UNIT 2 KNOWLEDGE - CALCULUS 12 – DIFFERENTIATION: DEFINITION AND FUNDAMENTAL PROPERTIES

2.1
- Defining Average and Instantaneous Rates of Change at a Point ✓

2.2
- Defining the Derivative of a Function and Using Derivative Notation ✓

2.3
- Estimating Derivatives of a Function at a Point ✓

2.4
Connecting Differentiability and Continuity: Determining When Derivatives Do and Do Not Exist ✓

2.5
- Applying the Power Rule ✓

2.6
- Derivative Rules: Constant, Sum, Difference, and Constant Multiple ✓

2.7
- Derivatives of cos x, sin x, e^x , and ln x ✓

2.8
- The Product Rule ✓

2.9
- The Quotient Rule ✓

2.10
- Finding the Derivatives of Tangent, Cotangent, Secant, and/or Cosecant Functions

Turn and Talk

1. Chain Rule

Differentiate:

$$y = \sin(3x^2 + 2x)$$

This requires using the **chain rule** because there's a function inside another function.

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2. Product Rule

Differentiate:

$$y = x^2 e^x$$

This is a product of x^2 and e^x, so you need the **product rule:** $(uv)' = u'v + uv'$.

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3. Quotient Rule

Differentiate:

$$y = \frac{\ln x}{x^3 + 1}$$

This is a fraction of two functions, so you use the **quotient rule:** $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$.

1. Chain Rule

$$y = \sin(3x^2 + 2x) \quad \Rightarrow \quad y' = \cos(3x^2 + 2x) \cdot (6x + 2)$$

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2. Product Rule

$$y = x^2 e^x \quad \Rightarrow \quad y' = 2xe^x + x^2 e^x = e^x(2x + x^2)$$

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3. Quotient Rule

$$y = \frac{\ln x}{x^3 + 1} \quad \Rightarrow \quad y' = \frac{(1/x)(x^3 + 1) - (\ln x)(3x^2)}{(x^3 + 1)^2} = \frac{(x^3 + 1)/x - 3x^2 \ln x}{(x^3 + 1)^2}$$

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1. Derivative – 导数
2. Differentiation – 微分
3. Chain Rule – 链式法则
4. Product Rule – 乘积法则
5. Quotient Rule – 商法则
6. Sine – 正弦
7. Cosine – 余弦
8. Tangent – 正切
9. Secant – 正割
10. Cotangent – 余切

What Will We Learn?

- What are the basic formulas for differentiating the remaining four trigonometric functions - $\tan x, \cot x, \sec x, \csc x$?

RECIPROCAL IDENTITIES

$$\sin(\theta) = \frac{1}{\csc(\theta)} \qquad \csc(\theta) = \frac{1}{\sin(\theta)}$$

$$\cos(\theta) = \frac{1}{\sec(\theta)} \qquad \sec(\theta) = \frac{1}{\cos(\theta)}$$

$$\tan(\theta) = \frac{1}{\cot(\theta)} \qquad \cot(\theta) = \frac{1}{\tan(\theta)}$$

Recall from Precalculus!

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$

Recall the derivatives of our two previously-taught trigonometric function.

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\frac{d}{dx}[\cos x] = -\sin x$$

If the derivatives of these two functions are known, then what about the other four trigonometric functions?
Do each of those have defined derivatives?

Let's take a closer look at $\frac{d}{dx}[\tan x] = \sec^2 x$.

$$\frac{d}{dx}[\tan x] = \frac{d}{dx}\left[\frac{\sin x}{\cos x}\right]$$

Recall the Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Recall Two Trig Identities

$$\sin^2 x + \cos^2 x = 1$$

$$\frac{1}{\cos x} = \sec x$$

Proving Others

$$\frac{d}{dx}(\csc x) =$$

$$\frac{d}{dx}(\cot x) =$$

Four New Trigonometric Derivative Formulas

$$\frac{d}{dx}[\tan x] = \sec^2 x$$

$$\frac{d}{dx}[\cot x] = -\csc^2 x$$

$$\frac{d}{dx}[\sec x] = \sec x \cdot \tan x$$

$$\frac{d}{dx}[\csc x] = -\csc x \cdot \cot x$$

At this point, you have seen the derivative formulas to ALL six trigonometric functions. You will now want to work on memorizing them.

