Normal Distribution

The weights of apples in a grocery store are approximately normally distributed. The mean weight is 200 grams.

Which of the following is **most likely true**?

A) Most apples weigh close to 200 grams.
B) Most apples weigh much less than 200 grams.
C) Most apples weigh much more than 200 grams.
D) The weights are evenly spread from very small to very large.

A set of data is perfectly normally distributed. If you pick a value at random, it is **most likely to be**:

A) Far above the mean
B) Exactly at the mean
C) Close to the mean
D) Far below the mean

Normal Distribution

1. **Mean (μ)** – 均值 *(jūn zhí)*
2. **Standard deviation (σ)** – 标准差 *(biāo zhǔn chā)*
3. **Symmetric** – 对称的 *(duì chèn de)*
4. **Bell-shaped curve** – 钟形曲线 *(zhōng xíng qū xiàn)*
5. **z-score** – 标准分数 *(biāo zhǔn fēn shù)*

1. **Shape** – Bell-shaped, symmetric, unimodal
2. **Center** – Mean = median = mode
3. **Spread** – Controlled by standard deviation (σ)
4. **Empirical Rule (68–95–99.7)** – Predicts data within 1, 2, 3 standard deviations
5. **Standardization** – Using z-scores to find probabilities

1. **Bell-Shaped Curve** – The graph is symmetric and bell-shaped, with the peak at the mean.
2. **Symmetry** – The distribution is perfectly symmetric about the mean. The left and right halves are mirror images.
3. **Mean = Median = Mode** – In a normal distribution, these three measures of central tendency are all equal.
4. **Defined by Two Parameters** – The **mean (μ)** determines the center, and the **standard deviation (σ)** determines the spread.
5. **Tails Never Touch the Axis** – The curve extends infinitely in both directions; extreme values are possible but rare.
6. **68–95–99.7 Rule** –
   - ~68% of data falls within 1σ of the mean
   - ~95% within 2σ
   - ~99.7% within 3σ
7. **Probability Interpretation** – The area under the curve represents probability. The total area equals 1.
8. **Standard Normal Distribution** – A normal distribution with $\mu = 0$ and $\sigma = 1$ is called the standard normal distribution.
9. **Z-Scores Measure Relative Position** – A z-score tells how many standard deviations a value is from the mean.
10. **Many Real-World Phenomena** – Heights, weights, test scores, and measurement errors often approximate a normal distribution.

Z score

1. **Definition:** A z-score tells **how many standard deviations a value is from the mean.**
2. **Interpretation:**
   - **Positive z-score:** value is **above the mean**
   - **Negative z-score:** value is **below the mean**
   - **z = 0:** value is **exactly at the mean**
3. **Formula:**

$$z = \frac{\text{value} - \text{mean}}{\text{standard deviation}}$$

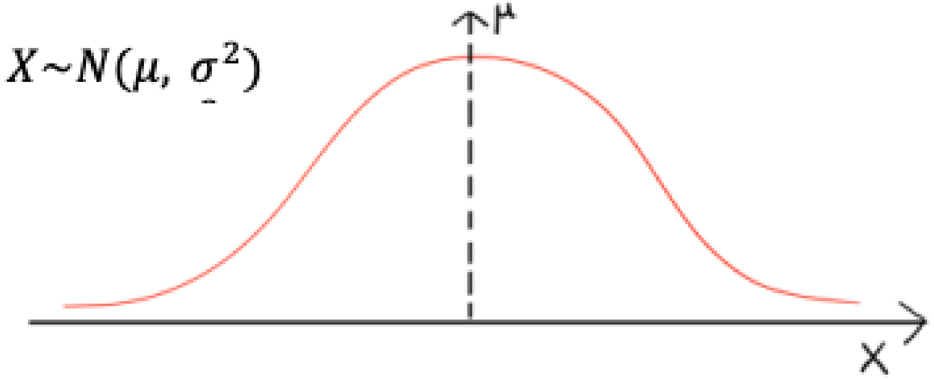
(Shows relative position compared to the average and spread.)

4. **Purpose:**
   - Standardizes values from different distributions
   - Helps compare scores from different scales
5. **Probability Link:**
   - The z-score corresponds to a point on the **standard normal curve**, used to find probabilities.

