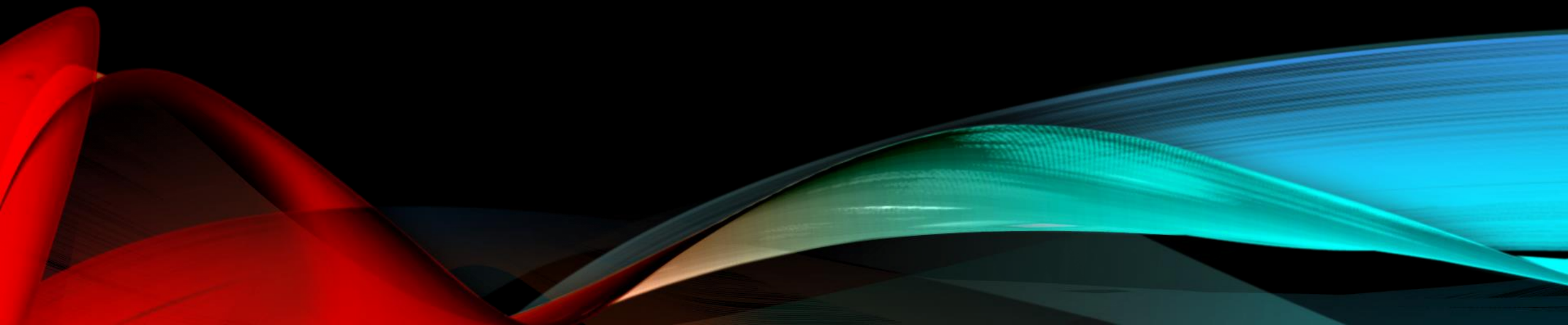
2-02 QUADRATIC EQUATIONS

In this section, you will:

- Recognize characteristics of parabolas.
- Understand how the graph of a parabola is related to its quadratic function.
- Determine a quadratic function's minimum or maximum value.
- Solve problems involving a quadratic function's minimum or maximum value.

TURN AND TALK?

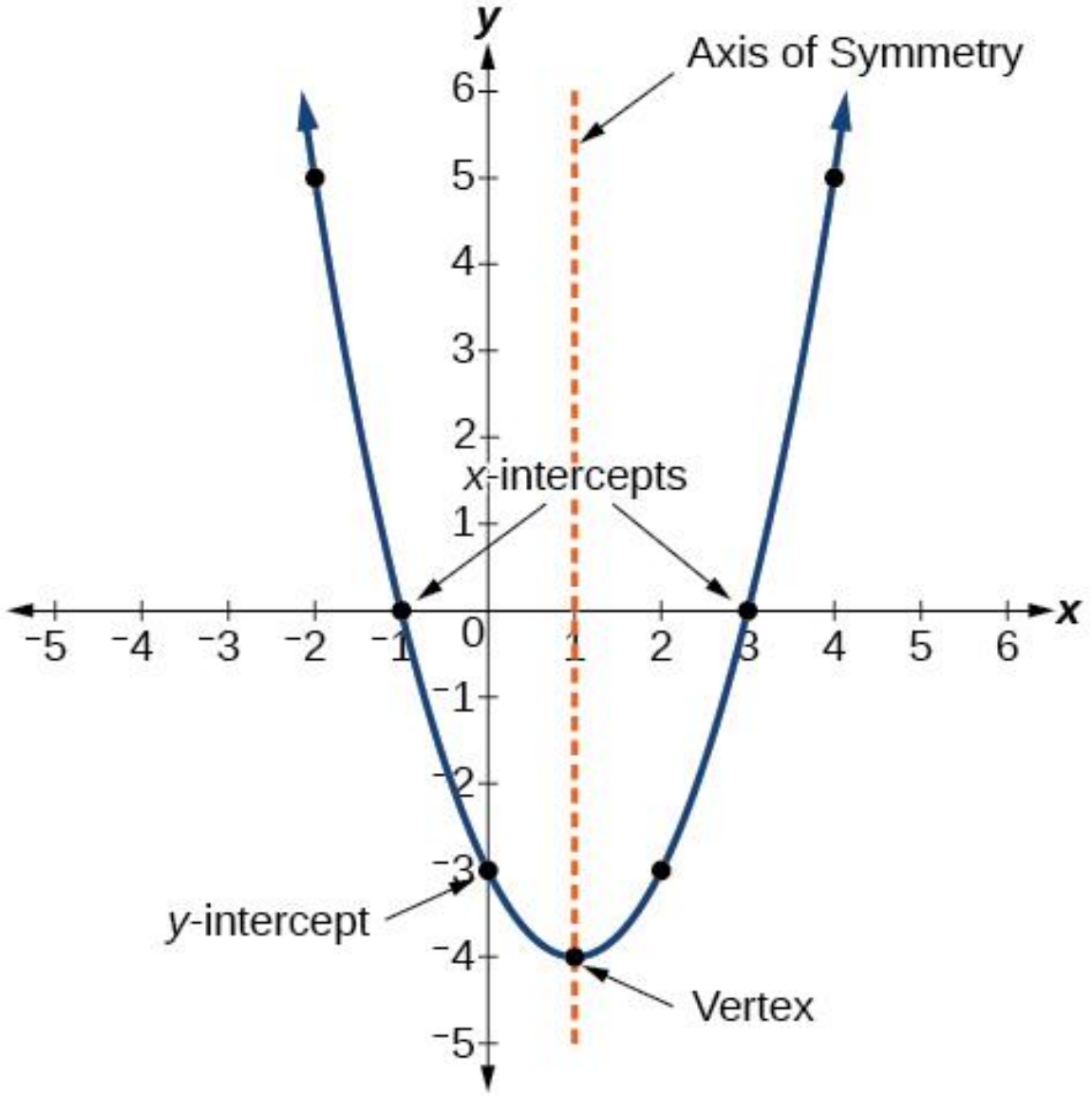
How can you tell a quadratic equation apart from a linear equation?

