A quadratic equation has **one root** at $x = 4$, and its **discriminant** is $D = 16$. The coefficient of x^2 is $a = 2$.

Find the quadratic equation.

- Quadratic equations:

$$\boxed{2x^2 - 12x + 16 = 0 \quad \text{or} \quad 2x^2 - 20x + 48 = 0}$$

Solve the system of inequalities:

$$\begin{cases} x^2 - 5x + 6 > 0 \\ x^2 - 3x - 4 < 0 \end{cases}$$

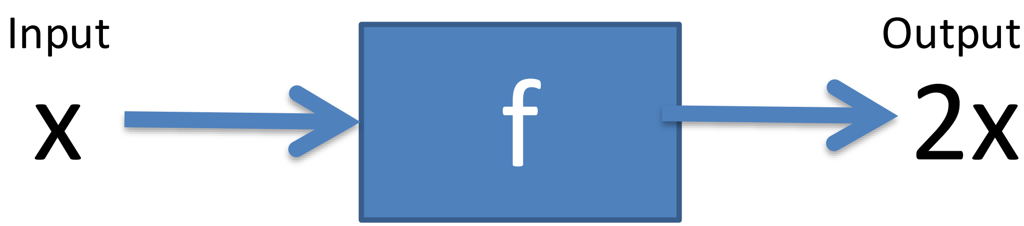
$$\boxed{-1 < x < 2 \quad \text{or} \quad 3 < x < 4}$$

Recap of functions

A function is something which provides a rule on how to map inputs to outputs.

Input Output

$$f(x) = 2x$$



Check Your Understanding

$$f(x) = x^2 + 2$$

Q1 What does this function do?

?

Q2 What is f(3)?

?

Q3 What is f(-5)?

?

Q4 If f(a) = 38, what is a?

?

Transformations of Functions

We saw that whatever is between the f( ) brackets is the input. If we were to replace x with say 3, we saw that we just substitute x with 3 on the RHS to find the output.

Given that the function f is defined as $f(x) = x^2 + 2$, determine:

$f(x + 1)$	?
$f(x) + 3$	?
$f(2x)$	?
$2f(x)$	?

Test Your Understanding

Given $f(x) = x^2 + 2x - 1$

Find:

$f(x) + 1 =$?

$f(x+1) =$?

$=$

$=$

$f(2x) =$?

$=$

Exercise 3

1 Given that $f(x) = \cos(x)$, find:

$f(2x)$	=	?
$f(x+1)$	=	?
$f(x) - 3$	=	?
$9f(x)$	=	?
$f(0)$	=	?

2 Given that $f(x) = x^2$, find:

$f(2x)$	=	?
$f(x+1)$	=	?
$f(x) - 3$	=	?
$9f(x)$	=	?
$f(4)$	=	?

Given $f(a) = 25$, find a

?

3 Given that $f(x) = \frac{1}{x}$, find:

$f(2x)$	=	?
$f(x+1)$	=	?
$f(x) - 3$	=	?
$9f(x)$	=	?
$f(4)$	=	?

4 Given that $f(x) = 2x - 1$, find:

$f(2x)$	=	?
$f(x+1)$	=	?
$f(x) - 3$	=	?
$9f(x)$	=	?
$f(4)$	=	?

Given $f(a) = 11$, find a

?

Transformations of Functions

Suppose $f(x) = x^2$

Sketch $y = f(x)$:

?

Then $f(x + 2) =$?

Sketch $y = f(x + 2)$:

?

What do you notice about the relationship between the graphs of $y = f(x)$ and $y = f(x + 2)$?

Transformations of Functions

This is all you need to remember when considering how transforming your function transforms your graph...

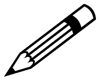
	Affects which axis?	What we expect or opposite?
Change **inside** f( )	?	?
Change **outside** f( )	?	?

Therefore...

$f(x+2)$ →	?
$f(x)+4$ →	?
$f(5x)$ →	?
$2f(x)$ →	?

Effect of transformation on specific points

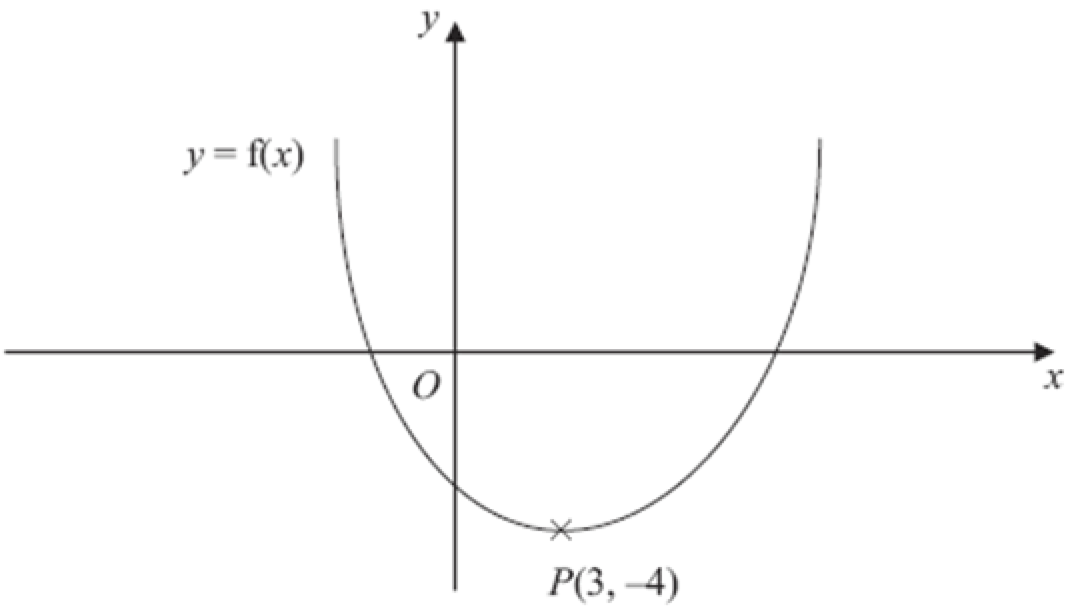
What effect will the following transformations have on these points?



$f(x)$	$(4, 3)$	$(1, 0)$	$(6, -4)$
$f(x+1)$	?	?	?
$f(2x)$	?	?	?
$3f(x)$	?	?	?
$f(x)-1$	?	?	?
$f\left(\frac{x}{4}\right)$	?	?	?
$f(-x)$	?	?	?
$-f(x)$	?	?	?

Exam Example

24. This is a sketch of the curve with the equation $y = f(x)$.
The only minimum point of the curve is at $P(3, -4)$.



(a) Write down the coordinates of the minimum point of the curve with the equation $y = f(x - 2)$.

