PRECALCULUS · POLYNOMIALS & RATIONALS

Complex Numbers & Rational Functions

The imaginary unit and conjugates · polynomial zeros · asymptotes · sign charts

i	2-01 to 2-09	sign charts
complex arithmetic	source booklets	inequalities

14	3	8
questions worked	sections	true/false checks

Built from the 2-01 Complex Numbers through 2-09 Nonlinear Inequalities booklets

How this lesson runs

Key points, worked examples, questions to try — then true or false

Starter	Two quick recall questions on the imaginary unit and complex arithmetic — attempt before the reveal.
Key points	Each of the three sections opens with a summary of the ideas you need, main point highlighted first.
Question	Questions appear on their own slides at full size. Attempt each before turning the page.
Solution	The next slide gives the condensed solution: the answer and the key steps only, plus a trap box.
True or false	The lesson ends with eight statements — one per slide, answer on the slide that follows.

Key topic · From real to complex, and from polynomial to rational

What this lesson is about, in one page

The new number	$i^2 = -1$. Every quadratic can now be solved: a negative discriminant gives a conjugate pair of complex roots.
The conjugate	For $a + bi$ the conjugate is $a - bi$. Their product is $a^2 + b^2$ — that is how we divide by complex numbers.
The zeros	End behaviour from degree and leading coefficient; the Remainder, Factor and Rational Zero Theorems; the FTA counts zeros.
The edges	Rational functions live beside vertical and horizontal asymptotes, and sign charts solve nonlinear inequalities.

Vocabulary

Key terms with Chinese translations

imaginary unit i	**虚数单位**	the number with $i^2 = -1$
complex number	**复数**	a + bi, with real part a and imaginary part b
complex conjugate	**共轭复数**	a − bi is the conjugate of a + bi
complex plane	**复平面**	plot a + bi at (a, b): real axis, imaginary axis
end behaviour	**端点趋势**	what y does as $x \to \pm\infty$, set by degree and leading coefficient
remainder	**余数**	f(k) when dividing by (x − k) — the Remainder Theorem
rational zero	**有理零点**	a zero of the form p/q, p a factor of the constant, q of the lead
multiplicity	**重数**	how many times a zero repeats as a factor
vertical asymptote	**垂直渐近线**	a line x = c the graph approaches after reducing, c a zero of the denominator
horizontal asymptote	**水平渐近线**	y = 0, y = lead ratio, or none — compare degrees N and D

S

Starter · The imaginary unit and complex arithmetic

What you must remember before the main sections

- Powers of i repeat in a cycle of four
- Add, subtract and multiply complex numbers term by term

Key points · the imaginary unit

Four facts that power every complex question

1	**The definition**	$i = \sqrt{(-1)}$, so $i^2 = -1$. $\sqrt{(-n)} = i\sqrt{n}$ — pull the negative out first, then simplify the root.
2	**The cycle**	$i^1 = i$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$. Divide the exponent by 4 and use the remainder.
3	**Conjugates**	The conjugate of $a + bi$ is $a - bi$, and $(a + bi)(a - bi) = a^2 + b^2$ — always real, never negative.
4	**Equality**	Two complex numbers are equal exactly when their real parts match and their imaginary parts match.

Question S1

4 marks

Starter · Powers of i

THE QUESTION

Simplify, giving your answers in the set { 1, i, −1, −i }:

(a) i^{24} (b) i^{47} (c) i^{61}

(d) $3i^{7} + 2i^{5}$

Question S1 — solution

4 marks

Starter · Powers of i

CONDENSED SOLUTION

- (a) i^{24}: 24 leaves remainder 0, so $i^{24} = 1$
- (b) i^{47}: remainder 3, so $i^{47} = i^3 = -i$
- (c) i^{61}: remainder 1, so $i^{61} = i$
- (d) $3i^7 + 2i^5 = 3(-i) + 2(i) = -i$

TRAP For i^{47} the remainder is 3, giving $i^3 = -i$ — not i. Divide by 4 and trust the remainder, not a pattern you think you see.

