What is the key vocabulary

A. A study claims that **less than 40% of adults exercise daily**, but a sample shows **38%**. This is the statement being tested.

B. A student claims that **the sample proves the null hypothesis is false.**

C. Out of **50 respondents**, 35 report a behavior; the proportion 0.7 is calculated for the test.

D. After a hypothesis test, the researcher writes: *"The p-value is very small, so the difference is unlikely to be due to chance."*

E. A teacher sets the **maximum probability of Type I error at 0.01** before conducting a test.

1 → A *(Null Hypothesis: statement being tested)*

2 → B *(Alternative Hypothesis is what we suspect; here, the student's claim is* ***misconception*** *– could be used to discuss why H_0 cannot be "proven false")*

3 → C *(Sample Proportion: 35/50 = 0.7)*

4 → D *(P-value: probability of observing extreme data under H_0; small p-value indicates unusual under H_0)*

5 → E *(Significance Level: $\alpha = 0.01$)*

Concluding a test for a population parameter

1. Null Hypothesis (H_0) — 原假设
2. Alternative Hypothesis (H_a) — 备择假设
3. P-value — p值
4. Significance Level (α) — 显著性水平
5. Statistically Significant — 统计显著

To **conclude a test for a population proportion**, compare the **p-value to the significance level (α)**.

If the **p-value ≤ α**, reject the null hypothesis and state that there is **convincing statistical evidence** for the alternative claim.

If the **p-value > α**, fail to reject the null hypothesis and state that there is **not convincing statistical evidence** for the alternative claim.

Your conclusion must be written **in context**, referring to the population proportion being tested.

Do **not** say you "accept" the null hypothesis—only **reject** or **fail to reject** it.

Students in an online course are each randomly assigned to receive either standard practice exercises or adaptive practice exercises. For the adaptive practice exercises, the next question asked is determined by whether the student got the previous question correct. The teacher of the course wants to determine whether there is a difference between the two practice exercise types by comparing the proportion of students who pass the course from each group. The teacher plans to test the null hypothesis that $p_S = p_A$ versus the alternative hypothesis $p_S \neq p_A$, where p_S represents the proportion of students who would pass the course using standard practice exercises and p_A represents the proportion of students who would pass the course using adaptive practice exercises.

The teacher knows that the 99 percent confidence interval for the difference in proportion of students passing the course for the two practice exercise types (standard minus adaptive) is $(0.03, 0.24)$ and the 90 percent confidence interval for the difference in the proportion of students passing the course for the two practice exercise types (standard minus adaptive) is $(0.06, 0.21)$. Which of the following is the most reasonable conclusion for the teacher's hypothesis test at the $\alpha = 0.05$ level?

(A) The p-value is less than 0.01, so there is convincing statistical evidence that the proportion of students who would pass the course for the two practice exercise types differs at the 5% level.

(B) The p-value is between 0.01 and 0.10, so there is convincing statistical evidence that the proportion of students who would pass the course for the two practice exercise types differs at the 5% level.

(C) The p-value is between 0.01 and 0.10, so the researchers cannot determine whether the proportion of students who would pass the course for the two practice exercise types differs at the 5% level.

A

(D) The p-value is between 0.01 and 0.10, so there is not convincing statistical evidence that the proportion of students who would pass the course for the two practice exercise types differs at the 5% level.

(E) The p-value is greater than 0.10, so there is not convincing statistical evidence that the proportion of students who would pass the course for the two practice exercise types differs at the 5% level.

Stretch and Challenge questions

Question 1 — Interpreting the Conclusion

A school claims that **60% of students prefer online homework**.

A sample of **150 students** finds **78 prefer online homework**.

A hypothesis test gives **p-value = 0.018** at **α = 0.05**.

a) State the decision.

b) Write a correct statistical conclusion in context.

c) Explain whether this proves that the true proportion is different from 0.60.