?

(b) Write down the coordinates of the minimum point of the curve with the equation $y = f(x + 5) + 6$

?

Exercise B

1 Describe the affects of the following graph transformations.

$y = f(x+10)$?
$y = 3f(x)$?
$y = f(2x)$?
$y = f(x) - 4$?
$y = f\left(\frac{x}{2}\right)$?
$y = f(3x) + 4$?
$y = f(-x)$?
$y = -f(x)$?

2 To what point will (4, -1) on the curve y = f(x) be transformed to under the following transformations?

$y = f(2x)$?
$y = 5f(x)$?
$y = 2f(4x)$?
$y = f(x+1) + 1$?
$y = f(-x)$?
$y = -f(x)$?

3 The point (0, 0) on a curve y = f(x) is mapped to the following points. Find the equation for the translated curve.

(4, 0) $y =$?
(0, 3) ?
(-5, 0) ?
(0, -1) ?
(5, -3) ?
(-5, 2) ?

4 **To what points will (-2, 0) on the curve y = f(x) be transformed to under the following transformations?**

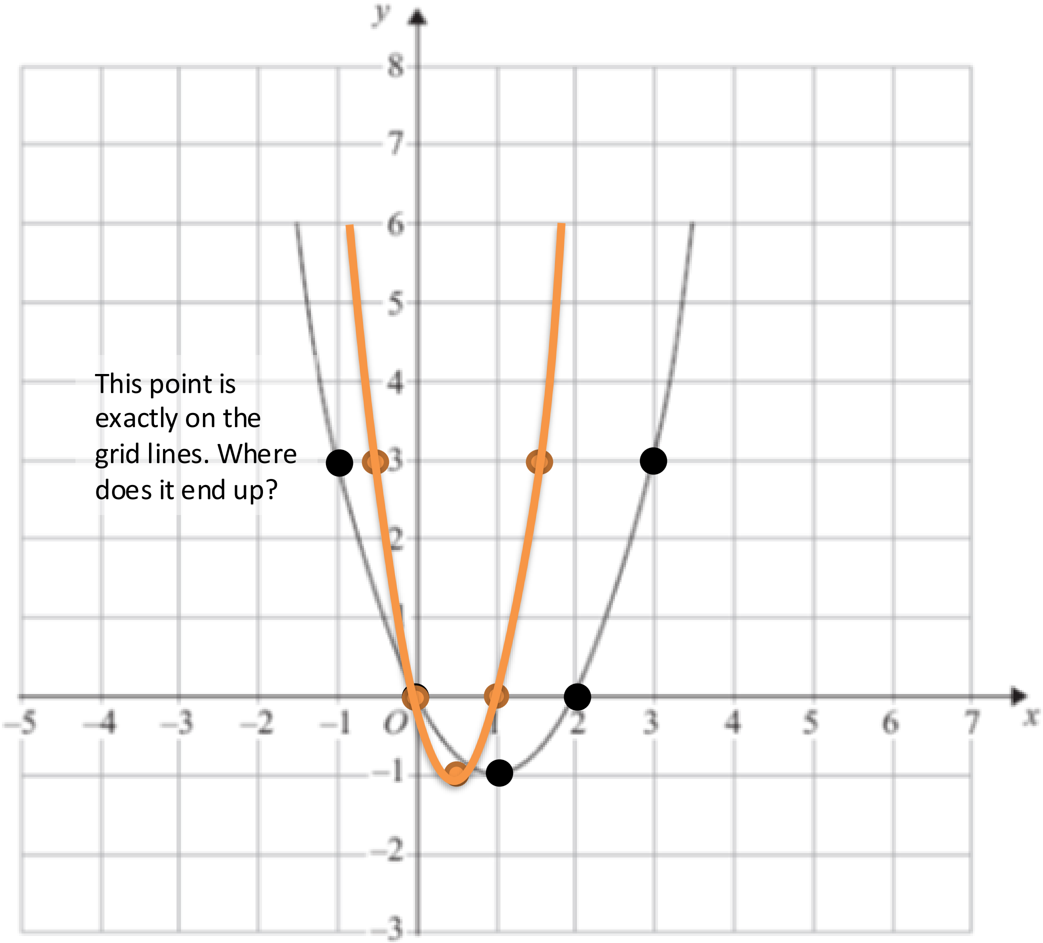
$y = f(2x)$?
$y = 2f(x)$?
$y = f\left(\frac{x}{3}\right) + 1$?
$y = f(x-1) - 1$?
$y = f(-x) + 1$?
$y = -f(x) + 1$?

5 **Find the equation of the curve obtained when $y = x^2 + 3x$ is:**

a) Translated 5 units up. ?
b) Translated 2 units right. ?
c) Reflected in x-axis. ?

Drawing Transformed Graphs

The graph shows the line with equation $y = f(x)$. On the same axis, sketch $y = f(2x)$



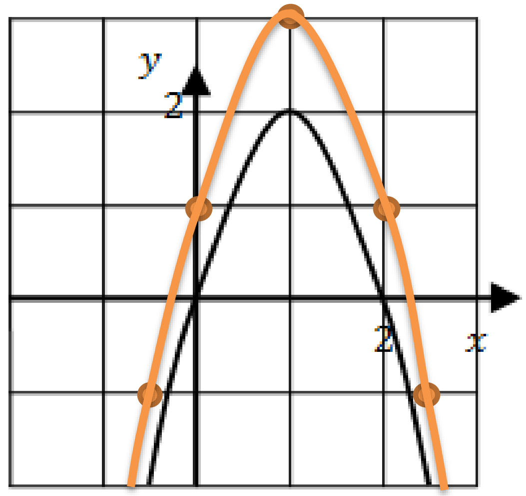
$y = f(2x)$ has the effect of:

?

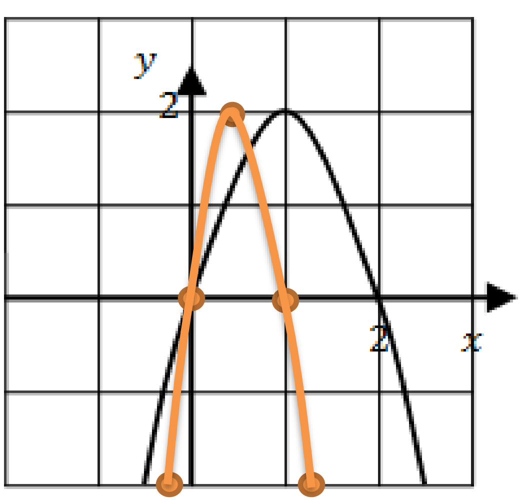
The mark scheme will check you have certain key points correct, so the key is to **<u>identify points exactly on the grid and transform one at a time</u>**.