Question S2

3 marks

Starter · Complex arithmetic

THE QUESTION

Simplify:

(a) $(6 - 2i) - (-3 + 5i)$
(b) $(1 + 2i)(4 - i)$
(c) Write down the conjugate of $7 - i$.

Question S2 — solution

3 marks

Starter · Complex arithmetic

CONDENSED SOLUTION

- (a) $9 - 7i$ — subtract real with real, i with i
- (b) $4 - i + 8i - 2i^2 = 6 + 7i$
- (c) conjugate of $7 - i$ is $7 + i$

TRAP Subtracting $-3 + 5i$ changes BOTH signs: the 6 gains 3 and the $-2i$ gains $-5i$. Dropping one sign is the classic slip.

01

Complex numbers and quadratic solutions

Arithmetic with i, division by a conjugate, complex roots

- $\sqrt{(-n)} = i\sqrt{n}$, and powers of i cycle with period 4
- Discriminant < 0 gives a conjugate pair of complex roots

Key points · complex arithmetic

Multiply, divide, and solve quadratics over the complex numbers

$$(a+bi) \pm (c+di) = (a \pm c) + (b \pm d)i \qquad (a+bi)(c+di) = (ac-bd) + (ad+bc)i$$

Divide by $c+di$: multiply numerator and denominator by the conjugate $c-di$

$$\text{Discriminant } \Delta = b^2 - 4ac < 0 \;\Rightarrow\; x = \left(-b \pm \sqrt{|\Delta|}\, i \right) / 2a \;\text{ — a conjugate pair}$$

1. **Multiply with FOIL** — Treat i like a letter, then replace every i^2 with −1: $(2-5i)(3+4i) = 26 - 7i$ after collecting.
2. **Divide by the conjugate** — $(4-3i)/(2+i)$: multiply above and below by $2-i$. The denominator becomes a^2+b^2, a real number.
3. **Solve over $\mathbb{C}$** — Every quadratic $ax^2+bx+c = 0$ has two solutions in $\mathbb{C}$. Negative Δ just means they are complex conjugates.
4. **Read the pair** — Complex roots arrive as $p + qi$ and $p - qi$ together. If your pair is not symmetric, re-check the algebra.

Worked example · dividing by a complex number

the question

Question first — write the quotient in a + bi form

THE QUESTION

Write the quotient below in the form a + bi, where a and b are real:

$(3 - 4i) / (1 + 2i)$

Show the multiplication by the conjugate and state the final answer.

Worked example · dividing by a complex number — working

the working

Data and conditions, then the calculation

THE WORKING

Conjugate of $1 + 2i$ is $1 - 2i$

$$(3 - 4i)(1 - 2i) = 3 - 6i - 4i + 8i^2 = -5 - 10i$$

$$(1 + 2i)(1 - 2i) = 1^2 + 2^2 = 5$$

$$(3 - 4i)/(1 + 2i) = (-5 - 10i)/5 = -1 - 2i$$

Interpretation: the denominator became the real number 5, so the quotient is a single complex number $-1 - 2i$. Check by multiplying $(1 + 2i)(-1 - 2i)$ — you must get back to $3 - 4i$.

Try it yourself

your turn

Attempt before turning the page — answer on the next slide

YOUR TURN

Simplify completely:

$(2 + i)(2 - i) + (3 - i)^2$

Give your answer in the form a + bi.

Question Y1 — solution

3 marks

Try it · complex arithmetic

CONDENSED SOLUTION

- $(2 + i)(2 - i) = 4 + 1 = 5$
- $(3 - i)^2 = 9 - 6i + i^2 = 8 - 6i$
- Sum: $5 + 8 - 6i = 13 - 6i$

TRAP Squaring $3 - i$ gives $8 - 6i$, because the i^2 term is -1. Writing $9 - 6i + 1$ instead is the error to avoid.