Observation…	
LHS	**RHS**
The 3 real basic trigonometric derivative formulas	➢ The trigonometric words start with c & each of the derivative begin with –ve. ➢ So, the derivative of a trig. that starts with c is always -ve

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\frac{d}{dx}[\tan x] = \sec^2 x$$

$$\frac{d}{dx}[\sec x] = \sec x \cdot \tan x$$

$$\frac{d}{dx}[\cos x] = -\sin x$$

$$\frac{d}{dx}[\cot x] = -\csc^2 x$$

$$\frac{d}{dx}[\csc x] = -\csc x \cdot \cot x$$

Look carefully at the list of formulas on the left and then on the right. What patterns do you notice?

Derivative Function	Domain of the Derivative Function
$\frac{d}{dx}\sin x = \cos x$	
$\frac{d}{dx}\cos x = -\sin x$	
$\frac{d}{dx}\tan x = \sec^2 x$	
$\frac{d}{dx}\cot x = -\csc^2 x$	
$\frac{d}{dx}\csc x = -\csc x \cot x$	
$\frac{d}{dx}\sec x = \sec x \tan x$	

Example: Finding the Derivatives of Tangent, Cotangent, Secant, and Cosecant Functions

Let $f(x) = \sin\pi - \frac{1}{2\cos x} + \frac{1}{3\tan x}$. Which of the following is $f'\left(\frac{\pi}{6}\right)$?

(A) -2 (B) $-\frac{5}{3}$ (C) $\frac{5}{\sqrt{3}}$ (D) $5\sqrt{3}$

Steps	Solution

Reciprocal Identities

$\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

$\sin\theta = \frac{1}{\csc\theta}$ $\cos\theta = \frac{1}{\sec\theta}$ $\tan\theta = \frac{1}{\cot\theta}$

$$\frac{d}{dx}[\tan x] = \sec^2 x$$

$$\frac{d}{dx}[\cot x] = -\csc^2 x$$

$$\frac{d}{dx}[\sec x] = \sec x \cdot \tan x$$

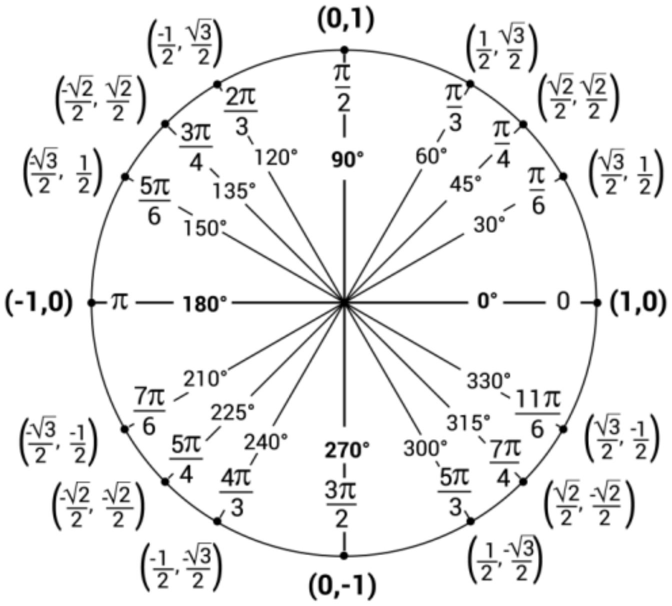
$$\frac{d}{dx}[\csc x] = -\csc x \cdot \cot x$$

Example: Finding the Derivatives of Tangent, Cotangent, Secant, and Cosecant Functions

Let $f(x) = \sin\pi - \frac{1}{2\cos x} + \frac{1}{3\tan x}$. Which of the following is $f'\left(\frac{\pi}{6}\right)$?

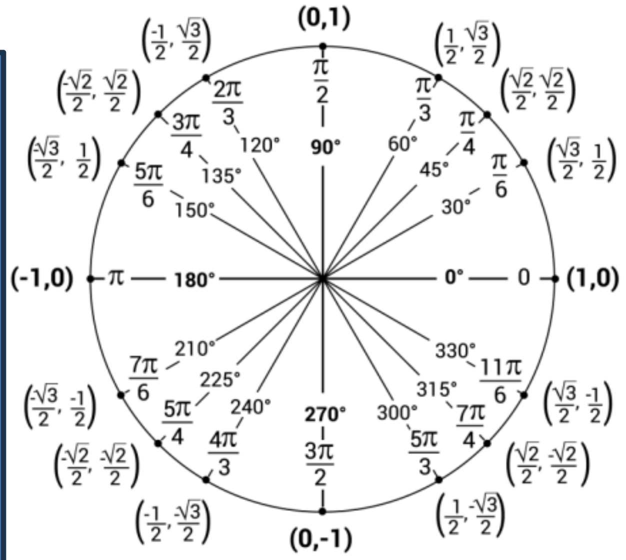
(A) -2 **(B) $-\frac{5}{3}$** (C) $\frac{5}{\sqrt{3}}$ (D) $5\sqrt{3}$