Quickfire Transforms

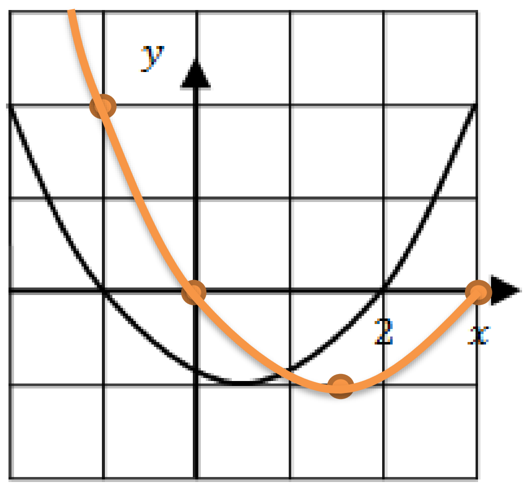
On provided sheet-ette



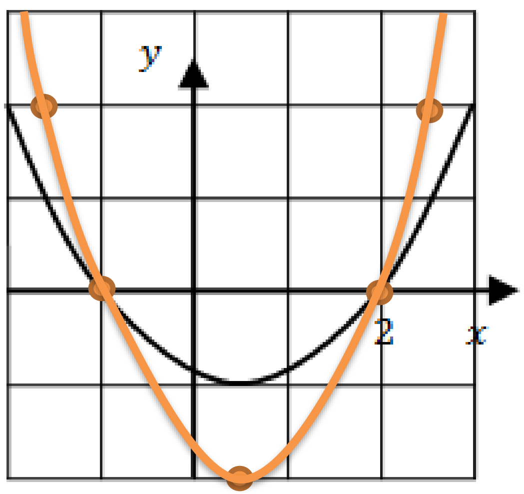
Sketch $y = f(x) + 1$



Sketch $y = f(2x)$



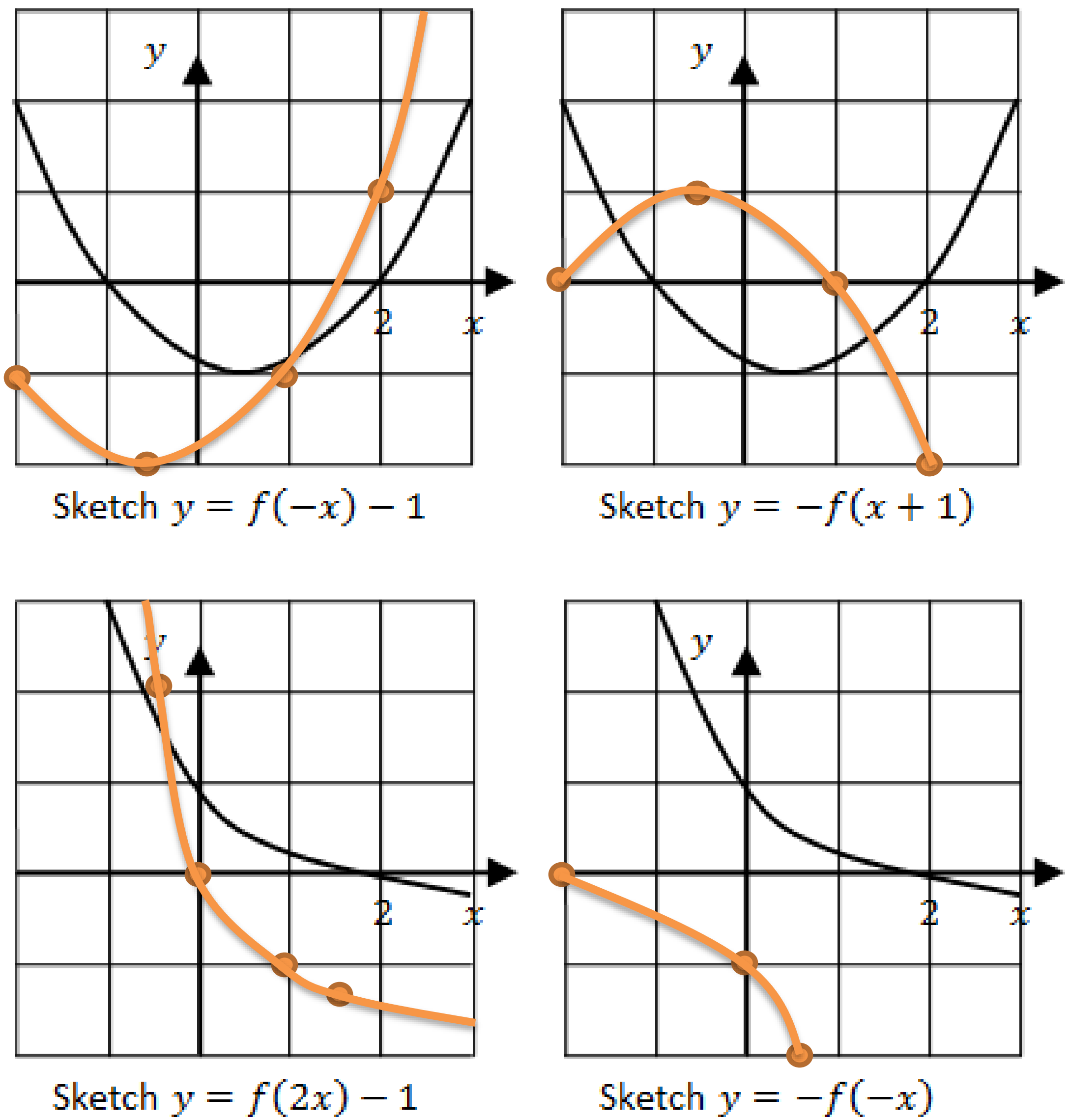
Sketch $y = f(x - 1)$



Sketch $y = 2f(x)$