The normal distribution is a continuous probability distribution that can be used to model a vast number of naturally occurring scenarios. As a result, it is one of the most important probability distributions in statistics. Examples of natural variables that follow the normal distribution are IQ scores, height and weight. The key difference between the normal distribution and the distributions you have previously met in Year 1 is that the normal distribution is continuous, while the others are discrete.

Characteristics

The normal distribution:



- has two parameters: the mean, μ, and variance σ^2.
- Is symmetrical (mean = median = mode).
- has a bell-shaped curve with asymptotes at each end.
- has total area under the curve equal to 1.
- has $P(X = a) = 0$ for any a. This is true for any continuous distribution.

If a variable X follows a normal distribution with mean μ and variance σ^2, we write $X \sim N(\mu, \sigma^2)$.
It is often very helpful to sketch the normal curve when solving normal distribution problems.

The heights of adult males in a population are approximately normally distributed with a mean of 70 inches and a standard deviation of 3 inches.

(a) What proportion of adult males are taller than 73 inches?

(b) What proportion of adult males are between 67 inches and 73 inches?

(c) Find the height that corresponds to the 90th percentile.

(d) If a random sample of 36 men is selected, what is the probability that the sample mean height is greater than 71 inches?

Finding probabilities

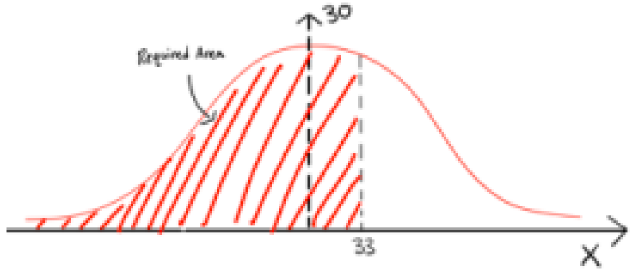
- You need to be able to use the normal cumulative distribution function on your calculator to find probabilities.

You will need to enter a lower and upper bound when using this function, as well as the mean and standard deviation of the distribution being used.

Example 1: The random variable $X \sim N(30, 2^2)$. Find $P(X < 33)$.

Using the cumulative function on your calculator, we enter our mean as 30 and our standard deviation as 2. Our upper bound will be 33 and for our lower bound we need to enter a really small value. Remember that the normal curve is asymptotic at both ends so to find an accurate approximation for the total area to the left of $x = 33$ we take our lower bound to be a small value. Take for example, -500.

$\Rightarrow P(X < 33) = 0.933\ (3\ s.f)$



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(d) If a random sample of 36 men is selected, what is the probability that the sample mean height is greater than 71 inches?

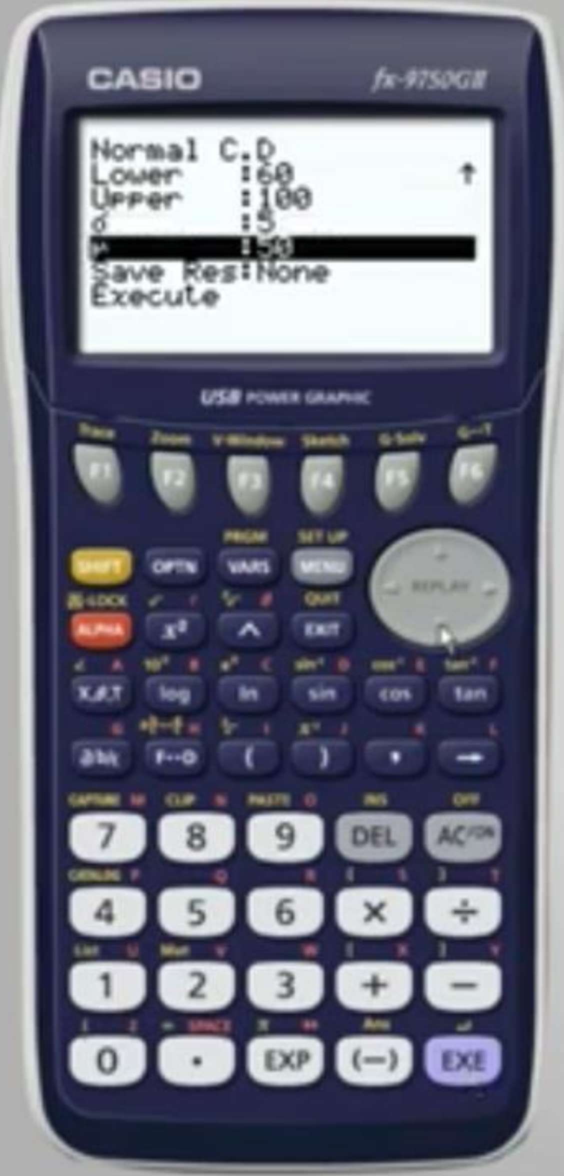
(a) $z = (73 - 70)/3 = 1 \Rightarrow P(Z > 1) \approx 0.159$