Question A1

3 marks

Complex numbers · arithmetic

THE QUESTION

Simplify, giving each answer in the form a + bi:

(a) (5 + 3i) + (2 − 7i) − (4 + i)
(b) (2 − 5i)(3 + 4i)

Question A1 — solution

3 marks

Complex numbers · arithmetic

CONDENSED SOLUTION

- (a) $(5 + 2 - 4) + (3 - 7 - 1)i = 3 - 5i$
- (b) $6 + 8i - 15i - 20i^2$
- $= 6 - 7i + 20 = 26 - 7i$

TRAP Replace every i^2 with -1 before collecting: $-20i^2$ turns into $+20$. Keeping it as -20 is the most common sign error in this topic.

Question A2

4 marks

Complex numbers · conjugates

THE QUESTION

(a) Write down the conjugate of $-3 + 8i$.

(b) Find the product $(-3 + 8i)(-3 - 8i)$.

(c) Write $(4 - 3i)/(2 + i)$ in the form $a + bi$.

Question A2 — solution

4 marks

Complex numbers · conjugates

CONDENSED SOLUTION

- (a) $-3 - 8i$
- (b) $(-3)^2 + (8)^2 = 9 + 64 = 73$
- (c) $(4 - 3i)(2 - i) / 5 = (8 - 4i - 6i + 3i^2)/5$
- $= (5 - 10i)/5 = 1 - 2i$

TRAP $(a + bi)(a - bi) = a^2 + b^2$ — the i^2 term always ADDS. A product of conjugates is never $a^2 - b^2$.

Question A3

4 marks

Complex solutions of quadratics

THE QUESTION

Consider the equation $2x^2 - 4x + 5 = 0$.

(a) Compute the discriminant and explain what its sign tells you.
(b) Solve the equation over the complex numbers.
(c) Describe the relationship between your two solutions.

Question A3 — solution

4 marks

Complex solutions of quadratics

CONDENSED SOLUTION

- (a) $\Delta = 16 - 40 = -24 < 0$, so the roots are complex
- (b) $x = (4 \pm \sqrt{24}\cdot i)/4 = 1 \pm (\sqrt{6}/2)i$
- (c) The pair are complex conjugates: $1 + (\sqrt{6}/2)i$ and $1 - (\sqrt{6}/2)i$

TRAP $\sqrt{(-24)} = 2\sqrt{6}\cdot i$ — pull -1 out and simplify $\sqrt{24} = 2\sqrt{6}$. Dropping the factor 2 under the root is where marks vanish.

02

Polynomials: behaviour, division and zeros

End behaviour, the Remainder Theorem, Rational Zero Theorem, FTA

- End behaviour is set by the degree and the leading coefficient
- Zeros: rational candidates, multiplicities, and the Fundamental Theorem of Algebra

Key points · polynomial behaviour and zeros

Four theorems that carry every polynomial question

Even degree + positive lead: both ends up · Odd degree: one end up, one down · sign of the lead decides which

Remainder Theorem: remainder on division by $(x - k)$ is $f(k)$ · Factor Theorem: $f(k) = 0 \Leftrightarrow (x - k)$ is a factor

Rational zeros: $\pm p/q$ with p a factor of the constant and q a factor of the leading coefficient

FTA: a degree-n polynomial has exactly n zeros over $\mathbb{C}$, counted with multiplicity — and n linear factors

1. **Describe the ends** — $f(x) = -3x^4 + 2x^3$: even degree, negative lead, so BOTH ends fall. Turning points: at most 3.
2. **Test a zero cheaply** — Remainder on dividing $f(x) = 2x^3 + x^2 - 7x - 3$ by $(x + 1)$ is $f(-1) = 3$ — no division needed.
3. **Build the candidate list** — For $2x^3 - 3x^2 - 11x + 6$: $\pm$(factors of 6)/(factors of 2) — includes halves like $\pm 1/2$ and $\pm 3/2$, not just integers.
4. **Count with multiplicity** — $(x - 2)^3$ contributes three zeros AT 2. Degree 5 means five zeros in total over $\mathbb{C}$, complex ones in conjugate pairs.