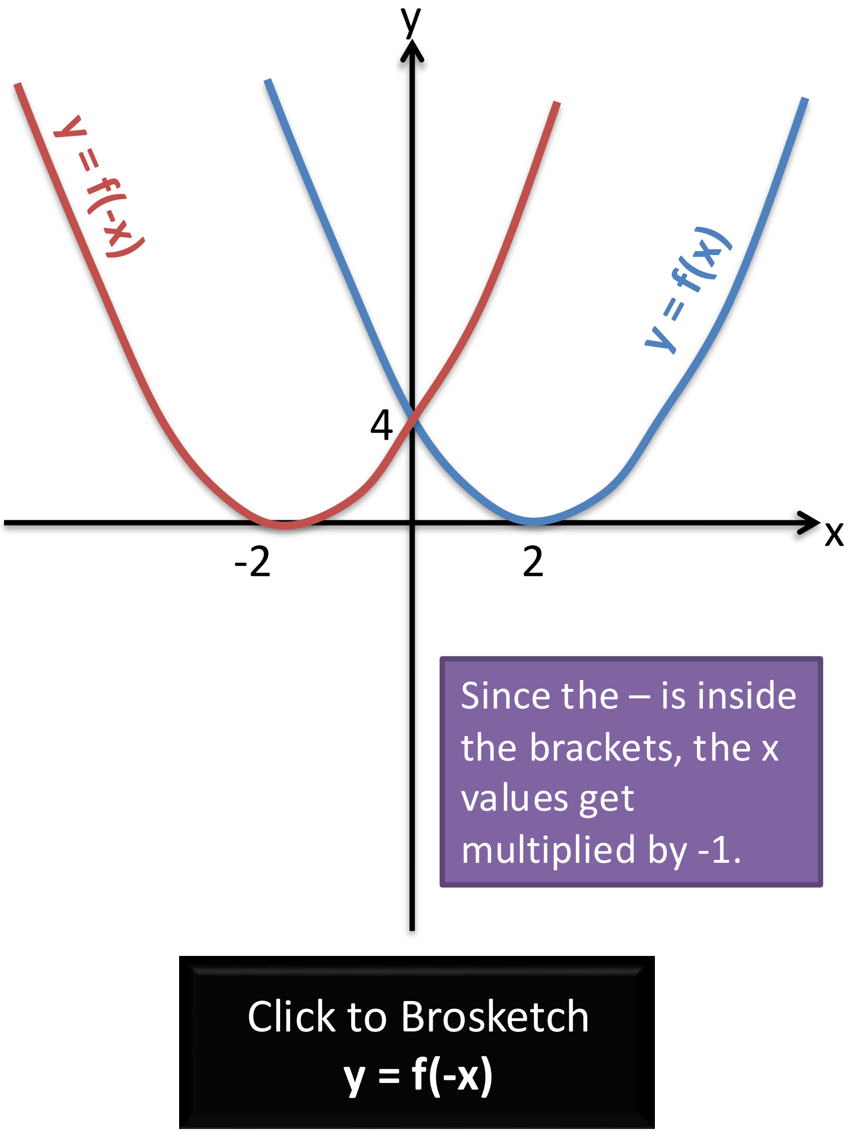
Quickfire Transforms

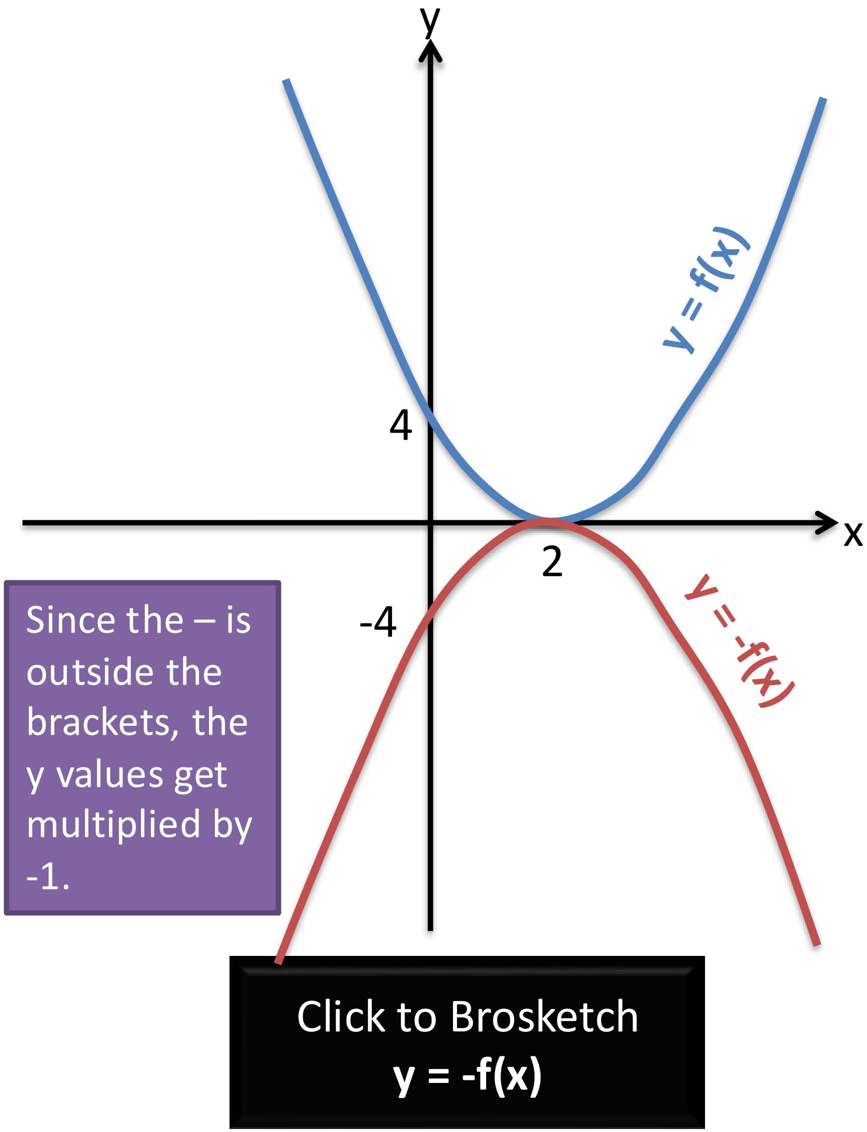
On provided sheet-ette



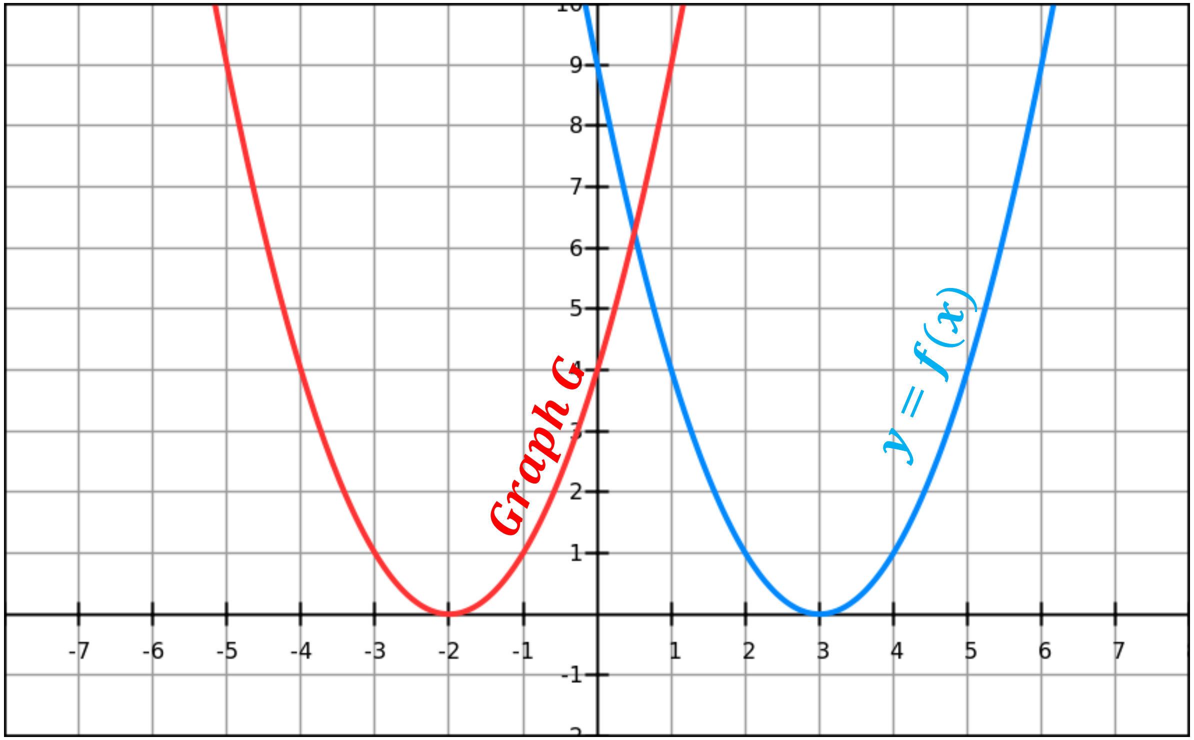
f(-x) and –f(x)

Below is a sketch of y= f(x) where $f(x) = (x – 2)^2$. Hence sketch the following.





