Set Operations: Union, Intersection, Difference and Complement

Let

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A = \{1, 2, 3, 4, 5\}$$

$$B = \{4, 5, 6, 7\}$$

Questions

1. Is A a subset of U?
2. Is B a subset of U?
3. Find $n(A)$.
4. Find $n(B)$.
5. Which elements are common to both A and B?

1. Yes
2. Yes
3. 5
4. 4
5. $\{4, 5\}$

English	Chinese
Set	集合
Union	并集
Intersection	交集
Difference	差集
Complement	补集
Universal set	全集
Element	元素
Venn diagram	韦恩图
Belongs to	属于
Does not belong to	不属于

There are four important operations:

Operation	Symbol	Meaning
Union	$A \cup B$	In A **or** B, or both
Intersection	$A \cap B$	In **both** A and B
Difference	$A - B$	In A, **but not** in B
Complement	A'	In the universal set but **not** in A

3. Union

Definition

The **union** of A and B contains every element that is in A, B, or both.

$$A \cup B$$

Using our sets:

$$A = \{1, 2, 3, 4, 5\}$$

$$B = \{4, 5, 6, 7\}$$

Therefore:

$$A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$$

Key idea

Union = OR

Students should remember:

> **Union collects everything from both sets without repeating elements.**

4. Intersection

Definition

The **intersection** contains elements that belong to **both** sets.

$$A \cap B$$

Therefore:

$$A \cap B = \{4, 5\}$$

Key idea

Intersection = AND

Students should remember:

> **Intersection means the elements the sets have in common.**

5. Difference

Definition

The difference $A - B$ contains elements that are in A but **not** in B.

$$A - B$$

Therefore:

$$A - B = \{1, 2, 3\}$$

But:

$$B - A = \{6, 7\}$$

Important Point

$$A - B \neq B - A$$

Set difference is **not commutative**.

6. Complement

Introduce the **universal set**:

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

If

$$A = \{1, 2, 3, 4, 5\}$$

then the complement of A is:

$$A' = \{6, 7, 8, 9, 10\}$$

Definition

The complement contains everything in the **universal set that is not in A**.

Key idea

> **Complement = everything outside the set.**

7. Venn Diagram Activity

Draw two overlapping circles labelled A and B.

Using:

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A = \{1, 2, 3, 4, 5\}$$

$$B = \{4, 5, 6, 7\}$$

Ask students to place each number in the correct region.

They should identify:

- **A only:** $\{1, 2, 3\}$
- **Both:** $\{4, 5\}$
- **B only:** $\{6, 7\}$
- **Neither:** $\{8, 9, 10\}$

Then ask:

1. Shade $A \cup B$.
2. Shade $A \cap B$.
3. Shade $A - B$.
4. Shade A'.

8. Guided Practice

Let

$$P = \{2, 4, 6, 8, 10\}$$

$$Q = \{1, 2, 3, 4, 5\}$$

Find:

1. $P \cup Q$
2. $P \cap Q$
3. $P - Q$
4. $Q - P$

Answers

$$P \cup Q = \{1, 2, 3, 4, 5, 6, 8, 10\}$$

$$P \cap Q = \{2, 4\}$$

$$P - Q = \{6, 8, 10\}$$

$$Q - P = \{1, 3, 5\}$$

9. Independent Practice

Let

$$U = \{1, 2, 3, \ldots, 12\}$$

$$A = \{1, 2, 3, 4, 5, 6\}$$

$$B = \{4, 5, 6, 7, 8\}$$

Find:

1. $A \cup B$
2. $A \cap B$
3. $A - B$
4. $B - A$
5. A'
6. B'
7. $(A \cup B)'$
8. $(A \cap B)'$