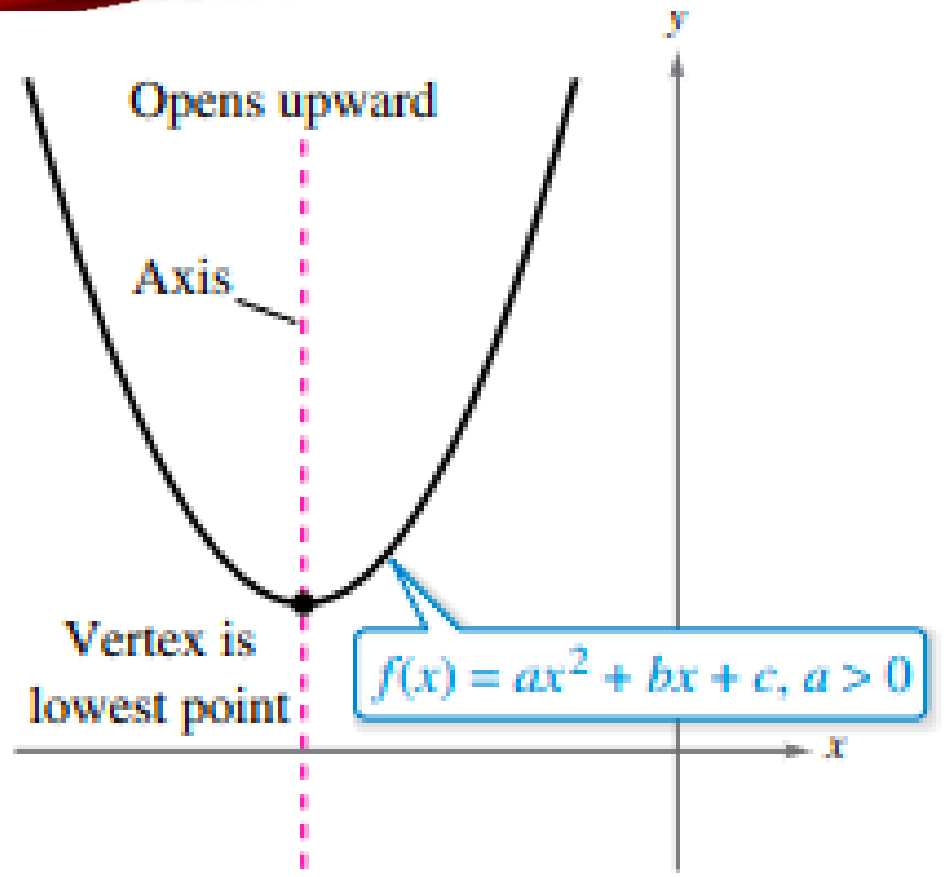


2-02 QUADRATIC EQUATIONS

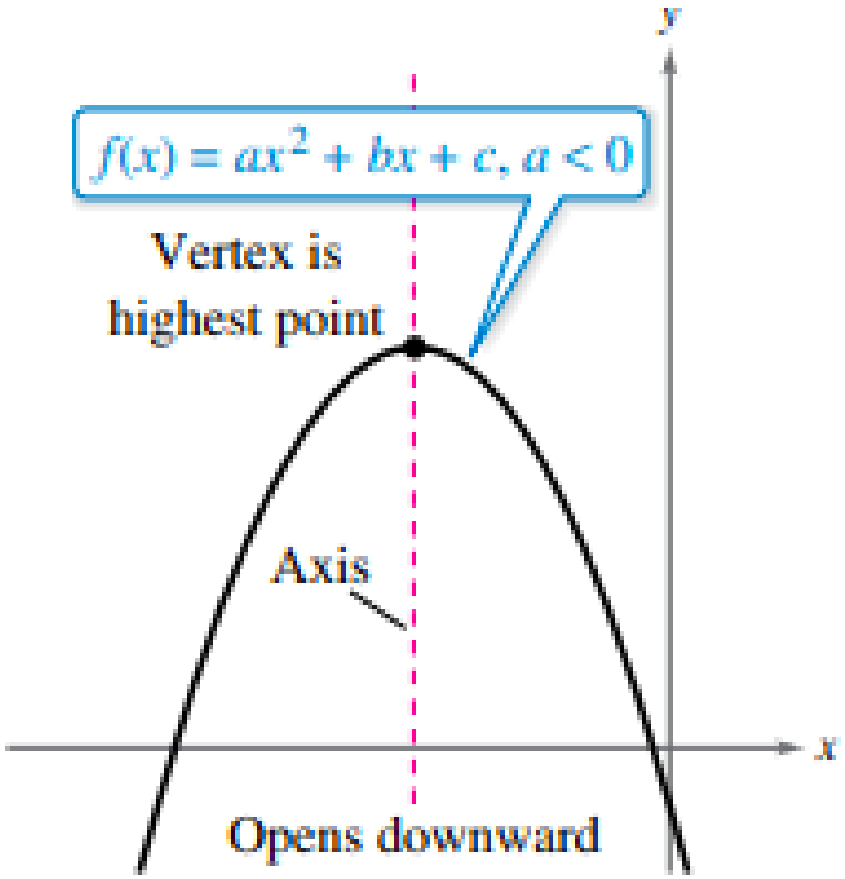
- $f(x) = ax^2 + bx + c$

- $|a| > 1$ → thinner
- $0 < |a| < 1$ → wider
- $a < 0$ → reflected over x-axis "opens down"
- $a > 0$ → "opens up"





Leading coefficient is positive.



Leading coefficient is negative.

COMPLETING THE SQUARE

Completing the square is used to change a quadratic from **general form**

$$y = ax^2 + bx + c$$

into **standard/vertex form**

$$\boxed{y = a(x - h)^2 + k}$$

Example 1: Basic

$$y = x^2 + 6x + 5$$

1. Complete the square:

$$x^2 + 8x + 7$$

2. Complete the square:

$$x^2 - 10x + 6$$

3. Complete the square:

$$x^2 + 4x - 12$$

4. Complete the square:

$$x^2 - 6x + 2$$

5. Complete the square:

$$2x^2 + 8x + 3$$

6. Complete the square:

$$3x^2 - 12x + 5$$

7. Complete the square:

$$-x^2 + 6x - 4$$

8. Complete the square:

$$-2x^2 - 12x + 7$$

1. $(x+4)^2 - 9$
2. $(x-5)^2 - 19$
3. $(x+2)^2 - 16$
4. $(x-3)^2 - 7$
5. $2(x+2)^2 - 5$
6. $3(x-2)^2 - 7$
7. $-(x-3)^2 + 5$
8. $-2(x+3)^2 + 25$

2-02 QUADRATIC EQUATIONS

- Standard Form
- $f(x) = a(x-h)^2 + k$
  - Vertex (h, k)
  - Axis of symmetry $x = h$
- To graph
  1. Find vertex
  2. Make table around vertex
  3. Draw parabola

- General Form
- $f(x) = ax^2 + bx + c$
  - Vertex $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$
  - Axis $x = -\frac{b}{2a}$

1. Expand:

$$y = 2(x-3)^2 + 5$$

2. Expand:

$$y = -(x+4)^2 + 7$$

3. Expand:

$$y = 3(x-2)^2 - 4$$

4. Expand:

$$y = -2(x-5)^2 + 9$$

5. Expand:

$$y = \frac{1}{2}(x+6)^2 - 3$$

1. $y = 2x^2 - 12x + 23$
2. $y = -x^2 - 8x - 9$
3. $y = 3x^2 - 12x + 8$
4. $y = -2x^2 + 20x - 41$
5. $y = \frac{1}{2}x^2 + 6x + 15$

B. Convert General Form → Standard Form

6. Rewrite in standard form:

$$y = x^2 - 6x + 11$$

7. Rewrite in standard form:

$$y = x^2 + 8x + 10$$

8. Rewrite in standard form:

$$y = 2x^2 - 12x + 7$$

9. Rewrite in standard form:

$$y = -x^2 + 10x - 3$$

10. Rewrite in standard form:

$$y = 3x^2 + 18x + 20$$

B. Convert General Form → Standard Form

6. Rewrite in standard form:

$$y = x^2 - 6x + 11$$

7. Rewrite in standard form:

$$y = x^2 + 8x + 10$$

8. Rewrite in standard form:

$$y = 2x^2 - 12x + 7$$

9. Rewrite in standard form:

$$y = -x^2 + 10x - 3$$

10. Rewrite in standard form:

$$y = 3x^2 + 18x + 20$$

6. $y = (x - 3)^2 + 2$
7. $y = (x + 4)^2 - 6$
8. $y = 2(x - 3)^2 - 11$
9. $y = -(x - 5)^2 + 22$
10. $y = 3(x + 3)^2 - 7$