(b) $z_{67} = -1, z_{73} = 1 \Rightarrow P(-1 < Z < 1) \approx 0.683$

(c) 90th percentile: $x = 70 + 1.28(3) \approx 73.8$ inches

(d) Sample mean SE = $3/\sqrt{36} = 0.5, z = (71 - 70)/0.5 = 2 \Rightarrow P \approx 0.023$

Conclusion: Percentages/probabilities calculated using standard normal distribution.



For a normal distribution with a mean of 50 and standard deviation of 5 ($\mu = 50$, $\sigma = 5$):

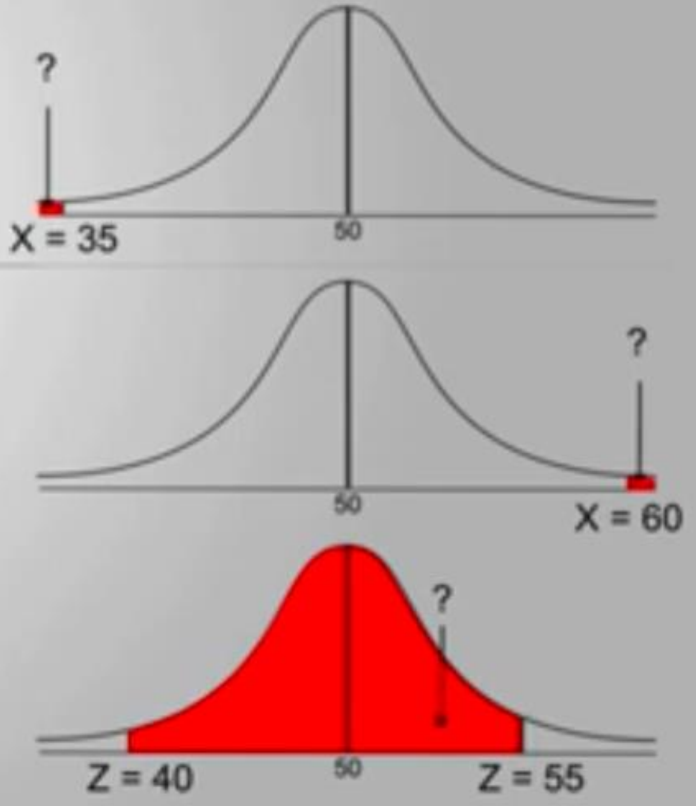
a) Find $P(X \leqslant 35) = ?$

b) Find $P(X \geqslant 60) = ?$

c) Find $P(40 \leqslant X \leqslant 55) = ?$

Normal C.D		
Lower	:	0
Upper	:	35
σ	:	5
μ	:	50

P = 1.35E-3



<u>The inverse normal distribution function</u>

- You can use the inverse normal function on your calculator to find the value of a such that $P(X < a) = p$.

Some problems might require you to instead find the value of a such that $P(X > a) = p$. Be aware that mos calculators (e.g. the Casio fx-991ex) will only return the value of a such that $P(X < a) = p$, so you will need to use the property $P(X > a) = 1 - P(X < a)$ in such situations. See example 3 for more detail.

<u>Example 2:</u> Given that $X \sim N(30, 2^2)$, find the value of a such that $P(X < a) = 0.4$.

<u>Example 3:</u> Given that $X \sim N(30, 2^2)$, find the value of a such that $P(X > a) = 0.22$.

The inverse normal distribution function

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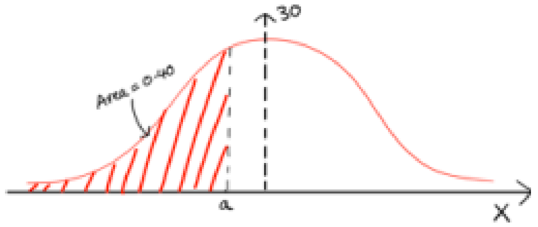
Example 2: Given that $X \sim N(30, 2^2)$, find the value of a such that $P(X < a) = 0.4$.