Worked example · rational zeros

the question

Question first — find every zero of a cubic

THE QUESTION

Find all zeros of $f(x) = x^3 + 2x^2 - 5x - 6$.

Use the Rational Zero Theorem to list candidates, test them, and factorise f(x) completely.

Worked example · rational zeros — working

the working

Candidates, tests, then the full factorisation

THE WORKING

Candidates: $\pm 1, \pm 2, \pm 3, \pm 6$

$f(1) = 1 + 2 - 5 - 6 = -8$ (no) $\qquad f(-1) = -1 + 2 + 5 - 6 = 0$ (yes)

Divide by $(x + 1)$: $x^3 + 2x^2 - 5x - 6 = (x + 1)(x^2 + x - 6)$

$x^2 + x - 6 = (x + 3)(x - 2) \Rightarrow$ zeros $-3, -1, 2$

Interpretation: three linear factors for a cubic, so exactly three zeros — −3, −1 and 2 — each with multiplicity 1, as the Fundamental Theorem of Algebra promises.

Try it yourself

your turn

Attempt before turning the page — answer on the next slide

YOUR TURN

Find all zeros of $f(x) = x^3 - 3x - 2$

and state the multiplicity of each.

Question Y2 — solution

3 marks

Try it · rational zeros

CONDENSED SOLUTION

- $f(-1) = -1 + 3 - 2 = 0 \Rightarrow (x + 1)$ is a factor
- $x^3 - 3x - 2 = (x + 1)^2(x - 2)$
- Zeros: -1 (multiplicity 2), 2 (multiplicity 1)

TRAP One candidate zero can appear TWICE: $(x + 1)$ dividing twice means multiplicity 2 — keep testing the same zero after each division.

Question A4

3 marks

Polynomial end behaviour

THE QUESTION

(a) Which of these polynomials falls at BOTH ends? Circle one and justify with degree and leading coefficient.

(A) $x^3 + 1$ (B) $-2x^4 + 3x^2 - 7$ (C) $x^2 - 4$ (D) $3x^3 - x$

(b) A polynomial graph has 3 turning points. What is its least possible degree?

Question A4 — solution

3 marks

Polynomial end behaviour

CONDENSED SOLUTION

- (a) B: even degree 4 with leading coefficient $-2 < 0$
- Both ends fall: $y \to -\infty$ as $x \to \pm\infty$
- (b) Least degree = 4 (a degree-n polynomial has at most $n - 1$ turning points)

TRAP Both ends falling needs TWO conditions at once: even degree AND a negative leading coefficient. Either one alone is not enough.

Question A5

4 marks

Polynomial division and factorisation

THE QUESTION

Let $f(x) = x^3 - 2x^2 - 5x + 6$.

(a) Divide f(x) by (x − 3), stating the quotient and remainder.
(b) Hence write f(x) as a product of three linear factors.
(c) State all zeros of f.

Question A5 — solution

4 marks

Polynomial division and factorisation

CONDENSED SOLUTION

- (a) Quotient $x^2 + x - 2$, remainder 0
- (b) $f(x) = (x - 3)(x^2 + x - 2) = (x - 3)(x + 1)(x - 2)$
- (c) Zeros: 3, −1, 2

TRAP Remainder 0 means the divisor is a factor — then factorise the quadratic quotient by any method. Check by multiplying the three factors back.

Question A6

3 marks

The Remainder Theorem

THE QUESTION

Let $f(x) = 2x^3 + x^2 - 7x - 3$.

(a) Without dividing, find the remainder when f(x) is divided by (x + 1).
(b) Is (x + 1) a factor of f(x)? Justify.
(c) Without dividing, find the remainder when f(x) is divided by (x − 1).

Question A6 — solution

3 marks

The Remainder Theorem

CONDENSED SOLUTION

- (a) Divisor $x + 1 = x - (-1)$, so the remainder is $f(-1)$
- $f(-1) = -2 + 1 + 7 - 3 = 3$
- (b) Not a factor — the remainder is not 0.
- (c) $f(1) = 2 + 1 - 7 - 3 = -7$

TRAP For $(x + 1)$ the value to substitute is $k = -1$, not $+1$. The sign flip between divisor and substitution is the classic error here.