Question 2 — Identifying an Incorrect Conclusion

A student writes:

> "Because the p-value is smaller than α, we know the null hypothesis is false."

a) Explain why this statement is **incorrect**.

b) Rewrite the conclusion **correctly using statistical language**.

Q1

- Reject H_0
- Evidence that the true proportion of students preferring online homework **differs from 0.60**
- Does **not prove**, only **provides evidence**

Q2

- Hypothesis tests **do not prove H_0 false**
- Correct: *There is sufficient evidence to reject the null hypothesis*

Confidence Interval for a population proportion

1. **Population Proportion (p) — 总体比例**
2. **Sample Proportion ($\hat{p}$) — 样本比例**
3. **Margin of Error (ME) — 误差幅度**
4. **Confidence Level — 置信水平**
5. *Critical Value (z)** — 临界值（z值）

1. **Estimate the Population Proportion** – A confidence interval provides a **range of plausible values** for the true proportion p.
2. **Use Sample Proportion and Standard Error** – The interval is calculated using $\hat{p}$ and the standard error $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$.
3. **Choose a Confidence Level** – Common levels are **90%, 95%, or 99%**, which determine the critical value z^*.
4. **Interpret Correctly** – A 95% CI means the method would capture the true proportion in **95% of repeated samples**, not that there's a 95% chance the specific interval contains p.
5. **Check Conditions** – Random sample, independent observations, and enough successes/failures ($n\hat{p} \geq 10$ and $n(1-\hat{p}) \geq 10$) to use the normal approximation.

A university claims that **70% of its students participate in at least one extracurricular activity**. A random sample of **150 students** finds that **95 students participate** in at least one activity.

a) Construct a **95% confidence interval** for the proportion of all students who participate in at least one activity.

b) Interpret the confidence interval in **context**.

c) Based on the interval, evaluate whether the university's claim of 70% is reasonable.

d) State any **conditions** needed for the confidence interval to be valid.

a) Sample proportion:

$$\hat{p} = 95/150 = 0.633$$

b) Standard error:

$$SE = \sqrt{\frac{0.633(1-0.633)}{150}} \approx 0.041$$

c) Margin of error (95% CI):

$$ME = 1.96 \times 0.041 \approx 0.080$$

d) Confidence interval:

$$0.633 \pm 0.080 \implies (0.553, 0.713)$$

e) Interpretation:

95% confident that the true proportion of students participating is **between 55.3% and 71.3%**.

f) Evaluate university claim:

70% is in the interval → claim is **plausible**.

g) Conditions for validity:

- Random sample ✓
- Independent observations ✓
- Normal approximation ✓ ($n\hat{p} \geq 10$, $n(1-\hat{p}) \geq 10$)

True or False

A 95% confidence interval for a population proportion means:

"There is a 95% probability that the true population proportion lies within this specific interval calculated from the sample."

False

True or False

Increasing the **sample size** while keeping the confidence level the same will **decrease the width** of the confidence interval.

TRUE

True or False

Using a **99% confidence level** instead of a 95% confidence level will **always make the interval narrower**.

FALSE

Confidence interval for the difference between two proportions – what is the key word

- The interval is constructed by taking the difference between two sample proportions and adding and subtracting a value that depends on the critical value and the variability of the estimator.

[redacted]

- When constructing the interval, the variability is calculated using the individual sample proportions rather than combining the two samples into one estimate.

[redacted]

- If many samples of the same size were taken and an interval were built from each one, about 95% of those intervals would contain the true difference in population proportions.

[redacted]

- The quantity p1−p2. represents the true but unknown value that the interval is designed to estimate.

[redacted]

A school district wants to compare the proportion of students who pass a standardized math test in **School A** and **School B**.