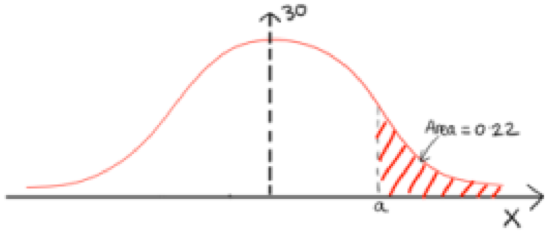
Using the inverse normal function with $p = 0.4$, mean = 30 and standard deviation = 2 gives us $a = 29.5$.



Example 3: Given that $X \sim N(30, 2^2)$, find the value of a such that $P(X > a) = 0.22$.

We first manipulate our expression:

$P(X > a) = 1 - P(X < a) = 0.22$
$\therefore P(X < a) = 0.78$



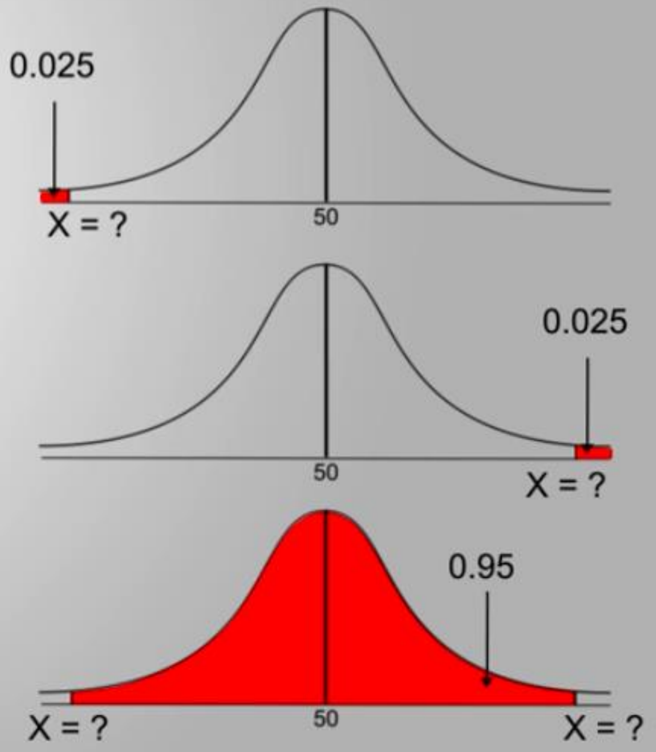
Now using the inverse function with $p = 0.78, \mu = 30$ and $\sigma = 2$ gives us that $a = 31.5$.

For a normally distributed population with a mean of 50 and standard deviation of 5 ($\mu = 50$, $\sigma=5$):

a) Find P ($X \leqslant$?) = 0.025

b) Find P ($X \geqslant$?) = 0.025

c) Find P (? $\leqslant X \leqslant$?) = 0.95



The standard normal distribution

We can use a coding to standardise our data, making it much easier to analyse and work with. For problems where the mean and/or variance are unknown, it is very useful to code data via the standard normal distribution.

- The standard normal distribution denoted $Z \sim N(0, 1^2)$, has mean 0 and standard deviation 1.

- If $X \sim N(\mu, \sigma^2)$, you can use the coding $Z = \frac{X-\mu}{\sigma}$ to convert your variable into a standard normal variable.

- The notation $\Phi(a)$ is equivalent to $P(Z < a)$.

Example 4: The random variable $X \sim N(50, 4^2)$. Write $P(X \geq 55)$ in terms of $\Phi(z)$ for some value z.

The standard normal distribution

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- The notation $\Phi(a)$ is equivalent to $P(Z < a)$.

Example 4: The random variable $X \sim N(50, 4^2)$. Write $P(X \geq 55)$ in terms of $\Phi(z)$ for some value z.

Since the normal distribution is continuous: $P(X \geq 55) = P(X > 55)$

$P(X > 55) = 1 - P(X < 55)$

Converting to standard normal:

$$P(X < 55) = P\left(Z < \frac{55-\mu}{\sigma}\right) = P\left(Z < \frac{55-50}{4}\right) = P(Z < 1.25)$$

So $P(X \geq 55) = 1 - P(Z < 1.25) = 1 - \Phi(1.25)$

Finding μ and σ

You need to be able to use the standard normal distribution to solve problems where you must find the mean and/or variance. You will be given either one or two probabilities which you must standardise in order to find the unknown parameters. We will go through two examples; one in which one parameter is missing and the other where both parameters are missing.