Steps	**Solution**
3rd: solve for the derivative (N.B. π is a constant, so its derivative $= 0$)	$f'(x) = -\frac{1}{2}\sec x\tan x - \frac{1}{3}\csc^2 x$
4th: plug in $\frac{\pi}{6}$ *for x*	$f'(x) = -\frac{1}{2}\sec\left(\frac{\pi}{6}\right)\cdot\tan\left(\frac{\pi}{6}\right) - \frac{1}{3}\csc^2\left(\frac{\pi}{6}\right)$
5th: solve	$= -\frac{1}{2}\cdot\frac{2}{\sqrt{3}}\cdot\frac{1}{\sqrt{3}} - \frac{1}{3}\cdot\left(\frac{2}{1}\right)^2$ $= -\frac{1}{3} - \frac{4}{3} = -\frac{5}{3}$



Example: Finding the Derivatives of Tangent, Cotangent, Secant, and Cosecant Functions

Find $f'\left(\frac{\pi}{6}\right)$ *if* $f(x) = \frac{x}{\sec x}$



$$Quotient\ rule: \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

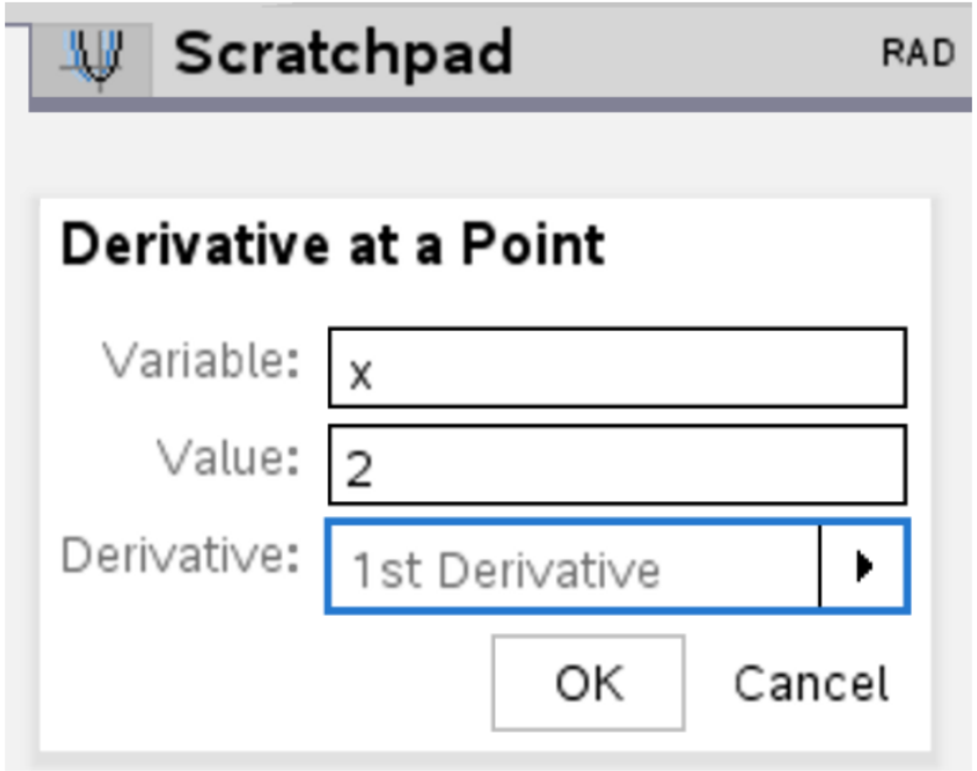
Example: Finding the Derivatives of Tangent, Cotangent, Secant, and Cosecant Functions