2-02 QUADRATIC EQUATIONS

- Graph $f(x) = x^2 - 10x + 25$ and identify the vertex and axis of symmetry

2-02 QUADRATIC EQUATIONS

- Write the standard form of the equation of parabola with vertex $(-4, 11)$ and passes through $(-6, 15)$

2-02 QUADRATIC EQUATIONS

- Maximum and minimum
- Occurs at the vertex

- Solve $8x^2 + 14x + 9 = 0$

- Quadratic formula
- $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

QUADRATIC FORMULA

1. $x^2 - 5x + 6 = 0$
2. $x^2 + 7x + 10 = 0$
3. $2x^2 + 5x - 3 = 0$
4. $3x^2 - 7x - 6 = 0$
5. $4x^2 + 4x - 15 = 0$

1. $x = 2, 3$
2. $x = -5, -2$
3. $x = \frac{1}{2}, -3$
4. $x = 3, -\frac{2}{3}$
5. $x = \frac{3}{2}, -\frac{5}{2}$

TRUE OR FALSE

- A quadratic equation always has an x^2 term.

TRUE

TRUE OR FALSE

- All quadratic equations have exactly two solutions.

FALSE

TRUE OR FALSE

- The highest power of the variable in a quadratic equation is 2.

TRUE

2-03 POLYNOMIAL EQUATIONS

In this section, you will:

- Identify polynomial functions.
- Identify the end behavior.
- Graph polynomial functions.
- Write polynomial functions.

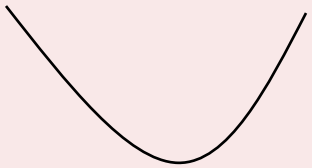
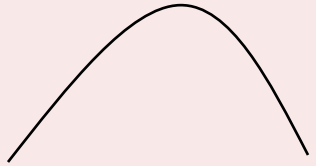
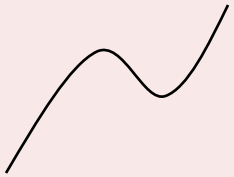
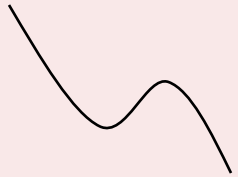
2-03 POLYNOMIAL EQUATIONS

- Polynomial Function
- $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$

- a_n are coefficients
- $a_n x^n$ are terms
- a_0 is constant term
- Degree is highest exponent
- Leading coefficient is coefficient of term with highest exponent
- Graphs are continuous, smooth, rounded turns

2-03 POLYNOMIAL EQUATIONS

- End Behavior
  - Polynomial functions always go towards ∞ or -∞ at either end of the graph

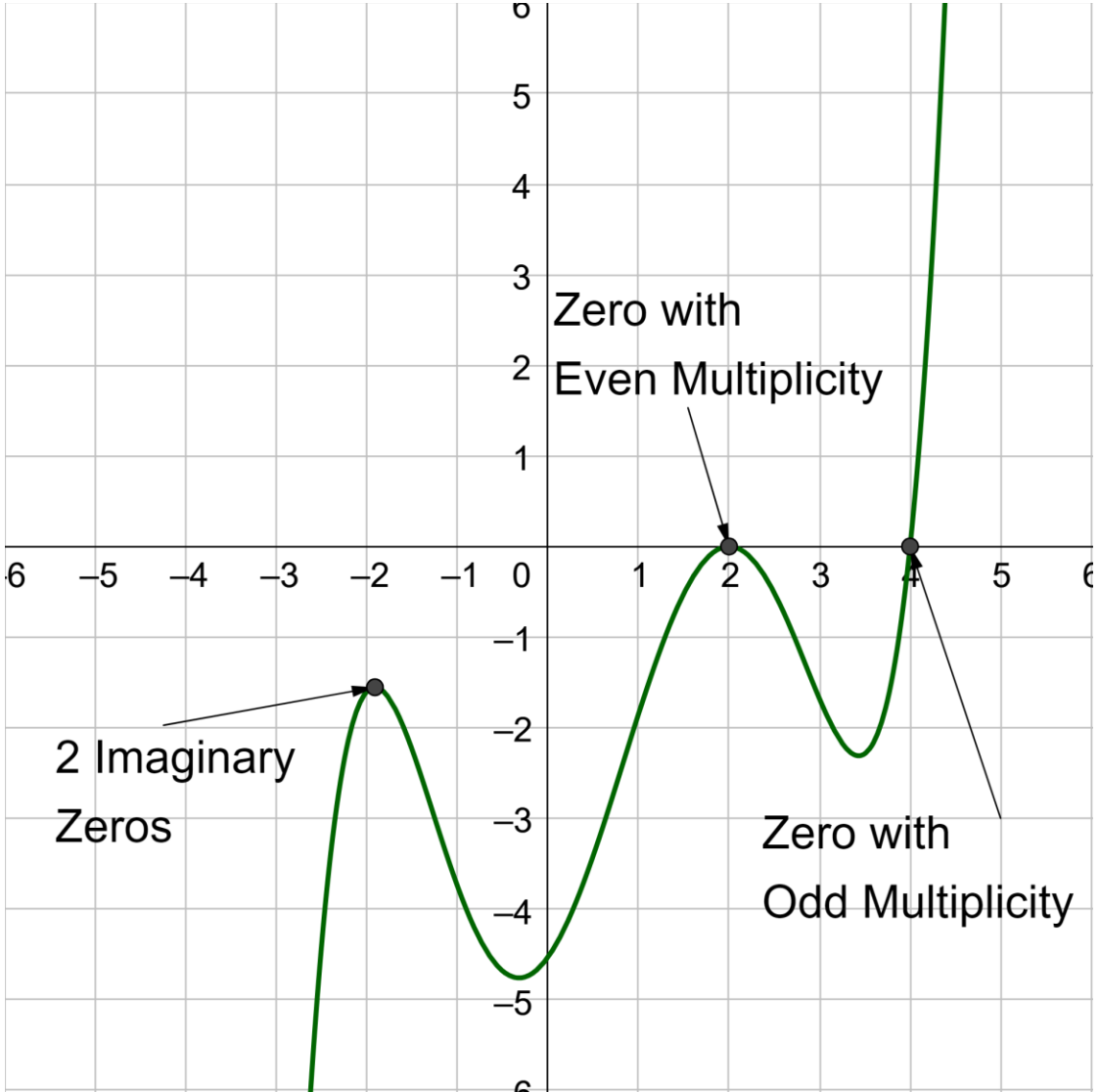
	Leading Coefficient +	Leading Coefficient -
Even Degree		
Odd Degree		

- What is the end behavior of $f(x) = \frac{1}{3}x^3 + 5x$?