- In a random sample of **120 students from School A, 78 students pass.**
- In a random sample of **150 students from School B, 90 students pass**.

a) Construct a **95% confidence interval** for the difference in the proportions of students passing the test between School A and School B ($p_A - p_B$).

b) Interpret the confidence interval in context.

c) Based on your interval, determine whether there is evidence of a difference between the two schools' passing rates.

d) State the conditions necessary for this confidence interval to be valid.

a) Sample proportions:

$$\hat{p}_A = 78/120 = 0.65, \quad \hat{p}_B = 90/150 = 0.60$$

b) Standard error:

$$SE = \sqrt{\frac{0.65(1-0.65)}{120} + \frac{0.60(1-0.60)}{150}} \approx 0.059$$

c) Margin of error (95% CI):

$$ME = 1.96 \times 0.059 \approx 0.116$$

d) Confidence interval:

$$0.05 \pm 0.116 \implies (-0.066, 0.166)$$

e) Interpretation:

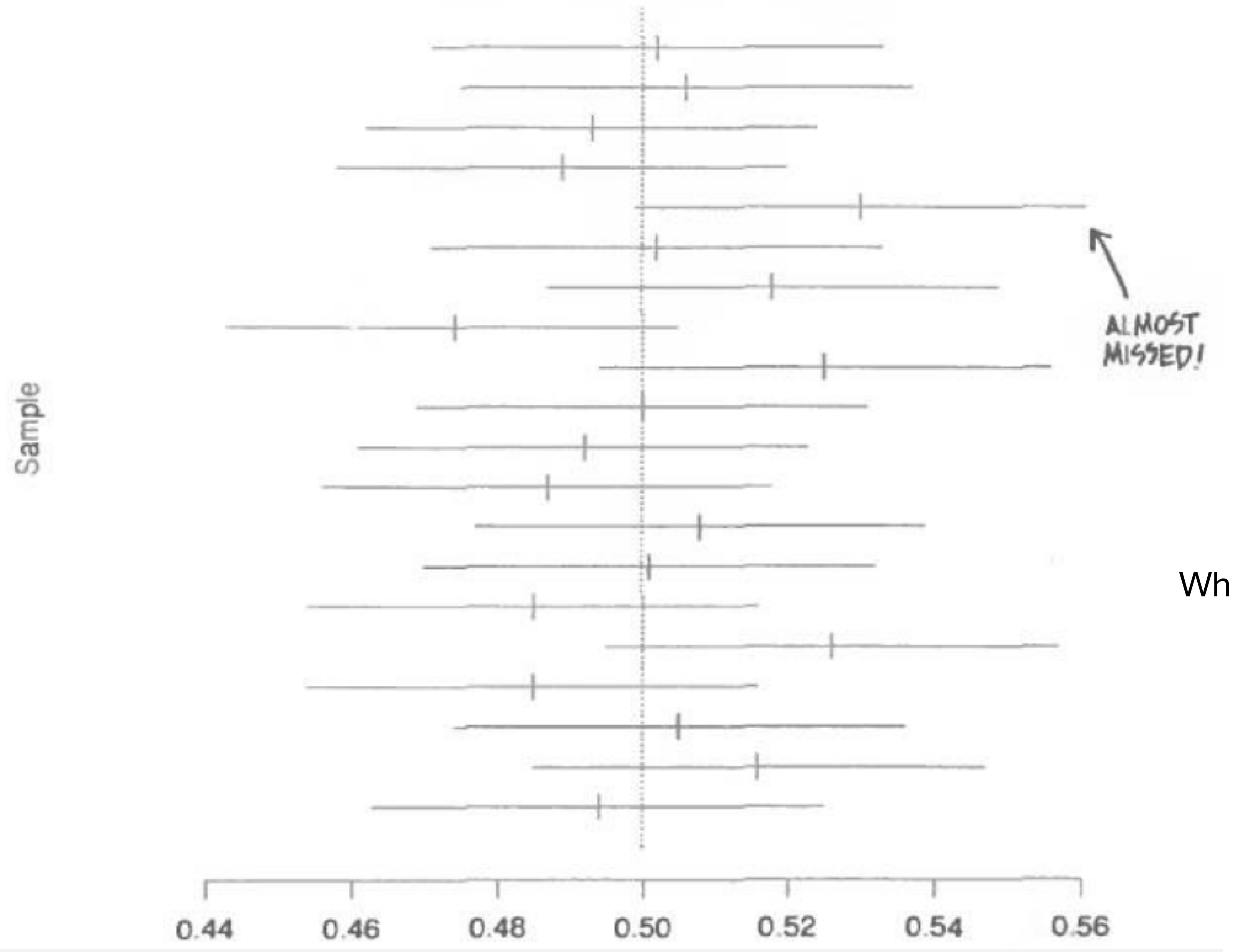
95% confident that the proportion of students passing in School A is **between 6.6% lower and 16.6% higher than School B**.