Question A7

4 marks

Rational zeros and the FTA

THE QUESTION

Let $f(x) = 2x^3 - 3x^2 - 11x + 6$.

(a) List all possible rational zeros using the Rational Zero Theorem.
(b) Find all three zeros of f.
(c) Write f(x) as a product of linear factors.

Question A7 — solution

4 marks

Rational zeros and the FTA

CONDENSED SOLUTION

- (a) ± p/q: p | 6, q | 2 ⇒ ±1, ±2, ±3, ±6, ±1/2, ±3/2
- (b) Testing gives f(−2) = 0, f(1/2) = 0, f(3) = 0
- Zeros: −2, 1/2, 3
- (c) f(x) = (x + 2)(2x − 1)(x − 3)

TRAP The candidate list must include fractions: p over EVERY factor of the leading coefficient 2. Forgetting ±1/2 and ±3/2 hides the zero you need.

03

Rational functions and nonlinear inequalities

Domains, asymptotes, intercepts — and sign charts

- Factor first, then reduce: cancelled zeros become holes
- Compare degrees N and D for horizontal asymptotes; sign charts solve inequalities

Key points · rational functions

Domain, asymptotes, intercepts — and the sign chart recipe

```
Domain: all real x with denominator ≠ 0   —   factor and reduce FIRST, then decide
```

```
VA: zeros of the reduced denominator  ·  x-int: zeros of the reduced numerator  ·  y-int: value at x = 0
```

```
HA:  N < D ⇒ y = 0    ·    N = D ⇒ y = ratio of leading coefficients    ·    N > D ⇒ no horizontal asymptote
```

```
Sign chart: make one side 0 → critical numbers (zeros + undefined) → plot on a number line → test each region
```

1	**Holes, not asymptotes**	A zero cancelled from BOTH numerator and denominator is a hole. Only zeros surviving in the reduced denominator give vertical asymptotes.
2	**Three HA cases**	N < D: y = 0. N = D: y = (lead of N)/(lead of D). N > D: no horizontal asymptote — the graph escapes.
3	**Intercepts survive reduction**	Take intercepts from the reduced form — but a shared zero that cancels gives a hole, not an x-intercept.
4	**Endpoints on the line**	Zeros of the numerator may be INCLUDED (the expression is 0 there); zeros of the denominator are always EXCLUDED.

Worked example · analysing a rational function

the question

Question first — one function, every feature

THE QUESTION

For the function $f(x) = (x^2 - 4)/(x^2 - x - 6)$:

(a) State the domain.
(b) Find any holes and vertical asymptotes.
(c) Find the horizontal asymptote and all intercepts.

Worked example · analysing a rational function — working

the working

Factor, cancel, then read off every feature

THE WORKING

```
f(x) = (x−2)(x+2) / [ (x−3)(x+2) ]   —  domain excludes x = −2 and x = 3
```

```
Cancel (x + 2):  hole at x = −2,  reduced form f(x) = (x − 2)/(x − 3)
```

```
VA: x = 3 (survives in the denominator)      HA: y = 1 (N = D, 1/1)
```

```
x-intercept x = 2  ·  y-intercept f(0) = −4/−6 = 2/3
```

Interpretation: $x = -2$ breaks the domain but cancels, so it is a hole with y-value $f(-2) = (-4)/(-5) = 4/5$. The graph approaches $y = 1$ as $x \to \pm\infty$.

Question A8

4 marks

Rational functions · domain and asymptotes

THE QUESTION

For $f(x) = (x^2 - 4)/(x^2 - x - 6)$:

(a) State the domain.
(b) Identify any holes and any vertical asymptotes, with reasons.
(c) State the horizontal asymptote, justifying with the degrees of numerator and denominator.