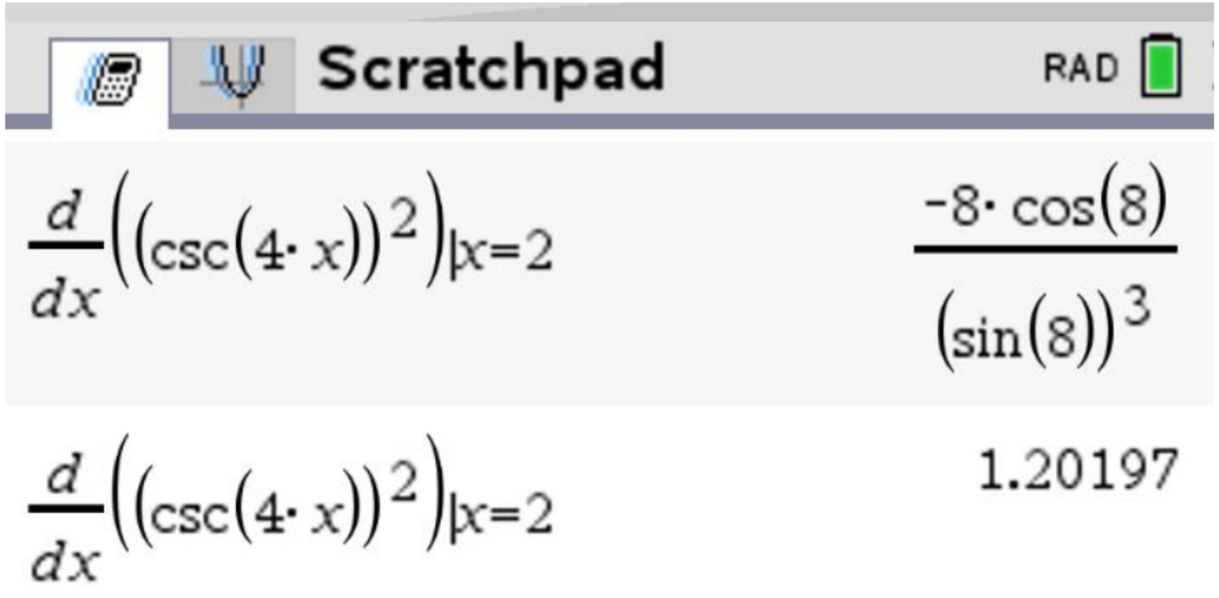
Find $f'\left(\frac{\pi}{6}\right)$ *if* $f(x) = \frac{x}{\sec x}$

Steps	Solution
3rd : solve	$f'\left(\frac{\pi}{6}\right) = \dfrac{\left(\frac{2}{\sqrt{3}}\right) - \left(\frac{\pi}{6}\right)\cdot\left(\frac{2}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right)}{\left(\frac{2}{\sqrt{3}}\right)^2}$
4th : rewrite & solve	$= \left[\left(\frac{2}{\sqrt{3}}\right) - \left(\frac{\pi}{6}\right)\left(\frac{2}{3}\right)\right] \times \left[\frac{3}{4}\right]$
	$= \left[\left(\frac{2}{\sqrt{3}}\right) \times \frac{3}{4}\right] - \left[\left(\frac{2\pi}{18}\right) \times \frac{3}{4}\right]$ $= \frac{6}{4\sqrt{3}} - \frac{2\pi}{24}$ $\boldsymbol{OR = \frac{3}{2\sqrt{3}} - \frac{\pi}{12}} \quad \boldsymbol{OR = \frac{\sqrt{3}}{2} - \frac{\pi}{12}}$

Example: Finding the Derivatives of Tangent, Cotangent, Secant, and Cosecant Functions

Estimate the derivative with a calculator of $g(x) = csc^2\, 4x$ at $x = 2$





$$g'(2) \approx 1.20197$$

Menu-> calculus-> derivative at a point (value is 2)->enter->
type the function-> press CRTL & enter to see the approx. value.

Summary/Key Takeaways

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\frac{d}{dx}[\cos x] = -\sin x$$

$$\frac{d}{dx}[\tan x] = \sec^2 x$$

$$\frac{d}{dx}[\cot x] = -\csc^2 x$$

$$\frac{d}{dx}[\sec x] = \sec x \cdot \tan x$$

$$\frac{d}{dx}[\csc x] = -\csc x \cdot \cot x$$

TRUE or FALSE

1. The derivative of $\tan(x^2)$ is $2x\sec^2(x^2)$.
2. The derivative of $\sin x + \cos x$ is $\cos x - \sin x$.
3. The derivative of $\sin^2 x$ is $2\sin x$.