Completing the Square

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Objectives: (from the specification)

2.9	Completing the square	Work out the values of a, b and c such that $2x^2 + 6x + 7 \equiv a(x + b)^2 + c$

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RECAP

Solve the equation:

$$x^4 - 5x^3 - 4x^2 + 20x = 0$$

Instructions:

1. Solve by factorising step by step.
2. Identify **all real solutions**, including repeated roots if any.
3. After factorisation, **check each solution** in the original equation.

Consider $x^3 - 5x^2 - 4x + 20$:

Group terms:

$$(x^3 - 5x^2) + (-4x + 20)$$

Factor each group:

$$x^2(x - 5) - 4(x - 5)$$

Factor out $(x - 5)$:

$$(x - 5)(x^2 - 4)$$

Step 3: Factor further (difference of squares)

$$x^2 - 4 = (x - 2)(x + 2)$$

Step 4: Full factorisation

$$x(x - 5)(x - 2)(x + 2) = 0$$

Step 5: Solve each factor

$$x = 0, \quad x - 5 = 0 \Rightarrow x = 5, \quad x - 2 = 0 \Rightarrow x = 2, \quad x + 2 = 0 \Rightarrow x = -2$$

1. Quadratic Expression – 二次表达式
2. Perfect Square – 完全平方
3. Binomial – 二项式
4. Vertex Form – 顶点形式
5. Constant Term – 常数项

What makes this topic Further Maths-ey?

2.9 Completing the square	Work out the values of a, b and c such that $2x^2 + 6x + 7 \equiv a(x + b)^2 + c$

You're used to expressing for example $x^2 + 4x - 3$ in the form $(x + 2)^2 - 7$

But you've (probably) never had to deal with the coefficient of x^2 not being 1!

Reminder

What the devil is ‘completing the square’?

What’s the point?

Recap of $(x+b)^2+c$

$x^2-2x=$?

$x^2-6x+4=$?

$x^2+8x+1=$?

$x^2+10x-3=$?

$x^2+4x+3=$?

$x^2-20x+150=$?

Reminder of method:

x^2-6x+4
$=(x-3)^2-9+4$
$=(x-3)^2-5$

x^2+8x+1
$=(x+4)^2-16+1$
$=(x+4)^2-15$

Remember we halve the coefficient of x, then square it and 'throw it away'.

$ax^2 + \cdots$

So far the coefficient of the x^2 term has been 1. What if it isn't?

Express $3x^2 + 12x - 6$ in the form $a(x + b)^2 + c$

$3x^2 + 12x - 6$

$=$?

$=$?

$=$?

Just factorise out the coefficient of the x^2 term. Now we have an expression just like before for which we can complete the square!

Now expand out the **outer** brackets. To be sure about your answer you could always expand and check you get the original expr.

Express $2 - 4x - 2x^2$ in the form $a - b(x + c)^2$

?

$=$?

$=$?

$=$?

$=$?

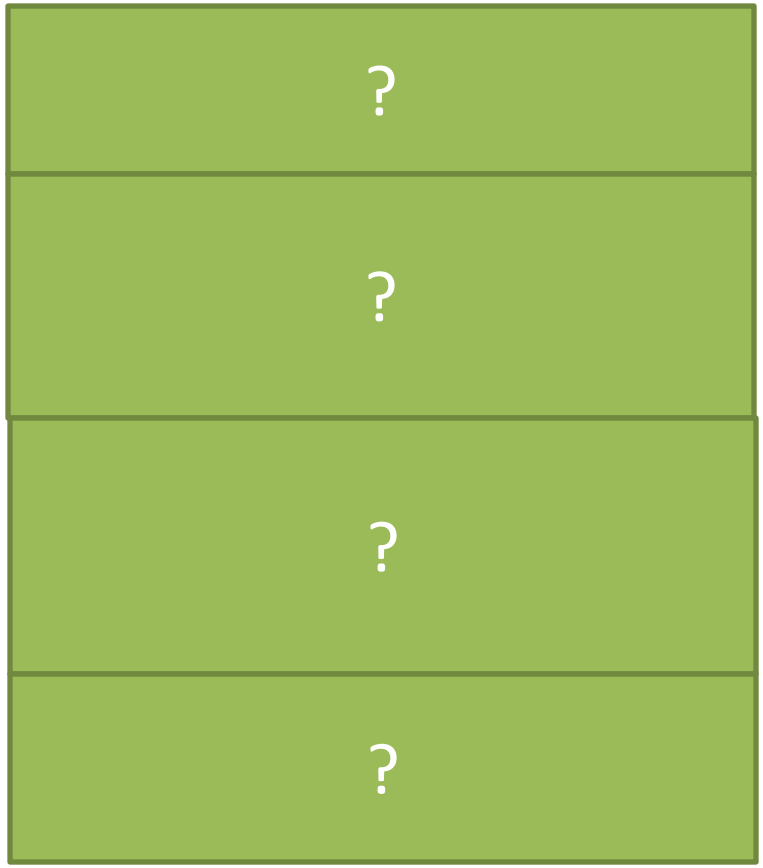
Bro Tip: Reorder the terms so you always start with something in the form $ax^2 + bx + c$

Bro Tip: Be jolly careful with your signs!