f) Evidence of difference:

0 is in the interval → **no strong evidence of a difference**.

g) Conditions for validity:

- Random samples ✓
- Independent observations ✓
- Normal approximation ✓ ($n\hat{p} \geq 10, n(1-\hat{p}) \geq 10$ for both groups)



What is the confidence interval?

Suppose we construct a 95% confidence interval for the true mean of a population. If the true mean is actually 0.5, which of the following statements is correct?

A) There is a 95% chance that the true mean is 0.5.

B) 95% of the time, the confidence interval will contain 0.5.

C) The probability that any particular interval contains the true mean is 0.5.

D) The confidence interval always contains the true mean.

B

Bernouilli Trials

Show that

$$X_i = \begin{cases} 1 & \text{if the } i^{th} \text{ observation is a success} \\ 0 & \text{if the } i^{th} \text{ observation is a failure} \end{cases}$$

$$E(X_i) = p$$
$$Var(X_i) = p(1-p)$$

$$E(X) = \sum_{i=1}^{n} x_i P(X = x_i)$$

$$E(X) = 1p + 0q = p$$

$$Var(X) = E(X^2) - E(X)^2$$

$$E(X^2) = \sum_{i=1}^{n} x_i^2 P(X = x_i)$$

$$E(X^2) = 1^2 p + 0^2 q = p$$

$$Var(x) = p - p^2 = pq$$

Main properties of variance

1. Non-negative

$$Var(X) \geq 0$$

2. Variance of a constant

$$Var(c) = 0$$

3. Adding a constant does not change variance

$$Var(X + c) = Var(X)$$

4. Scaling rule

$$Var(aX) = a^2 Var(X)$$

5. Sum of independent variables

$$Var(X + Y) = Var(X) + Var(Y)$$

6. Difference of independent variables

$$Var(X - Y) = Var(X) + Var(Y)$$

Show that the variance of the sample proportion

The **sample proportion** is the mean of these Bernoulli variables:

$$\hat{p} = \frac{X_1 + X_2 + \cdots + X_n}{n}$$

$$\frac{p(1-p)}{n}$$

$$Var\left(\sum_{i=1}^{n}\frac{x_i}{n}\right) = Var\left(\frac{(x_1 + x_2 + \ldots.. + x_n)}{n}\right)$$

$$Var\left(\sum_{i=1}^{n}\frac{x_i}{n}\right) = \frac{1}{n^2}Var\left((x_1 + x_2 + \ldots.. + x_n)\right)$$

$$Var\left(\sum_{i=1}^{n}\frac{x_i}{n}\right) = \frac{1}{n^2}\left[Var(x_1) + \ldots.Var(x_n)\right]$$

$$Var\left(\sum_{i=1}^{n}\frac{x_i}{n}\right) = \frac{1}{n^2}\left[Var(X) + \ldots.. + Var(X)\right]$$

$$Var\left(\sum_{i=1}^{n}\frac{x_i}{n}\right) = \frac{1}{n^2}\left[nVar(X)\right]$$

$$Var\left(\sum_{i=1}^{n}\frac{x_i}{n}\right) = \frac{Var(X)}{n} = \frac{pq}{n}$$