2-03 POLYNOMIAL EQUATIONS

- Zeros
- If $x = a$ is a zero of $f(x)$
  - $x = a$ is a solution to $f(x) = 0$
  - $(a, 0)$ is x-intercept
  - $(x - a)$ is a factor of $f(x)$
- Let $f(x)$ have degree n
  - At most n zeros (can be repeated)
  - At most $n - 1$ turning points

2-03 POLYNOMIAL EQUATIONS



2-03 POLYNOMIAL EQUATIONS

For $g(t) = t^5 - 6t^3 + 9t$

a. Find all zeros

b. Find multiplicity of zeros

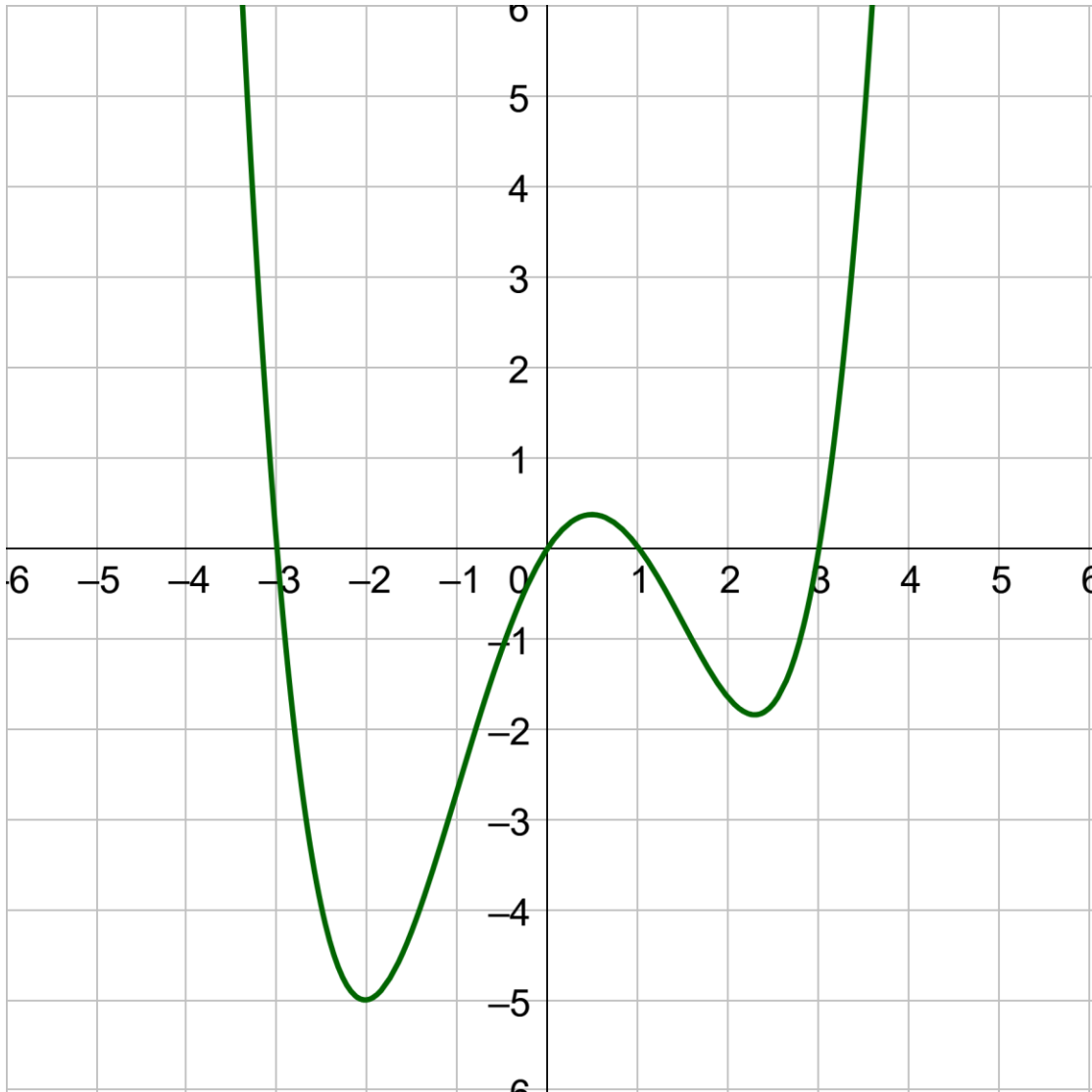
c. Graph

2-03 POLYNOMIAL EQUATIONS

- Find the intercepts of $f(x) = x(x+2)(x-3)$

2-03 POLYNOMIAL EQUATIONS

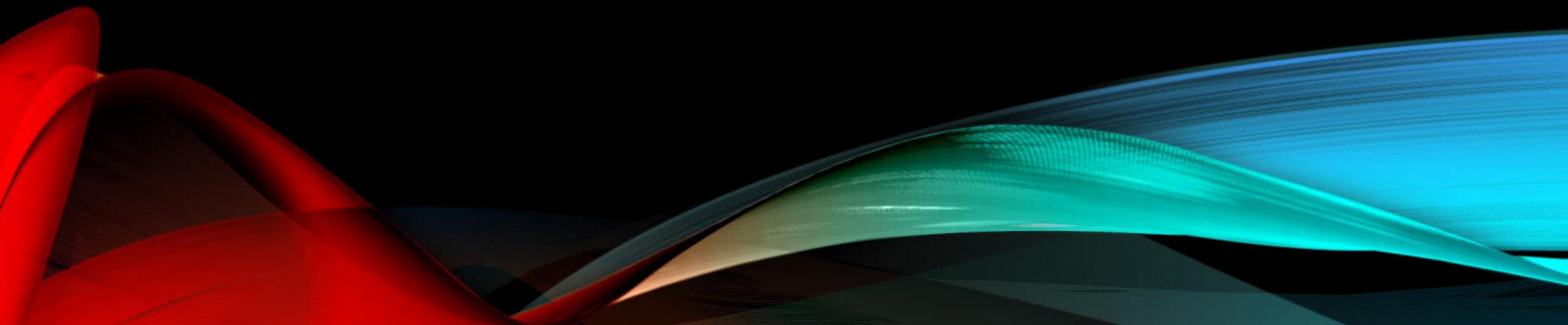
- Determine the least possible degree of the polynomial function shown.



2-04 DIVIDING POLYNOMIALS

In this section, you will:

- Divide polynomials with long division.
- Divide polynomials with synthetic division.