Bro Tip: You were technically done on the previous line, but it's nice to reorder the terms so it's more explicitly in the requested form.

One more example

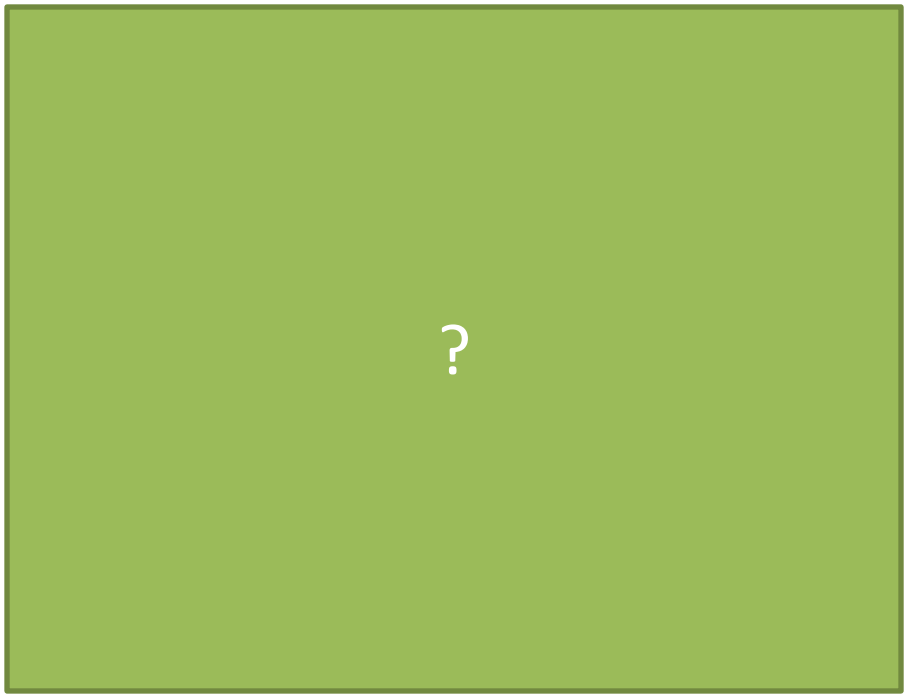
$$2x^2 + 6x + 7 =$$



This was the actual example on the specification!

Test Your Understanding

Put the expression $3x^2 - 12x + 5$ in the form $a(x + b)^2 + c$.



Proof of the Quadratic Formula!

by completing the square…

$ax^2 + bx + c = 0$

?

?

?

?

?

?

Exercises

1 Express $x^2 - 4x + 5$ in the form $(x - a)^2 + b$:

?

2 Work out the values of a and b such that

$$x^2 - 6x + 5 \equiv (x + a)^2 + b$$

?

3 [June 2013 Paper 1] Express $2x^2 - 12x - 7$ in the form $a(x + b)^2 + c$.

?

4

$$2x^2 - 4x + 5 \equiv a(x + b)^2 + c$$

Work out the values of a, b, c

?

5 Express the following in the form $a(x + b)^2 + c$

a $2x^2 + 16x =$?

b $5x^2 + 20x - 10 =$?

c $9x^2 - 18x + 27 =$?

d $3x^2 - 6x + 4 =$?

e $4x^2 + 16x - 1 =$?

6 Express the following in the form $a(x + b)^2 + c$:

a $3x^2 - x =$?

b $4x^2 + x - 1 =$?

7 Express the following in the form $a - b(x + c)^2$:

a $3 + 6x - x^2 =$?

b $10 - 8x - x^2 =$?

c $10x - 8 - 5x^2 =$?

d $1 - 36x - 6x^2 =$?

TRUE or FALSE

True or False: To complete the square for $2x^2 + 8x + 5 = 0$, you first need to divide the entire equation by 2 before proceeding.

TRUE or FALSE

True or False: The vertex form of a quadratic equation is $y = a(x - h)^2 + k$, where (h, k) is the vertex.

TRUE or FALSE

True or False: The vertex of the quadratic equation $y = 3x^2 - 12x + 7$ is $(2, -5)$.