Question A8 — solution

4 marks

Rational functions · domain and asymptotes

CONDENSED SOLUTION

- (a) All real x except x = −2 and x = 3
- (b) (x + 2) cancels: hole at x = −2. The surviving zero x = 3 is the VA.
- (c) N = D = 2, so HA: y = 1/1 = 1

TRAP Cancel the common factor BEFORE reading asymptotes. The cancelled zero is a hole; only zeros of the reduced denominator are vertical asymptotes.

Question A9

3 marks

Rational functions · intercepts and graph features

THE QUESTION

For $h(x) = (2x + 3)/(x^2 - 1)$:

(a) Find the x-intercept and the y-intercept.
(b) State the vertical asymptotes.
(c) State the horizontal asymptote, or explain why there is none.

Question A9 — solution

3 marks

Rational functions · intercepts and graph features

CONDENSED SOLUTION

- (a) x-int: $2x + 3 = 0 \Rightarrow x = -3/2$. y-int: $h(0) = 3/(-1) = -3$
- (b) $x^2 - 1 = (x - 1)(x + 1) = 0 \Rightarrow$ VA: $x = \pm 1$
- (c) N < D (degree 1 vs 2), so HA: $y = 0$

TRAP The numerator here has no common factor with the denominator, so every zero of the denominator is a true asymptote — nothing cancels.

Question A10

4 marks

Nonlinear inequalities · sign chart

THE QUESTION

Solve, using a sign chart:

$$(x+1)(x-4)/(x-2) \leq 0$$

State your answer in interval notation and explain your treatment of each critical number.

Question A10 — solution

4 marks

Nonlinear inequalities · sign chart

CONDENSED SOLUTION

- Critical numbers: $x = -1, 4$ (zeros); $x = 2$ (undefined)
- Sign chart: negative on $(-\infty, -1)$, positive on $(-1, 2)$, negative on $(2, 4)$, positive on $(4, \infty)$
- Solution: $(-\infty, -1] \cup (2, 4]$

TRAP ≤ includes zeros of the NUMERATOR (−1 and 4 get square brackets) but never the zero of the DENOMINATOR — x = 2 stays in a round bracket.

Mark scheme

Marks by question, with what earns each mark.

S1	4	1 per part: 1, −i, i, −i
S2	3	1 subtraction · 1 product 6 + 7i · 1 conjugate 7 + i
Y1	3	1 product 5 · 1 square 8 − 6i · 1 sum 13 − 6i
Y2	3	1 factor $(x + 1)^2$ · 1 zero 2 · 1 multiplicities stated
A1	3	1 sum 3 − 5i · 1 expansion · 1 product 26 − 7i
A2	4	1 conjugate · 1 product 73 · 1 conjugate multiplication · 1 quotient 1 − 2i
A3	4	1 discriminant −24 · 1 roots $1 \pm (\sqrt{6}/2)i$ · 1 simplification · 1 conjugate-pair statement
A4	3	1 choice B with degree · 1 justification (even, negative lead) · 1 least degree 4
A5	4	1 division with quotient $x^2 + x - 2$ · 1 remainder 0 · 1 full factorisation · 1 zeros 3, −1, 2
A6	3	1 remainder f(−1) = 3 · 1 not-a-factor decision · 1 remainder f(1) = −7
A7	4	1 complete candidate list incl. halves · 1 testing · 1 zeros −2, 1/2, 3 · 1 linear factorisation
A8	4	1 domain · 1 hole x = −2 with cancellation reason · 1 VA x = 3 · 1 HA y = 1
A9	3	1 both intercepts · 1 VA x = ±1 · 1 HA y = 0 with degree comparison
A10	4	1 critical numbers · 1 sign chart regions · 1 interval notation · 1 endpoint treatment

Main lesson total: 36 marks (A1–A10). Starter and try-it questions: 13 practice marks (not assessed).