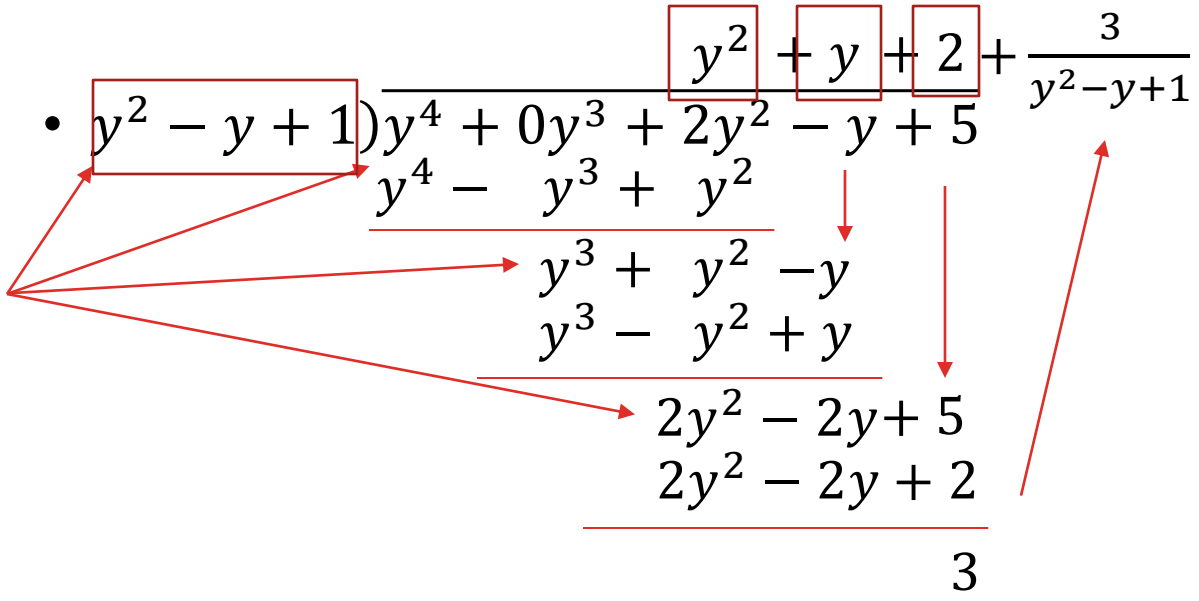
2-04 DIVIDING POLYNOMIALS

- Long Division
  - Done like long division with numbers
- $\frac{y^4+2y^2-y+5}{y^2-y+1}$

Divide the 1st terms

Multiply

Subtract



2-04 DIVIDING POLYNOMIALS

- $\frac{x^3+4x^2-3x+10}{x+2}$

Divide the 1st terms

Multiply

Subtract

$$x^2+2x-7+\frac{24}{x+2}$$
$$x+2\overline{)x^3+4x^2-3x+10}$$
$$x^3+2x^2$$
$$2x^2-3x$$
$$2x^2+4x$$
$$-7x+10$$
$$-7x-14$$
$$24$$

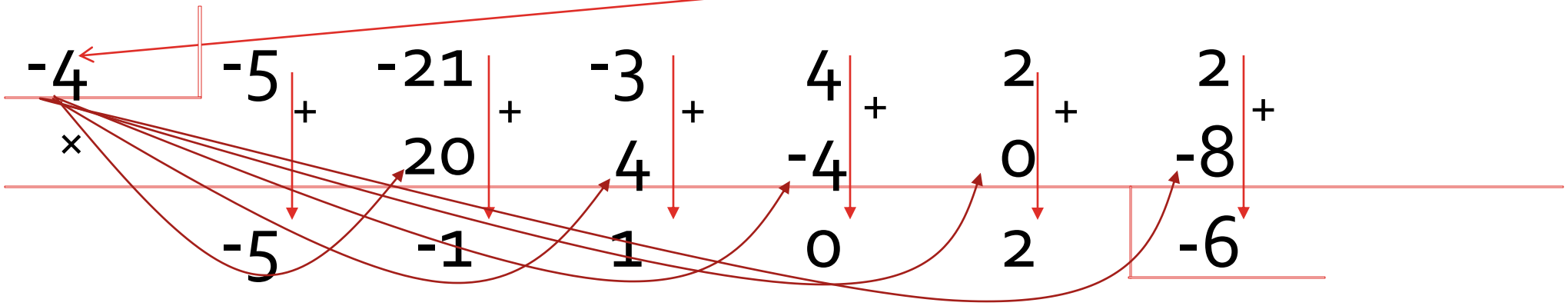
2-04 DIVIDING POLYNOMIALS

- Synthetic Division
  - Shortened form of long division for dividing by a **binomial**
  - Only when dividing by $(x - r)$

2-04 DIVIDING POLYNOMIALS

- Synthetic Division
  - $(-5x^5 - 21x^4 - 3x^3 + 4x^2 + 2x + 2)/(x + 4)$

Coefficients with placeholders



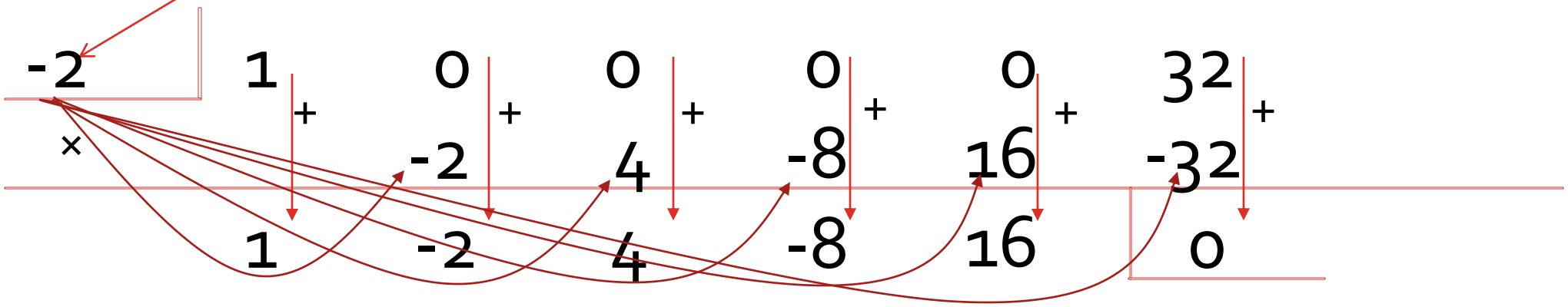
- The bottom row give the coefficients of the answer. This is called the depressed polynomial.
- The number in the box is the remainder.
- Start with an exponent one less than the original expression since you divided by x.

$$-5x^4 - x^3 + x^2 + 2 + \frac{-6}{x+4}$$

2-04 DIVIDING POLYNOMIALS

- $(y^5 + 32)(y + 2)^{-1}$

Coefficients with placeholders



- The degree was 5, now the depressed polynomial is one less degree. It's degree is 4.

$$y^4 - 2y^3 + 4y^2 - 8y + 16$$

2-04 DIVIDING POLYNOMIALS

- Factor Theorem
  - If $f(x)$ is divided by $(x - k)$ and remainder is 0, then $(x - k)$ is a factor of $f(x)$

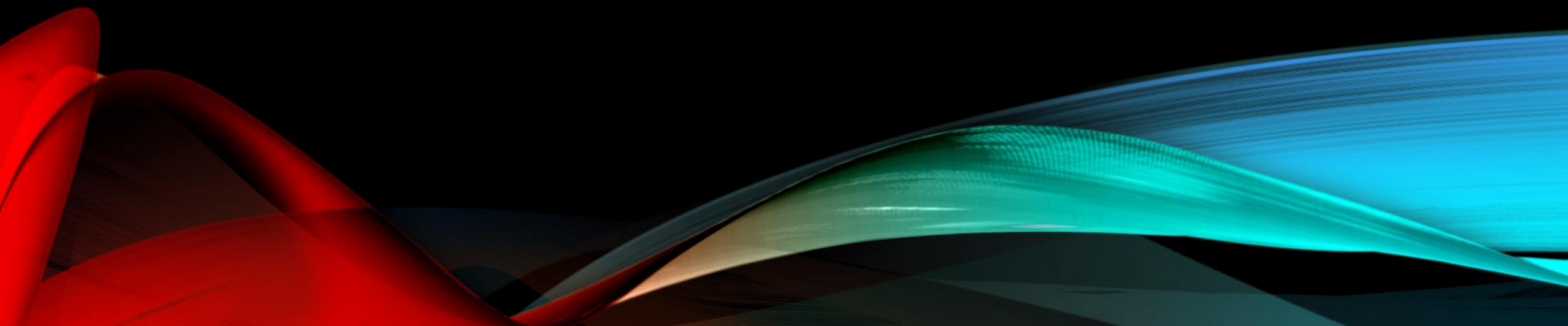
2-04 DIVIDING POLYNOMIALS

- Show that $(x + 3)$ is a factor of $x^3 - 19x - 30$. Then find the remaining factors.

