1. Interpreting a Confidence Interval

A 95% confidence interval for the population slope is $(2.4, 5.8)$. Which conclusion is appropriate?

A. The slope is definitely 4.1.
B. We are 95% confident that the population slope is between 2.4 and 5.8.
C. 95% of individual observations have slopes between 2.4 and 5.8.
D. There is no linear relationship.



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2. Testing a Claim of No Linear Relationship

A researcher claims that the population slope is 0. A 95% confidence interval for the slope is $(1.2, 4.6)$. What does this suggest?

A. The claim is supported.
B. The claim is not supported because 0 is not in the interval.
C. The slope must equal 1.2.
D. The correlation must be 0.



One-Sample Mean t-Test — Key Points

- **Purpose:** Test a claim about a population mean μ.
- **Hypotheses:**

$$H_0 : \mu = \mu_0, \qquad H_a : \mu \neq,\ >,\ \text{or}\ < \mu_0$$

- **Test statistic:**

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

- **Degrees of freedom:**

$$df = n - 1$$

- **Conditions:** Random, 10% condition, Normal/Large Sample.
- **P-value < α:** Reject H_0.
- **P-value > α:** Fail to reject H_0.
- **Two-tailed test:** Split α between two tails.
- **Type I error:** Reject a true H_0.
- **Type II error:** Fail to reject a false H_0.
- **Power:**

$$\text{Power} = 1 - \beta$$

FRQ

A company claims that its batteries have a mean lifetime of **10 hours**. A random sample of **36 batteries** has a mean lifetime of **10.8 hours** and a standard deviation of **2.4 hours**.

The company wants to determine whether there is evidence that the true mean battery lifetime is different from 10 hours.

(a) State the hypotheses.

(b) Check the conditions for performing a one-sample t-test.

(c) Calculate the test statistic.

(d) The P-value is approximately **0.025**. Interpret the P-value in context.

(e) Using $\alpha = 0.05$, state your conclusion in context.

(f) Describe a possible Type I error in this situation.

(a)

$$H_0 : \mu = 10$$

$$H_a : \mu \neq 10$$

(b) Random sample; $36 < 10\%$ of the population; sample size is sufficiently large.

(c)

$$t = \frac{10.8 - 10}{2.4/\sqrt{36}} = 2.00$$

(d) Assuming $\mu = 10$, there is about a **2.5% chance** of obtaining a sample mean at least as far from 10 as 10.8.

(e) Since $0.025 < 0.05$, **reject** H_0. There is convincing evidence that the true mean battery lifetime differs from 10 hours.

(f) Concluding that the true mean battery lifetime differs from 10 hours when it actually equals 10 hours.

- **Population slope (β_1)** — 总体斜率 (*zǒng tǐ xié lǜ*)
- **Sample slope (b_1)** — 样本斜率 (*yàng běn xié lǜ*)
- **Confidence interval (CI)** — 置信区间 (*zhì xìn qū jiān*)
- **Significance / statistical significance** — 显著性 / 统计显著性 (*xiǎn zhù xìng / tǒng jì xiǎn zhù xìng*)
- **Evidence supporting the claim** — 支持主张的证据 (*zhī chí zhǔ zhāng de zhèng jù*)

Population Slope β

Describes the true linear relationship between x and y.

Sample Slope b

Estimates the population slope using sample data.

Confidence Interval

Provides a range of plausible values for the population slope.

Zero in the Interval

If **0 is included**, a zero slope is plausible; there is not sufficient evidence of a linear relationship at that confidence level.

Zero Not in the Interval

If **0 is not included**, there is evidence that the population slope is different from zero.

Positive Slope

If the entire interval is above 0, there is evidence of a **positive linear association**.

Negative Slope

If the entire interval is below 0, there is evidence of a **negative linear association**.

Confidence Level

A higher confidence level produces a **wider interval**.

Context Matters

Interpret the slope using the actual variables and their units.

Causation

A confidence interval for the slope can provide evidence of an association, but **does not establish causation**.

A study examines the relationship between the number of hours students sleep per night (x) and their test scores (y). A regression analysis gives a sample slope of $b_1 = 2.5$ with a **95% confidence interval** for the population slope: $(1.2, 3.8)$.

Claim: Each additional hour of sleep **increases test scores**.

a. Does the confidence interval support this claim? Explain.

b. State the **null and alternative hypotheses** for testing whether the slope is different from 0.

