 3.372 | 3.852 | 4.221 |
| 14 | .692 | .868 | 1.076 | 1.345 | 1.761 | 2.145 | 2.264 | 2.624 | 2.977 | 3.326 | 3.787 | 4.140 |
| 15 | .691 | .866 | 1.074 | 1.341 | 1.753 | 2.131 | 2.249 | 2.602 | 2.947 | 3.286 | 3.733 | 4.073 |
| 16 | .690 | .865 | 1.071 | 1.337 | 1.746 | 2.120 | 2.235 | 2.583 | 2.921 | 3.252 | 3.686 | 4.015 |
| 17 | .689 | .863 | 1.069 | 1.333 | 1.740 | 2.110 | 2.224 | 2.567 | 2.898 | 3.222 | 3.646 | 3.965 |
| 18 | .688 | .862 | 1.067 | 1.330 | 1.734 | 2.101 | 2.214 | 2.552 | 2.878 | 3.197 | 3.611 | 3.922 |
| 19 | .688 | .861 | 1.066 | 1.328 | 1.729 | 2.093 | 2.205 | 2.539 | 2.861 | 3.174 | 3.579 | 3.883 |
| 20 | .687 | .860 | 1.064 | 1.325 | 1.725 | 2.086 | 2.197 | 2.528 | 2.845 | 3.153 | 3.552 | 3.850 |
| 21 | .686 | .859 | 1.063 | 1.323 | 1.721 | 2.080 | 2.189 | 2.518 | 2.831 | 3.135 | 3.527 | 3.819 |
| 22 | .686 | .858 | 1.061 | 1.321 | 1.717 | 2.074 | 2.183 | 2.508 | 2.819 | 3.119 | 3.505 | 3.792 |
| 23 | .685 | .858 | 1.060 | 1.319 | 1.714 | 2.069 | 2.177 | 2.500 | 2.807 | 3.104 | 3.485 | 3.768 |
| 24 | .685 | .857 | 1.059 | 1.318 | 1.711 | 2.064 | 2.172 | 2.492 | 2.797 | 3.091 | 3.467 | 3.745 |
| 25 | .684 | .856 | 1.058 | 1.316 | 1.708 | 2.060 | 2.167 | 2.485 | 2.787 | 3.078 | 3.450 | 3.725 |
| 26 | .684 | .856 | 1.058 | 1.315 | 1.706 | 2.056 | 2.162 | 2.479 | 2.779 | 3.067 | 3.435 | 3.707 |
| 27 | .684 | .855 | 1.057 | 1.314 | 1.703 | 2.052 | 2.158 | 2.473 | 2.771 | 3.057 | 3.421 | 3.690 |
| 28 | .683 | .855 | 1.056 | 1.313 | 1.701 | 2.048 | 2.154 | 2.467 | 2.763 | 3.047 | 3.408 | 3.674 |
| 29 | .683 | .854 | 1.055 | 1.311 | 1.699 | 2.045 | 2.150 | 2.462 | 2.756 | 3.038 | 3.396 | 3.659 |
| 30 | .683 | .854 | 1.055 | 1.310 | 1.697 | 2.042 | 2.147 | 2.457 | 2.750 | 3.030 | 3.385 | 3.646 |
| 40 | .681 | .851 | 1.050 | 1.303 | 1.684 | 2.021 | 2.123 | 2.423 | 2.704 | 2.971 | 3.307 | 3.551 |
| 50 | .679 | .849 | 1.047 | 1.299 | 1.676 | 2.009 | 2.109 | 2.403 | 2.678 | 2.937 | 3.261 | 3.496 |
| 60 | .679 | .848 | 1.045 | 1.296 | 1.671 | 2.000 | 2.099 | 2.390 | 2.660 | 2.915 | 3.232 | 3.460 |
| 80 | .678 | .846 | 1.043 | 1.292 | 1.664 | 1.990 | 2.088 | 2.374 | 2.639 | 2.887 | 3.195 | 3.416 |
| 100 | .677 | .845 | 1.042 | 1.290 | 1.660 | 1.984 | 2.081 | 2.364 | 2.626 | 2.871 | 3.174 | 3.390 |
| 1000 | .675 | .842 | 1.037 | 1.282 | 1.646 | 1.962 | 2.056 | 2.330 | 2.581 | 2.813 | 3.098 | 3.300 |
| ∞ | .674 | .841 | 1.036 | 1.282 | 1.645 | 1.960 | 2.054 | 2.326 | 2.576 | 2.807 | 3.091 | 3.291 |
| | 50% | 60% | 70% | 80% | 90% | 95% | 96% | 98% | 99% | 99.5% | 99.8% | 99.9% |

Confidence level $C$

# Experimental Design

1. Experiment（实验）
2. Treatment（处理）
3. Control Group（对照组）
4. Random Assignment（随机分配）
5. Confounding Variable（混杂变量）
6. Bias（偏差）
7. Placebo（安慰剂）
8. Placebo Effect（安慰剂效应）
9. Cause and Effect（因果关系）

## 🔑 Experimental Design & Causation — Key Points (Concise)

- Only experiments can show causation（只有实验能说明因果关系）
- Random assignment（随机分配） → balances confounding variables
- Control group（对照组） → provides comparison
- Placebo & blinding（安慰剂与盲法） → reduce bias
- Confounding variables（混杂变量） → must be controlled
- Well-designed experiment = control + randomization + replication（控制+随机+重复）
- Observational studies（观察性研究） → show association, NOT causation
- Large sample size（大样本） → more reliable results

---

## ⚡ One-line takeaway

👉 Causation requires random assignment and control

A school wants to test whether a new study app improves students' test scores.
They select 80 students and allow them to **choose** whether to use the app or not. After one month, students who used the app scored higher on average.

---

### Questions

**(a)** Is this an experiment or an observational study? Explain.

**(b)** Identify a potential **confounding variable**.

**(c)** Can the school conclude that the app **causes** higher scores? Explain.

**(d)** Describe how you would redesign the study to allow for a **causal conclusion**.

**(a)**

Observational study（观察性研究）

→ Students **choose** whether to use the app (no random assignment)

---

**(b)**

Example confounding variable（混杂变量）：

- Motivation（学习动机）

  (Students who choose the app may already be more motivated)

---

**(c)**

No, cannot conclude causation（不能得出因果关系）

→ No random assignment → confounding variables may affect results

---

**(d)**

Redesign:

- Randomly assign students to:
  - App group（实验组）
  - No-app group（对照组）
- Keep other conditions the same

# TRUE or FALSE

- The sampling distribution of $\bar{X}_1 - \bar{X}_2$ is approximately normal if both populations are normal, regardless of sample size.

TRUE

# TRUE or FALSE

- If the population distributions are not normal, the sampling distribution of $\bar{X}_1 - \bar{X}_2$ is approximately normal only for large sample sizes.

TRUE

# TRUE or FALSE

- For two independent samples with equal population variances, the degrees of freedom is $n_1 + n_2 - 2$.

TRUE

# TRUE or FALSE

- In a paired sample test, the degrees of freedom is equal to the total number of observations.

False $(n_d - 1)$