2-05 RATIONAL ZEROS OF POLYNOMIAL FUNCTIONS

In this section, you will:

- Evaluate a polynomial using the Remainder Theorem.
- Use the Factor Theorem to solve a polynomial equation.
- Use the Rational Zero Theorem to find rational zeros.



2-05 RATIONAL ZEROS OF POLYNOMIAL FUNCTIONS

- Remainder Theorem
  - If $f(x)$ is divided by $(x - k)$, then the remainder is $r = f(k)$

2-05 RATIONAL ZEROS OF POLYNOMIAL FUNCTIONS

- Use the remainder theorem to evaluate $f(x) = 4x^3 + 10x^2 - 3x - 8$ for $f(-1)$

2-05 RATIONAL ZEROS OF POLYNOMIAL FUNCTIONS

- Rational Zero Theorem
  - If polynomial
    $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$
    The rational zeros are in the form $\frac{p}{q}$

    where p = factors of a_0

    q = factors of a_n

2-05 RATIONAL ZEROS OF POLYNOMIAL FUNCTIONS

- Find the rational zeros of $f(x) = x^3 - 5x^2 + 2x + 8$ given that $x + 1$ is a factor.

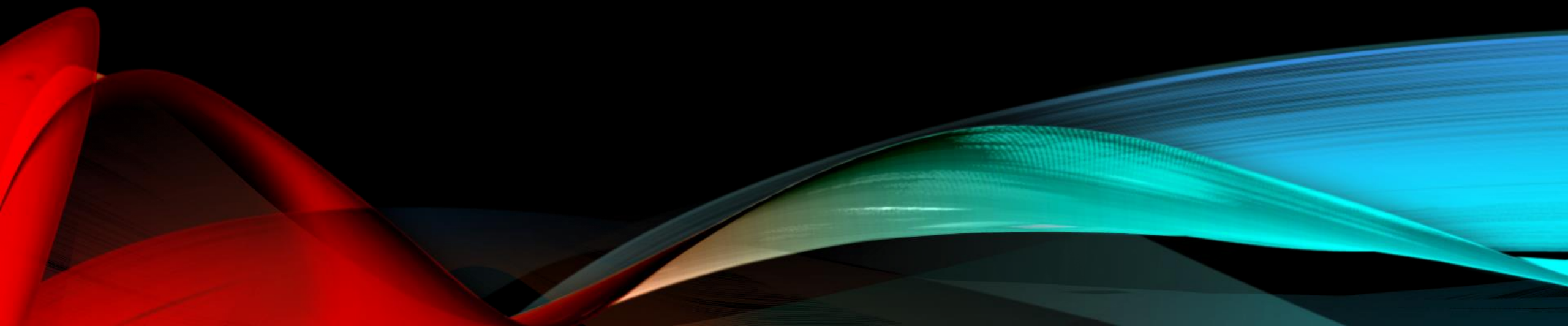
2-05 RATIONAL ZEROS OF POLYNOMIAL FUNCTIONS

- Find the real zeros of $f(x) = x^3 - 7x^2 - 11x + 14$ given that $x + 2$ is a factor.

2-06 ZEROS OF POLYNOMIAL FUNCTIONS

In this section, you will:

- Find zeros of a polynomial function.
- Use the Linear Factorization Theorem to find polynomials with given zeros.
- Use Descartes' Rule of Signs.



2-06 ZEROS OF POLYNOMIAL FUNCTIONS

- Fundamental Theorem of Algebra
  - If $f(x)$ is polynomial of degree n, then there is at least 1 zero
  - There are exactly n zeros
  - There are n linear factors (Linear Factorization Theorem)

2-06 ZEROS OF POLYNOMIAL FUNCTIONS

- Find all zeros of $f(x) = x^4 - 16$

2-06 ZEROS OF POLYNOMIAL FUNCTIONS

- Find all the zeros of

$$f(x) = 2x^4 - 9x^3 - 18x^2 + 71x - 30$$

2-06 ZEROS OF POLYNOMIAL FUNCTIONS

- Descartes's Rule of Signs
- Let $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$ be a polynomial with real coefficients and $a_0 \neq 0$
  - The number of positive real zeros is equal to the number of variations in sign of $f(x)$ or less by even integer
  - The number of negative real zeros is equal to the number of variations in sign of $f(-x)$ or less by even integer.

2-06 ZEROS OF POLYNOMIAL FUNCTIONS

- Describe the possible real zeros of

$$f(x) = -2x^3 + 5x^2 - x + 8$$

2-06 ZEROS OF POLYNOMIAL FUNCTIONS

- Complex Conjugate Theorem
  - If a complex number $a + bi$ is a zero, then $a - bi$ is also a zero.

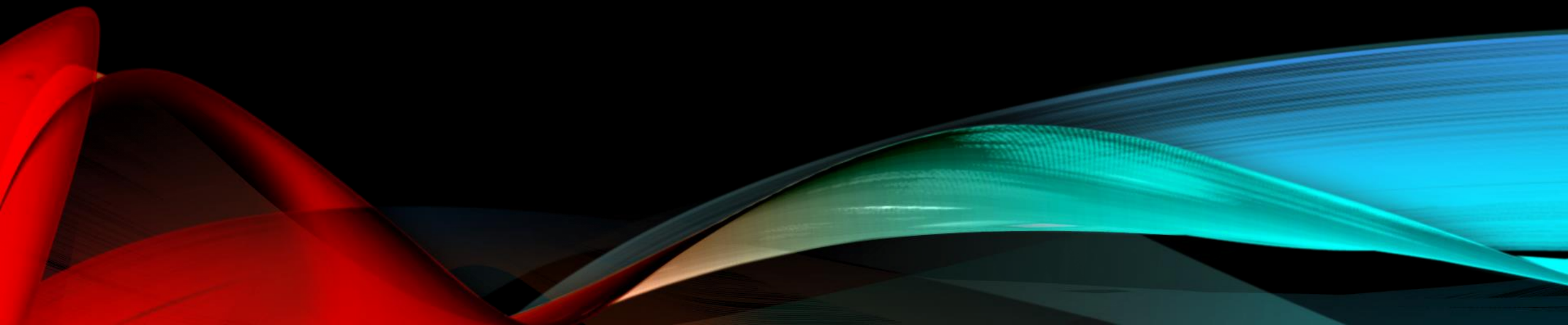
2-06 ZEROS OF POLYNOMIAL FUNCTIONS

- Find a polynomial with real coefficients with zeros $\frac{2}{3}, -\sqrt{2}i$

2-07 ASYMPTOTES OF RATIONAL FUNCTIONS

In this section, you will:

- Find the domains of rational functions.
- Identify vertical asymptotes.
- Identify horizontal asymptotes.