- CI = (1.2, 3.8) → all positive → **supports claim**
- Interpretation: Each extra hour of sleep increases scores **by 1.2–3.8 points**
- Hypotheses: $H_0 : \beta_1 = 0, H_1 : \beta_1 > 0$
- Conclusion: Reject H_0; evidence supports a positive slope

Stretch and Challenge

1. Zero on the Boundary

A 95% confidence interval for the slope is $(0,\ 4.2)$.

Question: Can you conclude that the population slope is positive? Explain.

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2. Changing Confidence Levels

A 90% confidence interval for the slope is $(1.8,\ 4.6)$.

Question: Would you expect the corresponding 99% confidence interval to include 0? Explain what information you would need to know.

Stretch and Challenge

3. Comparing Two Intervals

Two studies produce the following 95% confidence intervals:

- Study A: $(2.1,\ 5.4)$
- Study B: $(-0.8,\ 4.9)$

Question: What can you conclude about evidence for a nonzero population slope in each study?

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4. Claim vs. Evidence

A researcher claims that the population slope is **greater than 5**. The 95% confidence interval is $(3.7,\ 6.8)$.

Question: Does this confidence interval provide sufficient evidence to support the claim that the slope is greater than 5? Explain.

1. **Zero on the Boundary:** No; 0 is included, so a zero slope is plausible.
2. **Changing Confidence Levels:** Not necessarily; the 99% interval will be wider.
3. **Comparing Two Intervals:** A: evidence of a nonzero slope. B: insufficient evidence of a nonzero slope.
4. **Claim vs. Evidence:** No; the interval includes values below 5.

Hypothesis testing for the Slope

Population Slope (β)	总体斜率
Sample Slope (b)	样本斜率
Null Hypothesis (H_0)	原假设
Alternative Hypothesis (H_a)	备择假设
Test Statistic (t)	检验统计量
Standard Error of the Slope	斜率的标准误
Degrees of Freedom (df)	自由度
P-value	P值
Significance Level (α)	显著性水平
Statistically Significant	具有统计显著性

Hypothesis testing for the Slope

1. **Population Slope** β — the true slope of the population regression line.
2. **Null Hypothesis** — usually $H_0 : \beta = 0$, meaning no linear relationship in the population.
3. **Alternative Hypothesis** — $H_a : \beta \neq 0$, $H_a : \beta > 0$, or $H_a : \beta < 0$, depending on the claim.
4. **Test Statistic** — compares the sample slope b with the hypothesized population slope.
5. **t-Statistic** — used to test whether the population slope differs significantly from the hypothesized value.
6. **Degrees of Freedom** — for simple linear regression, $df = n - 2$.
7. **P-value** — the probability of obtaining a result at least as extreme as the sample result, assuming H_0 is true.
8. **Significance Level** α — the cutoff used to decide whether the evidence is statistically significant.
9. **Reject** H_0 — if $p\text{-value} < \alpha$.
10. **Fail to Reject** H_0 — if $p\text{-value} \geq \alpha$.

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(a) State the hypotheses.

(b) Check the conditions for performing a one-sample t-test.

(c) Calculate the test statistic.

(d) The P-value is approximately **0.025**. Interpret the P-value in context.

(e) Using $\alpha = 0.05$, state your conclusion in context.

(f) Describe a possible Type I error in this situation.

1. Study Time and Exam Scores

A teacher investigates whether the number of hours students spend studying is related to their final exam scores. A random sample of 25 students is selected. The least-squares regression line gives a sample slope of $\mathbf{b = 4.8}$, with a standard error of **1.6**.

(a) State the appropriate hypotheses for testing whether there is a linear relationship between study time and exam score.

(b) Calculate the test statistic.

(c) State the degrees of freedom.

(d) The P-value is **0.005**. Using $\boldsymbol{\alpha = 0.05}$, make a decision about the null hypothesis.

(e) Interpret the conclusion in context.

(a) $H_0: \beta = 0$; $H_a: \beta \neq 0$

(b) $t = (4.8 - 0)/1.6 = \mathbf{3.00}$

(c) $df = 25 - 2 = \mathbf{23}$

(d) $0.005 < 0.05 \rightarrow$ **Reject H_0**

(e) Convincing evidence of a positive linear relationship between study time and exam score.

TRUE or FALSE

1. Null Hypothesis

H_o: $\beta = 0$ means there is no linear relationship between the two population variables.

TRUE or FALSE

2. Degrees of Freedom

For a simple linear regression, the degrees of freedom for testing the slope is $n - 2$.

TRUE or FALSE

3. P-value

If the P-value is less than α, we reject the null hypothesis.