Describing Transforms

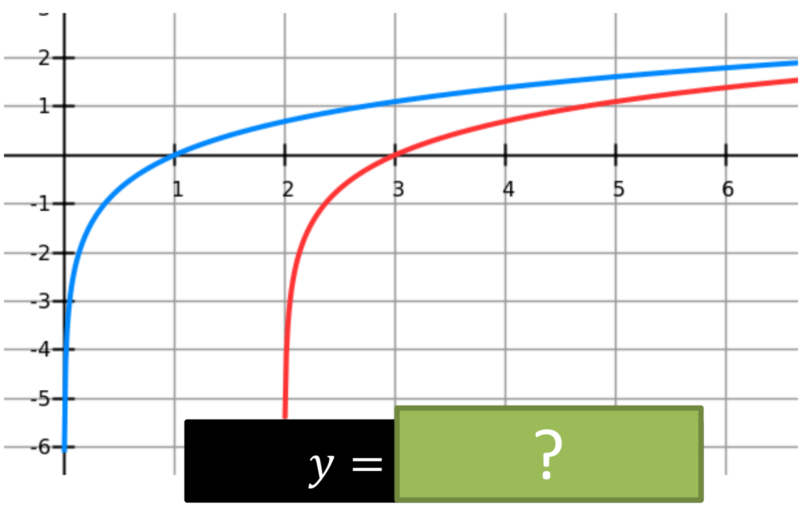


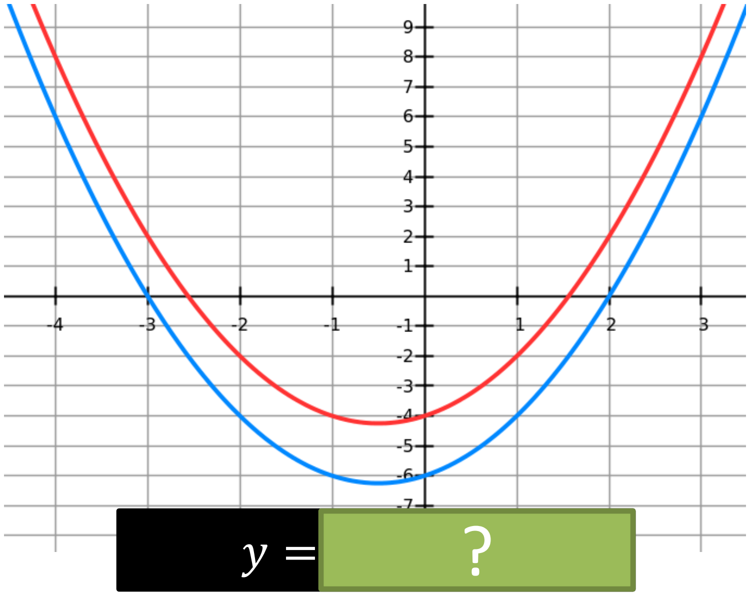
The blue graph shows the line with equation $y = f(x)$.
What is the equation of graph G, in terms of f?

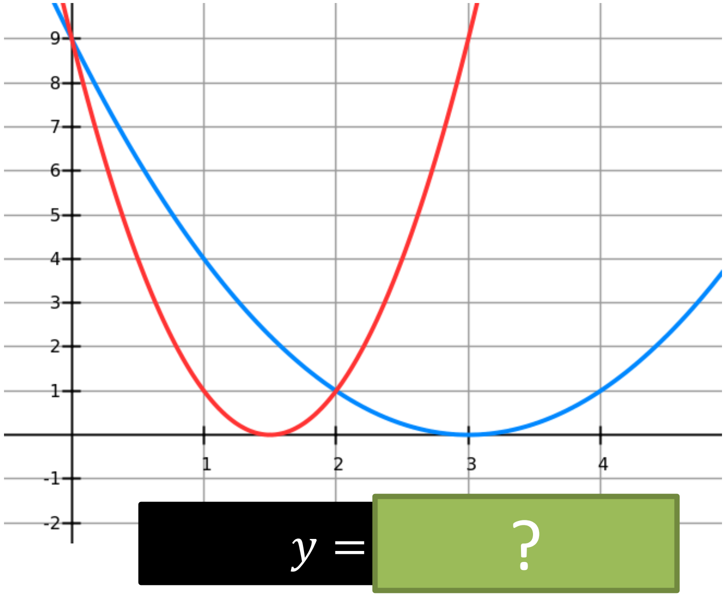
?