2-07 ASYMPTOTES OF RATIONAL FUNCTIONS

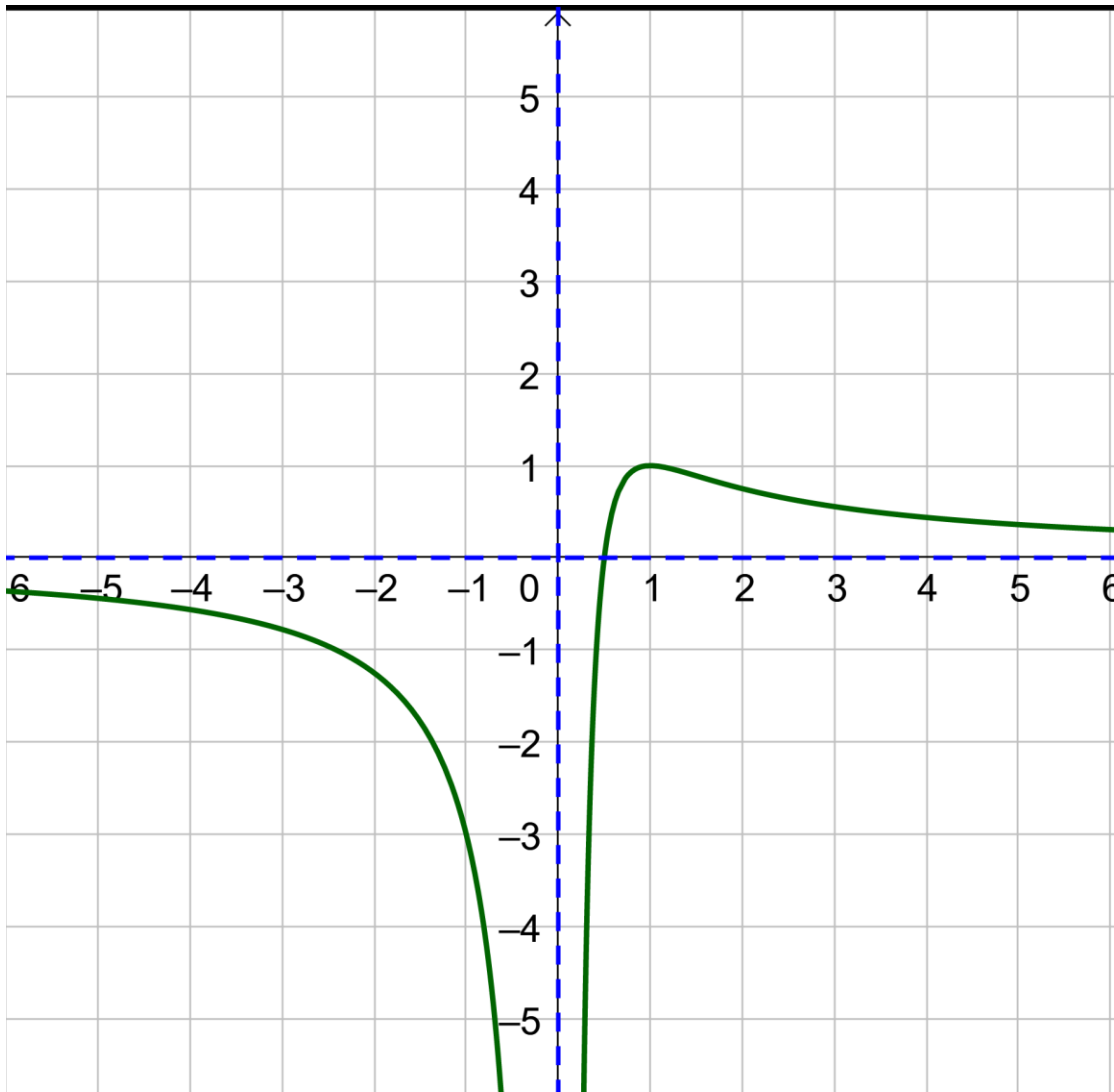
- Rational Function
  - Fraction

- $f(x) = \frac{2x+1}{3x-4}$

- Domain:
  - Denominator ≠ 0

2-07 ASYMPTOTES OF RATIONAL FUNCTIONS

- Asymptotes describe behavior of the graph at the edges



2-07 ASYMPTOTES OF RATIONAL FUNCTIONS

- Vertical Asymptotes
  - Factor and reduce
  - Set denominator = 0 and solve for *x*
- Horizontal Asymptotes
  - Plug in huge number for *x* and simplify
  - OR
  - Find degree of numerator (N) and denominator(D)
  - If N<D, y = 0
  - If N=D, y = leading coeff
  - If N>D, No HA

2-07 ASYMPTOTES OF RATIONAL FUNCTIONS

- Find the asymptotes of $f(x) = \frac{5x^2}{x^2-1}$

2-07 ASYMPTOTES OF RATIONAL FUNCTIONS

- For $f(x) = \frac{2x^2 - x}{2x^2 + x - 1}$
  a. Find the domain
  b. Find the removable discontinuity
  c. Find the asymptotes

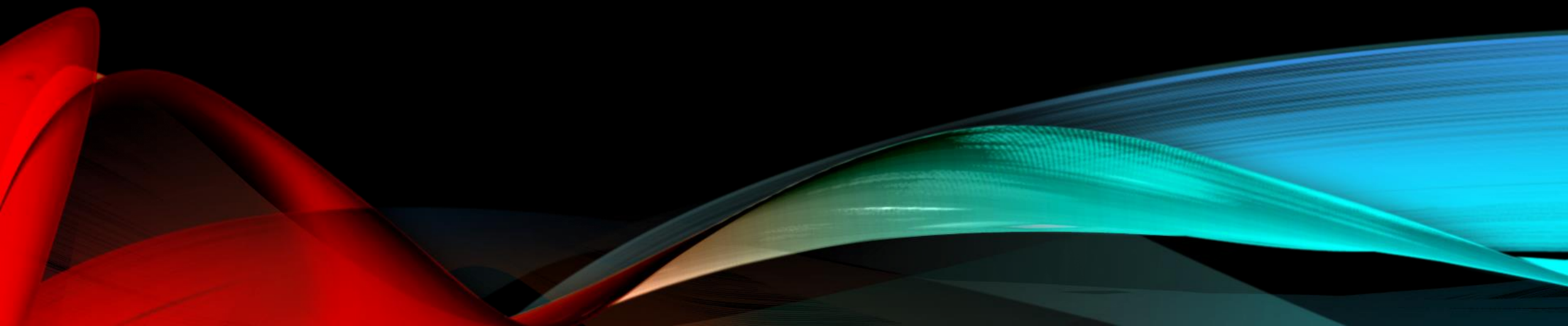
2-07 ASYMPTOTES OF RATIONAL FUNCTIONS

- Slant Asymptote
  - If $N = D + 1$, Divide and ignore remainder
- Find the asymptotes of $f(x) = \frac{3x^2+1}{x}$

2-08 GRAPHS OF RATIONAL FUNCTIONS

In this section, you will:

- Find the intercepts of rational functions.
- Graph rational functions.
- Solve applied problems involving rational functions.