Answers

1. $\{1, 2, 3, 4, 5, 6, 7, 8\}$
2. $\{4, 5, 6\}$
3. $\{1, 2, 3\}$
4. $\{7, 8\}$
5. $\{7, 8, 9, 10, 11, 12\}$
6. $\{1, 2, 3, 6, 9, 10, 11, 12\}$
7. $\{9, 10, 11, 12\}$
8. $\{1, 2, 3, 7, 8, 9, 10, 11, 12\}$

10. Stretch and Challenge

Let

$$U = \{1, 2, \dots, 20\}$$

A = multiples of 2

B = multiples of 3

Ask students:

1. Find $A \cap B$.
2. Find $A \cup B$.
3. Find $A - B$.
4. Describe $A \cap B$ in words.
5. What does A' represent?
6. Find $n(A \cup B)$.
7. Find $n(A \cap B)$.

Challenge Question

Explain why:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

This provides an excellent bridge into **Venn diagrams and the Inclusion-Exclusion Principle**.

Given $U = \{1, 2, \ldots, 20\}$:

- $A = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$ — multiples of 2
- $B = \{3, 6, 9, 12, 15, 18\}$ — multiples of 3

1. $A \cap B$

$$\{6, 12, 18\}$$

2. $A \cup B$

$$\{2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20\}$$

3. $A - B$

$$\{2, 4, 8, 10, 14, 16, 20\}$$

4. **Describe $A \cap B$**
   Multiples of **both 2 and 3**, i.e. multiples of **6**.
5. **What does A' represent?**
   Numbers in U that are **not multiples of 2**:

$$A' = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$$

6. $n(A \cup B)$

$$\boxed{13}$$

7. $n(A \cap B)$

$$\boxed{3}$$

Challenge

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$= 10 + 6 - 3 = \boxed{13}$$

11. Exit Ticket

Given:

$$U = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$A = \{1, 2, 3, 4\}$$

$$B = \{3, 4, 5, 6\}$$

Find:

1. $A \cup B$
2. $A \cap B$
3. $A - B$
4. A'

Answers

1. $\{1, 2, 3, 4, 5, 6\}$
2. $\{3, 4\}$
3. $\{1, 2\}$
4. $\{5, 6, 7, 8\}$

TRUE or FALSE

If $A - B = \emptyset$, then every element of A is also in B.

TRUE or FALSE

If $A = \{1, 2, 3\}$ and $B = \{2, 3\}$, then $A - B = \{1\}$.

TRUE or FALSE

If $A \cap B = \emptyset$, then A and B have no common elements.

Power Set

2^n	子集数量公式
$\emptyset$	空集
$\subseteq$	子集
$\subset$	真子集

Set	集合
Element	元素
Subset	子集
Proper Subset	真子集
Empty Set (Null Set)	空集
Universal Set	全集
Power Set	幂集
Cardinality	基数（集合大小
Member Of (∈)	属于
Not a Member Of (∉)	不属于
Contains	包含
Enumeration	列举法
Binary Choice	二元选择
Number of Subsets	子集个数
Set Builder Notation	描述法

Question

Let

$$A = \{1, 2, 3, 4\}$$

(a) Define the power set $P(A)$.

(b) Determine the number of elements in $P(A)$.

(c) Explain why the number of subsets of A is 2^4.

(d) List all the **proper subsets** of A that contain the element 1.

(e) A set B has 7 elements. Determine the number of elements in $P(B)$.

(f) **Stretch:** A set C has n elements. Explain why

$$|P(C)| = 2^n.$$

(a) $P(A)$ is the set containing **all subsets of** A.

(b)

$$|P(A)| = 2^4 = \boxed{16}$$

(c) Each of the 4 elements has 2 choices: **included or excluded**.

(d)

$$\{1\}, \{1,2\}, \{1,3\}, \{1,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}$$

(e)

$$2^7 = \boxed{128}$$

(f) Each of the n elements has 2 independent choices, giving

$$\underbrace{2 \times 2 \times \cdots \times 2}_{n \text{ times}} = \boxed{2^n}.$$