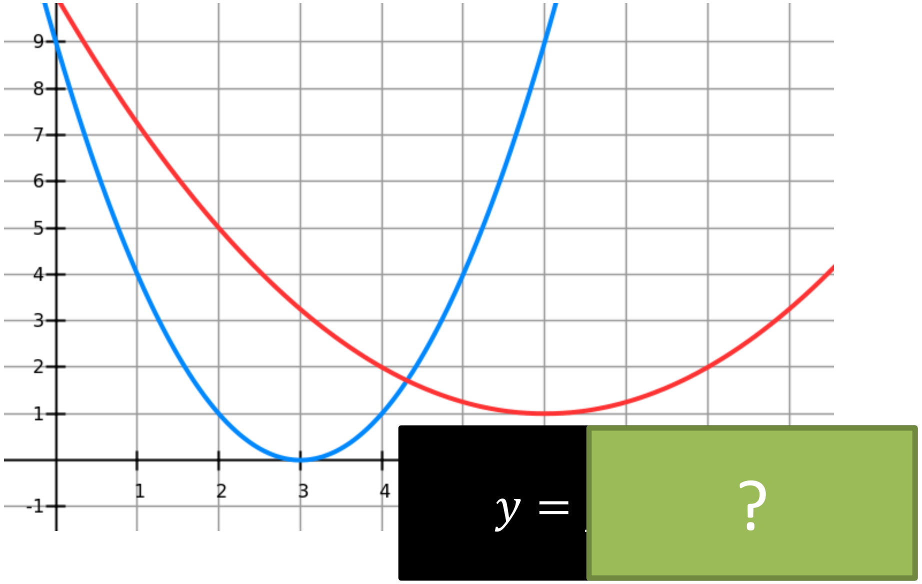
Quickfire Describing Transforms

Given the blue graph has equation $y = f(x)$, determine the equation of the red graph.







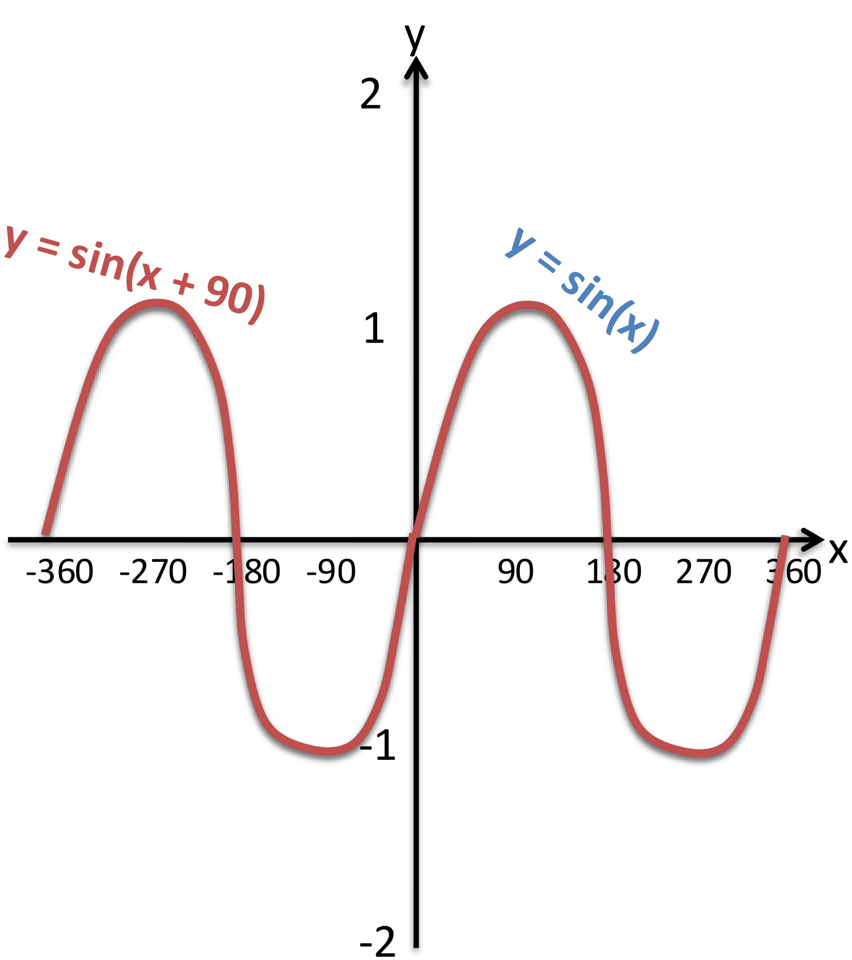


GCSE: Transformations of Trig Functions

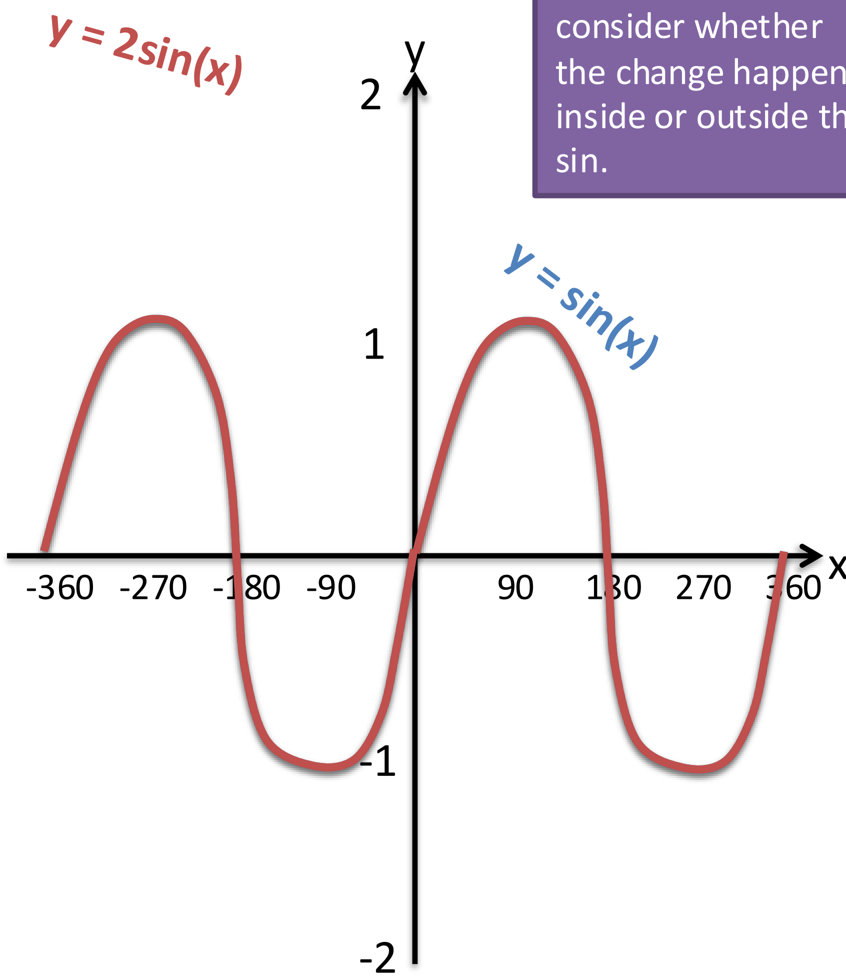
Dr J Frost (jfrost@tiffin.kingston.sch.uk)

Example

Below is a sketch of y = sin(x). Hence sketch the following.