2-08 GRAPHS OF RATIONAL FUNCTIONS

- Intercepts
  - x-int: let $y = 0$
    - Numerator = 0
  - y-int: let $x = 0$

- Find the intercepts of $f(x)$

$$\frac{3x^2-1}{x}$$

2-08 GRAPHS OF RATIONAL FUNCTIONS

- To graph rational functions
  1. Find domain
  2. Find asymptotes
  3. Graph asymptotes as dotted lines
  4. Create table of values around vertical asymptotes
  5. Plot points
  6. Draw curves starting near an asymptote and ending near another asymptote
     Don't cross VA
  7. Put any required holes. Check the domain

2-08 GRAPHS OF RATIONAL FUNCTIONS

- Graph $f(x) = \frac{3x^2+1}{x}$

2-08 GRAPHS OF RATIONAL FUNCTIONS

- Graph $f(x) = \frac{3x}{x^2+x-2}$

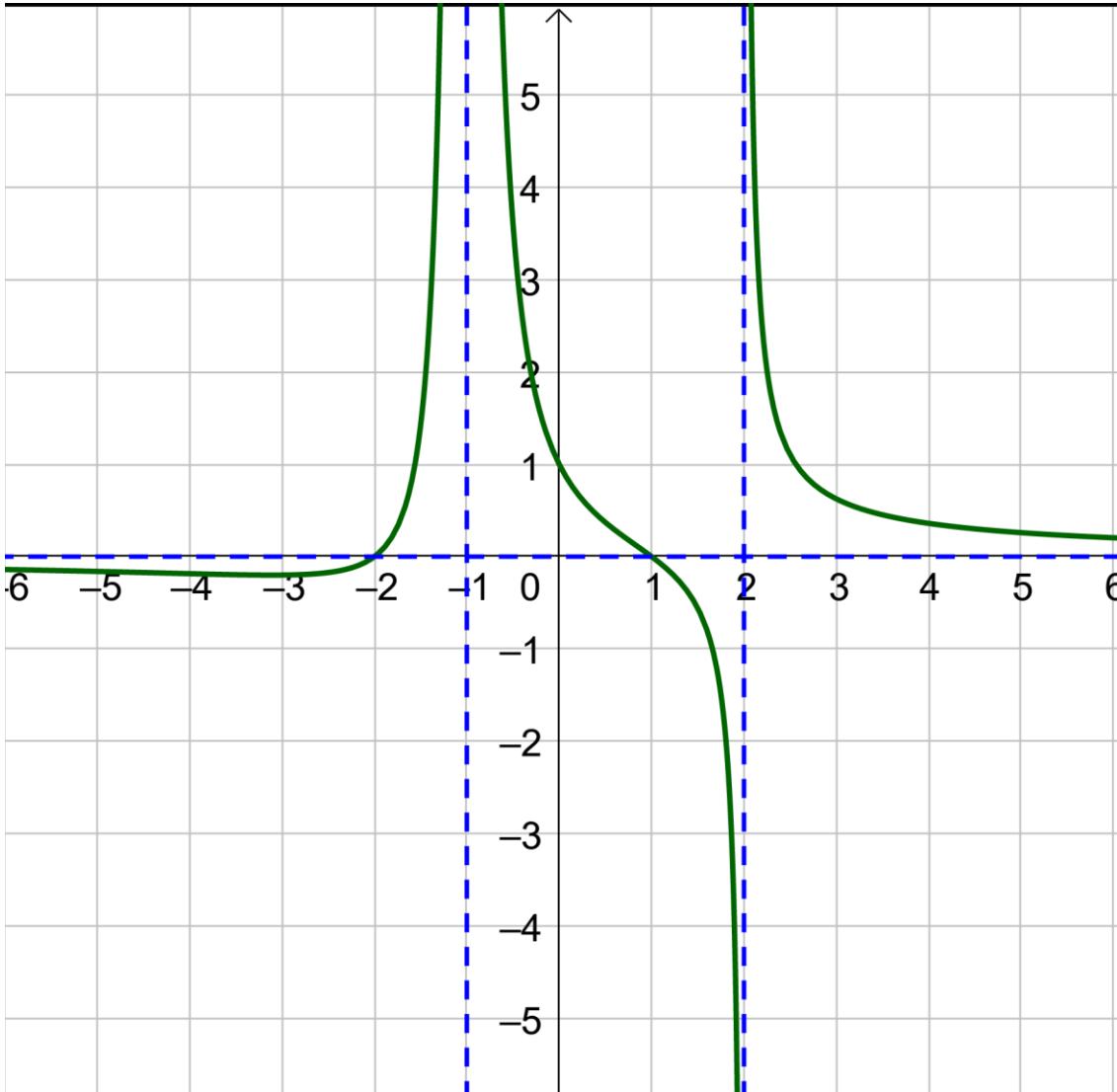
2-08 GRAPHS OF RATIONAL FUNCTIONS

- Find the function given a graph

1. Use the x-intercepts and multiplicity to get factors of numerator
   a. If cross x-axis: multiplicity 1 or 3
   b. If touch but not cross: multiplicity 2 or 4
2. Use VA to get factors of denominator
   a. If 1 end goes up and 1 down: multiplicity 1
   b. If both ends go same direction: multiplicity 2
3. Use any other point to get stretch factor, a

2-08 GRAPHS OF RATIONAL FUNCTIONS

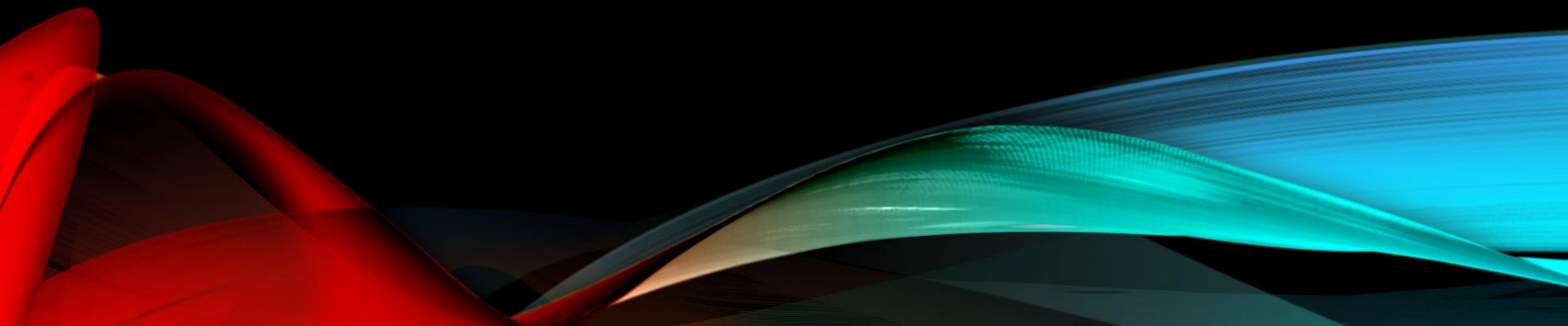
- Find the function



2-09 NONLINEAR INEQUALITIES

In this section, you will:

- Find critical numbers of nonlinear inequalities.
- Solve one-variable nonlinear inequalities algebraically.
- Solve one-variable nonlinear inequalities by graphing.



2-09 NONLINEAR INEQUALITIES

- Steps to solve

1. Make one side 0
2. Find critical #'s
   - Zeros
   - Undefined
3. Plot on #-line
4. Pick test points and test them

- Solve $x^2 - 5x - 6 < 0$

2-09 NONLINEAR INEQUALITIES

- Solve $3x^3 - 4x^2 - 12x > -16$

2-09 NONLINEAR INEQUALITIES

- Solve $\frac{3x-5}{x-3} \leq 1$