Answer key

One line per question

ANSWERS

- S1 $1 \cdot -i \cdot i \cdot -i$
- S2 $9 - 7i \cdot 6 + 7i \cdot 7 + i$
- Y1 $13 - 6i$
- Y2 -1 (multiplicity 2), 2 (multiplicity 1)
- A1 $3 - 5i \cdot 26 - 7i$
- A2 $-3 - 8i \cdot 73 \cdot 1 - 2i$
- A3 $\Delta = -24$; $x = 1 \pm (\sqrt{6}/2)i$; conjugate pair

QUICK CHECKS

- A4 B (even degree, negative lead); least degree 4
- A5 $q = x^2 + x - 2$, $r = 0$; zeros 3, −1, 2
- A6 remainder 3 (not a factor); remainder −7
- A7 zeros -2, $1/2$, 3; $f(x) = (x + 2)(2x - 1)(x - 3)$
- A8 $x \neq -2, 3$; hole -2, VA 3; HA $y = 1$
- A9 x-int $-3/2$, y-int -3; VA ± 1; HA $y = 0$
- A10 $(-\infty, -1] \cup (2, 4]$

True or false · 1

T / F

Decide, then justify your decision — answer on the next slide

The powers of i repeat in a cycle of length 4.

True or false · 1 — answer

TRUE

TRUE

$i^1 = i$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$, and then it repeats. Divide the exponent by 4 and use the remainder.

True or false · 2

T / F

Decide, then justify your decision — answer on the next slide

Every quadratic equation has exactly two solutions over the complex numbers.

True or false · 2 — answer

TRUE

TRUE

Counting multiplicity, a quadratic always has two solutions: real when $\Delta \geq 0$, a complex conjugate pair when $\Delta < 0$.

True or false · 3

T / F

Decide, then justify your decision — answer on the next slide

If a polynomial with real coefficients has 2 + i as a zero, then 2 − i is also a zero.

True or false · 3 — answer

TRUE

TRUE

Complex zeros of real-coefficient polynomials come in conjugate pairs — and the FTA guarantees degree-many zeros in total.

True or false · 4

T / F

Decide, then justify your decision — answer on the next slide

The product $(a + bi)(a - bi)$ equals $a^2 - b^2$.

True or false · 4 — answer

FALSE

FALSE

It equals $a^2 + b^2$: the i^2 term contributes $-b^2 \cdot (-1)$. The product of conjugates is always a positive-style real sum.

True or false · 5

T / F

Decide, then justify your decision — answer on the next slide

A polynomial of degree 7 has exactly 7 distinct zeros.

True or false · 5 — answer

FALSE

FALSE

It has 7 zeros COUNTED WITH MULTIPLICITY. Some may repeat — degree 7 allows as few as 1 distinct zero.

True or false · 6

T / F

Decide, then justify your decision — answer on the next slide

The graph of a rational function never crosses a vertical asymptote.

True or false · 6 — answer

TRUE

TRUE

Near a vertical asymptote the function is undefined — the graph is split into separate branches on either side of the line.

True or false · 7

T / F

Decide, then justify your decision — answer on the next slide

If the numerator has greater degree than the denominator, the horizontal asymptote is y = 0.

True or false · 7 — answer

FALSE

FALSE

N > D means NO horizontal asymptote. y = 0 belongs to the case N < D;
N = D gives y = ratio of leading coefficients.

True or false · 8

T / F

Decide, then justify your decision — answer on the next slide

When solving an inequality with ≤, a zero of the denominator can never be included in the solution.

True or false · 8 — answer

TRUE

TRUE

The expression is undefined there, so it keeps a round bracket — even when the inequality sign would otherwise allow equality.

FIVE THINGS TO DO IN THE EXAM

Do these and complex numbers and rationals will not cost you marks

- Divide powers of i by 4 and use the remainder — never guess from the last digit.
- To divide by a complex number, multiply above and below by the conjugate of the denominator.
- A negative discriminant gives a conjugate pair: simplify $\sqrt{|\Delta|}$ and keep the ± symmetric.
- For rational functions: factor and reduce FIRST — holes vs vertical asymptotes depend on it.
- On sign charts, square-bracket the numerator zeros and round-bracket the denominator zeros.

Built from the 2-01 Complex Numbers through 2-09 Nonlinear Inequalities booklets.