Example 5: The random variable $X \sim N(\mu, 5^2)$ and $P(X < 18) = 0.9032$. Find the value of μ.

Finding μ and σ

You need to be able to use the standard normal distribution to solve problems where you must find the mean and/or variance. You will be given either one or two probabilities which you must standardise in order to find the unknown parameters. We will go through two examples; one in which one parameter is missing and the other where both parameters are missing.

Example 5: The random variable $X \sim N(\mu, 5^2)$ and $P(X < 18) = 0.9032$. Find the value of μ.

We are told that $P(X < 18) = 0.9032$. Standardising:

$$P(X < 18) = P\left(Z < \frac{18-\mu}{5}\right) = 0.9032$$

We can now use the inverse normal function (with $\mu = 0, \sigma = 1, area = 0.9032$) to find the value of a such that $P(X < a) = 0.9032$:

The calculator returns a value of $a = 1.30$

$$\therefore \frac{18-\mu}{5} = 1.30 \Rightarrow \mu = 11.5.$$

Example 6: The random variable $X \sim N(\mu, \sigma^2)$. Given that $P(X < 17) = 0.8159$ and $P(X < 25) = 0.9970$, find the value of μ and σ.

Hint: Try sketching the probability curve to help visualise the distribution.

Example 6: The random variable $X \sim N(\mu, \sigma^2)$. Given that $P(X < 17) = 0.8159$ and $P(X < 25) = 0.9970$, find the value of μ and σ.

Hint: Try sketching the probability curve to help visualise the distribution.

We are told $(X < 17) = 0.8159$ and $P(X < 25) = 0.9970$. We standardise both of these cases separately, following the same method as in example 4:

$$\Rightarrow P(X < 17) = P\left(Z < \frac{17-\mu}{\sigma}\right) = 0.8159$$
$$\Rightarrow P(X < 25) = P\left(Z < \frac{25-\mu}{\sigma}\right) = 0.9970$$

Using the inverse normal function for both probabilities, we acquire two equations:

$$\frac{17-\mu}{\sigma} = 0.8998 \Rightarrow 17 - \mu = 0.8998\sigma$$

$$\frac{25-\mu}{\sigma} = 2.748 \Rightarrow 25 - \mu = 2.748\sigma$$

We now have two equations with two unknowns, so we can solve for μ and σ. You could use your calculator for this part.

Solving gives us $\mu = 13.1$, $\sigma = 4.33$ $(3\ s.f)$

Table entry for z is the probability lying below z.

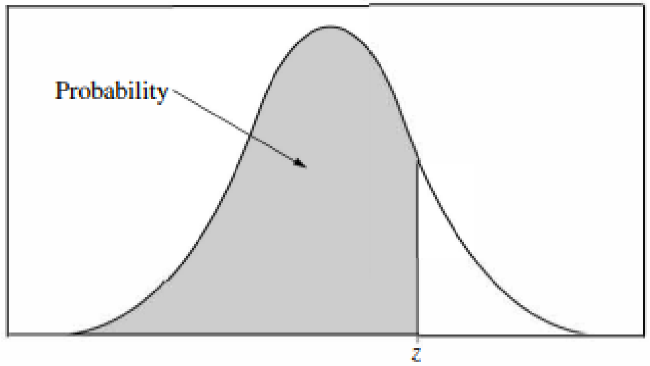

Table A ***(Continued)***

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

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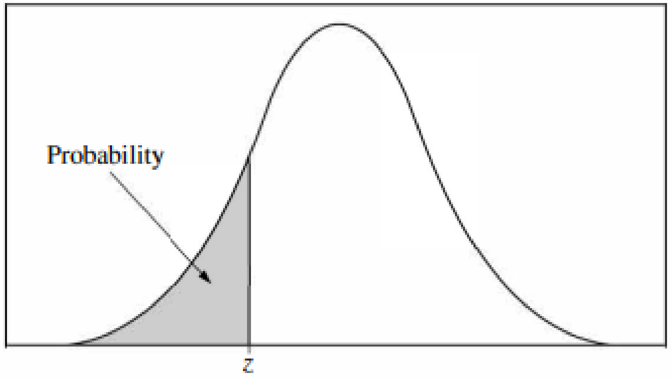

Table A Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

TRUE or FALSE

- Z-scores indicate how many standard deviations a value is from the mean.