Click to Brosketch
y = sin(x + 90)

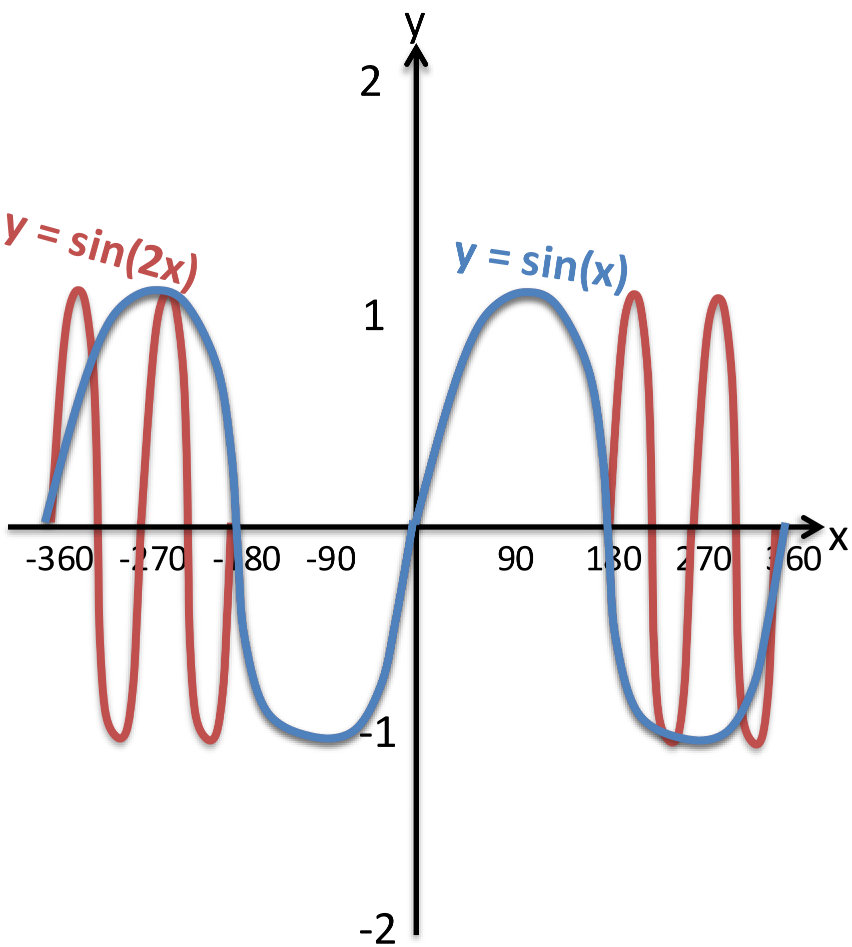


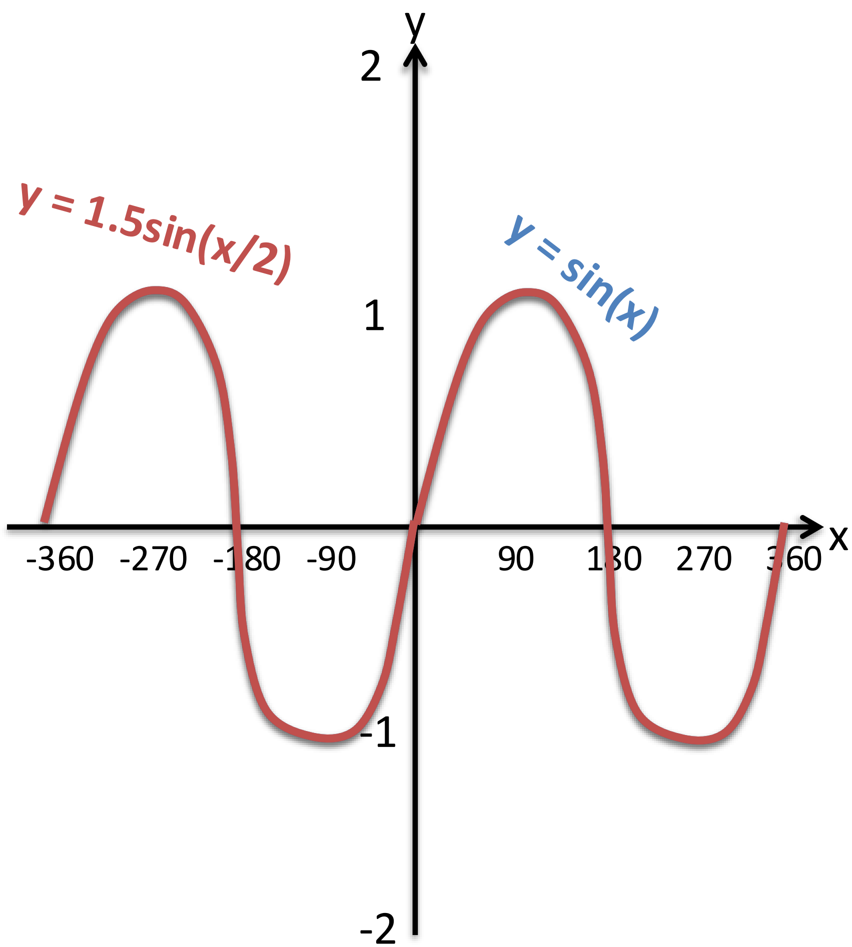
Bro Tip: The function here is the sin. So consider whether the change happens inside or outside the sin.

Click to Brosketch
y = 2sin(x)

Example

Below is a sketch of y = sin(x). Hence sketch the following.