TRUE or FALSE

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TRUE

TRUE or FALSE

- The area under the curve of a normal distribution can be greater than 1.

TRUE or FALSE

- The area under the curve of a normal distribution can be greater than 1.

FALSE

TRUE or FALSE

- The 95% of data in a normal distribution falls within two standard deviations of the mean.

TRUE or FALSE

- The 95% of data in a normal distribution falls within two standard deviations of the mean.

TRUE

Stretch and Challenge

The heights of a college population are approximately **normally distributed** with a mean of 170 cm and a standard deviation of 10 cm.

Tasks:

1. **Basic z-score calculation:**
   - Find the z-score for a student who is **185 cm tall**.
2. **Percentile interpretation:**
   - What percentage of students are **shorter than 185 cm**?
   - What percentage are **between 160 cm and 180 cm**?
3. **Reverse problem (challenge):**
   - A scholarship is awarded to the **top 5% tallest students**. What is the **minimum height required**?
4. **Contextual reasoning (stretch):**
   - Suppose the university reports only **rounded heights in whole numbers**.
   - How might this rounding **affect your percentile calculations** and interpretation?
5. **Multi-step synthesis (challenge):**
   - If another campus has a normal height distribution with the **same mean** but a **smaller standard deviation of 5 cm**, which campus has **more students taller than 185 cm**? Explain using z-scores and reasoning.

1. Z-score for 185 cm:

$$z = (185 - 170)/10 = 1.5$$

2. Percentiles:

- Shorter than 185 cm ≈ **93.3%**
- Between 160 cm and 180 cm ≈ **68.3%**

3. Top 5% minimum height:

$$X = 170 + 1.645(10) \approx 186.5 \text{ cm}$$

4. Effect of rounding:

Rounding to whole numbers causes **minor approximation errors**, but conclusions stay essentially the same.

5. Comparing campuses:

- $\sigma = 5 \rightarrow z = 3 \rightarrow P(Z>185) \approx 0.13\%$
- $\sigma = 10 \rightarrow z = 1.5 \rightarrow P(Z>185) \approx 6.7\%$

  Conclusion: Original campus (larger σ) has **more students taller than 185 cm.**

A pottery artist makes hand-made mugs. For one particular style of mug, the amounts of liquid the mugs will hold are approximately normally distributed with a mean of 260 mL and a standard deviation of 6 mL. Which of these is closest to the proportion of mugs that will hold less than 250 mL?

- (A) 0.01
- (B) 0.05
- (C) 0.16
- (D) 0.95
- (E) 0.99

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B

Central Limit Theorem

Central Limit Theorem	中心极限定理
Sampling distribution	抽样分布
Sample mean	样本均值
Standard error	标准误
Approximately normal	近似正态

1. **Distribution of sample means**: The CLT states that the distribution of sample means approaches a **normal distribution** as sample size increases, regardless of the population's shape.
2. **Mean of the sampling distribution**: The mean of the sample means equals the **population mean** ($\mu_{\bar{x}} = \mu$).
3. **Standard error**: The standard deviation of the sample means is called the **standard error** ($\sigma_{\bar{x}} = \sigma/\sqrt{n}$) and decreases as sample size increases.
4. **Sample size effect**: Larger samples produce a **sampling distribution that is closer to normal** and less variable.
5. **Foundation for inference**: The CLT allows us to use **normal probability methods** for confidence intervals and hypothesis tests on means, even if the population is not normal.

A bottling company fills soda bottles with a mean volume of 500 mL and a standard deviation of 12 mL. The distribution of individual bottle volumes is slightly right-skewed.

A random sample of 36 bottles is selected.

(a) Describe the shape of the sampling distribution of the sample mean.

(b) Calculate the mean and standard deviation of the sampling distribution of the sample mean.

(c) What is the probability that the sample mean is less than 496 mL?

(d) Explain why it is appropriate to use a normal distribution in this situation.

(a) Approximately normal (by the Central Limit Theorem, since $n = 36 \geq 30$)

(b)

Mean: $\mu_{\bar{x}} = 500$

Standard deviation: $\sigma_{\bar{x}} = \frac{12}{\sqrt{36}} = 2$

(c)

$z = \frac{496-500}{2} = -2$

Probability ≈ 0.0228

(d)

Sample size is large ($n \geq 30$), so the Central Limit Theorem applies even though the population is skewed.