Click to Brosketch
y = sin(2x)

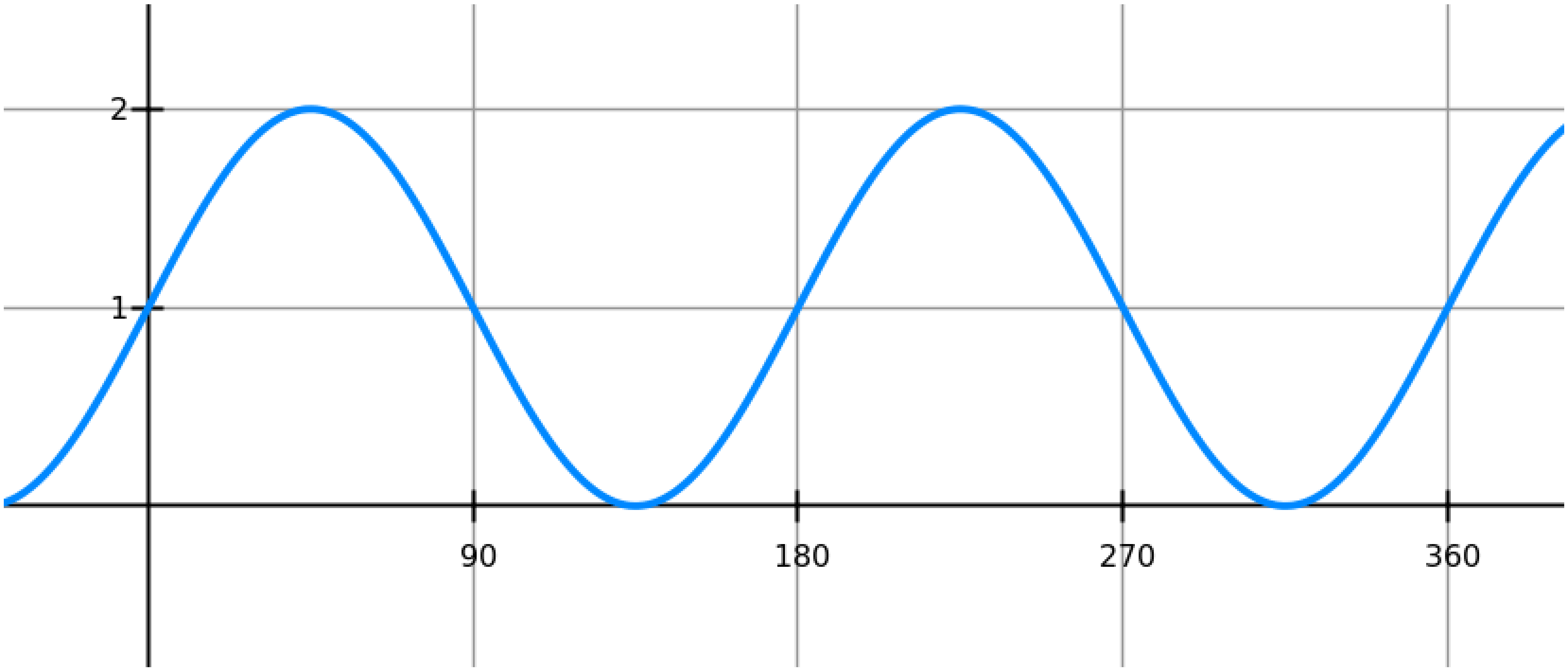
Click to Brosketch
y = 1.5sin(x/2)

Exercises

On printed sheets.

(File ref: GCSERevision-TrigGraphs)

Describing Transforms of Trig Graphs



The graph shows the line with equation $y = \sin(ax) + b$.
Determine the constants a and b.

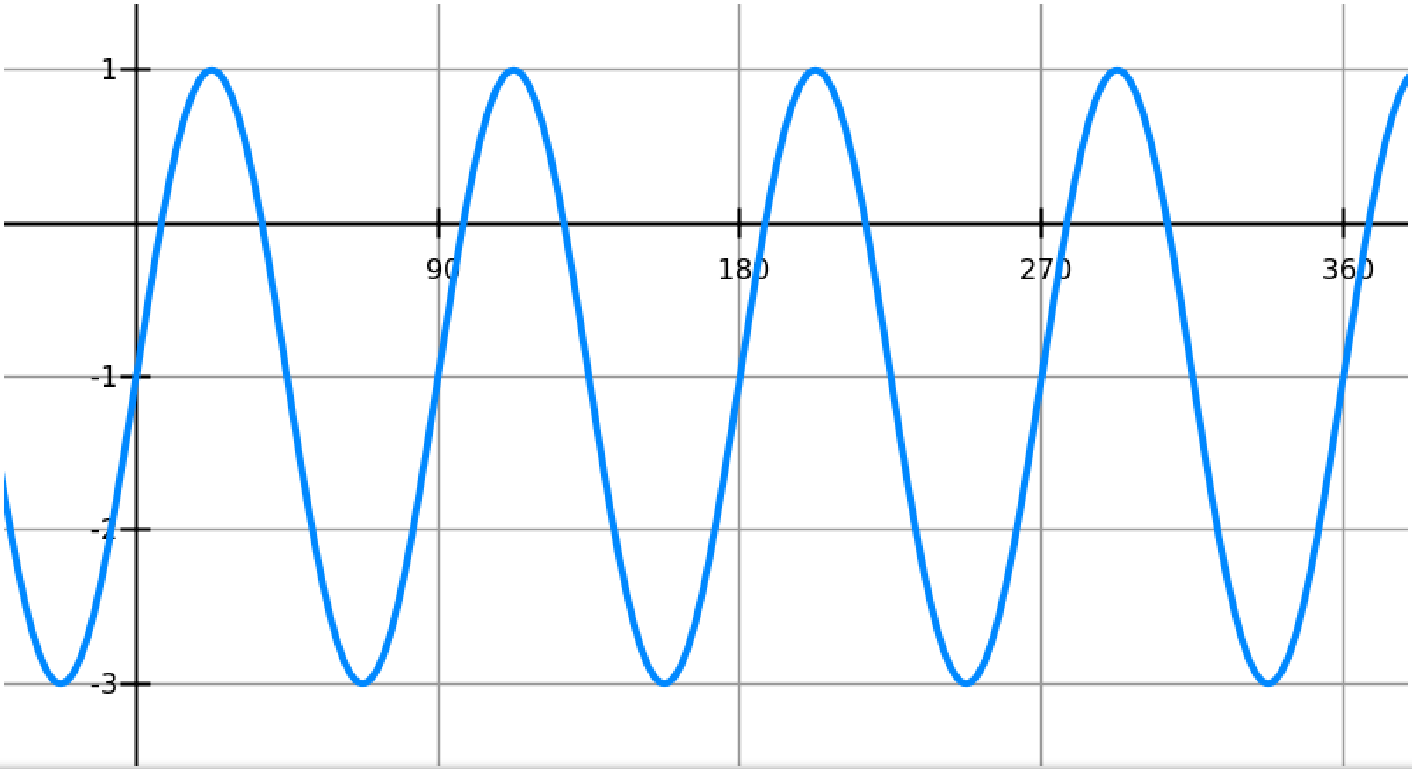
$a =$?

$b =$?

Helpful questions to ask yourself:

- Usually the sine graph makes one full oscillation every 360. How many oscillations per 360 is it making here?
- sin usually has a range on the y-axis of -1 to 1. What is it here?

Describing Transforms of Trig Graphs



The graph shows the line with equation $y = a\sin(bx) + c$.
Determine the constants a, b and c.

$a =$?
$b =$?
$c =$?