PRECALCULUS · PROBABILITY

Estimating Probabilities Using Simulation

Model the chance process, run many trials, count what happens

14
questions worked

4
sections

8
true/false checks

Built from the Estimating Probabilities Using Simulation source deck

How this lesson runs

Key concept, worked example, questions to try — then true or false

Starter	One quick recall question: which situations suit a formula and which need a model you can run.
Key points	Each section opens with a summary of the ideas you need, main point highlighted first.
Question	Questions appear on their own slides. Attempt each before turning the page.
Solution	The next slide gives the condensed solution: the answer and the key steps only.
True or false	The lesson ends with eight statements — one per slide, answer on the slide that follows.

Key topic · Estimating probabilities using simulation

What this lesson is about, in one page

The idea	A simulation uses a random process to model a real-world chance situation, so a probability that is hard to compute can be ESTIMATED instead.
The mechanism	Each outcome gets its own random device: digits, spinners, dice, coins, cards or a random number generator — chosen to match the real probabilities.
The estimate	Estimate = favourable trials ÷ total trials. More trials make the estimate more reliable (the Law of Large Numbers).
The design	A badly designed simulation stays wrong no matter how many trials you run. Correct assignments come first; extra trials come second.

Vocabulary

Key terms for simulation work

simulation	repeating a chance process many times to estimate a probability
trial	one complete run of the simulated process (e.g. 3 coin flips)
chance mechanism	the random device that stands in for the real outcome
favourable outcome	one trial result that counts as a success for the event
estimate	favourable trials ÷ total trials — an approximation of the probability
random number generator	produces digits or numbers with no pattern
digit assignment	mapping digits to outcomes, e.g. 0–6 = hit, 7–9 = miss
two-digit numbers	00–99 gives 100 equally likely values for finer probabilities
Law of Large Numbers	estimates settle closer to the true probability as trials increase
bias	a systematic flaw in the design that no number of trials can fix

S

Starter · Formula or simulation?

Sorting probability questions by how they should be tackled

- One question to answer before the reveal
- Some probabilities have exact formulas — others need a model you can run

Key points · when to simulate

Formula first — simulation when the formula is out of reach

1	**Exact formulas**	Single coins, dice and cards often have short formulas: P(at least one head) = 1 − P(all tails).
2	**When to simulate**	Many stages, awkward probabilities or real-world messiness — model the process and run it many times.
3	**The four steps**	Define the event · assign the mechanism · run many trials · count favourable trials.
4	**The estimate**	Estimated probability = favourable trials ÷ total trials.
5	**Trials and trust**	More trials make the estimate steadier — but never rescue a badly designed model.

Question S1

3 marks

Starter · Formula or simulation?

THE QUESTION

For each situation, decide whether an exact probability is easy to compute with a formula, or whether a simulation is the natural tool. Give a one-line reason.

(a) The probability of exactly two heads in three flips of a fair coin.
(b) The probability that a basketball player making 70% of free throws makes at least 8 of her next 10.
(c) The probability that a randomly chosen word in a long paragraph starts with a vowel.

Question S1 — solution

3 marks

Starter · Formula or simulation?

CONDENSED SOLUTION

- (a) Formula — fair coin, few flips: P(exactly 2 heads in 3) is a short calculation.
- (b) Simulation — many shots with a 0.7 success chance; the exact binomial sum is heavy, so run trials.
- (c) Simulation — the probability depends on real text you cannot model with a simple device.

TRAP “Hard to compute” does not mean impossible. Simulation trades difficulty for approximation — exact formulas still win where they exist.

01

Estimating a probability from trials

Run the model, count the successes, divide

- Estimated probability = favourable trials ÷ total trials
- More trials → the estimate steadies toward the true value

Key points · estimating from trials

The core arithmetic of every simulation

1	**One trial**	A trial is one complete run: three coin flips, ten free throws, one spinner spin plus a coin flip.
2	**Count favourable**	A trial is favourable when the event of interest happens — at least one head, 8 of 10 made, and so on.
3	**Estimate**	Estimated probability = favourable trials ÷ total trials. With 14 favourable in 50 trials, the estimate is 0.28.
4	**Law of Large Numbers**	As trials increase, the estimate gets closer to the true probability — variation shrinks.
5	**Keep it in context**	An estimate is only meaningful next to the event it came from: state the event, not just the number.

Worked example · estimating a probability

the question

Question first — at least one head in three flips

THE QUESTION

A student flips a fair coin 3 times and records whether she gets at least one head.

She repeats this for 100 trials and gets at least one head in 88 of them.

(a) What is one trial here?
(b) Estimate the probability of at least one head in 3 flips.
(c) The exact probability is $1 - (1/2)^3 = 0.875$. Comment on the estimate.

Worked example · estimating a probability — working

the working

Answers only — define, estimate, compare

THE WORKING

```
(a)  One trial = flip the coin 3 times; success = at least one head.
```

```
(b)  Estimate = 88/100 = 0.88
```

```
(c)  0.88 vs 0.875: close — 100 trials give a good but not exact estimate.
```

The estimate sits 0.005 above the true value. With 1,000 trials the gap would typically shrink — the Law of Large Numbers at work.

Try it yourself

your turn

Attempt before turning the page — answer on the next slide

YOUR TURN

A die is rolled 4 times in each trial; success means at least one six.

In 50 trials, 31 contained at least one six.

Estimate the probability, then compare with $1 - (5/6)^4 \approx 0.518$.

Question Y1 — solution

3 marks

Try it · estimating from trials

CONDENSED SOLUTION

- Estimate = 31/50 = 0.62
- Exact ≈ 0.518 — the estimate is high by about 0.10.
- 50 trials is a small run: repeat with many more trials to steady the estimate.

TRAP Divide by the number of TRIALS (50), not the number of rolls (200). A trial is the whole event, not a single stage of it.

Question A1

3 marks

Estimating from trials

THE QUESTION

A fair coin is flipped 4 times in each trial; success means at least one head.

(a) In 200 trials, 187 contained at least one head. Estimate the probability.
(b) The exact probability is $1 - (1/2)^4$. Work it out.
(c) Explain why the estimate and the exact value are not identical.

Question A1 — solution

3 marks

Estimating from trials

CONDENSED SOLUTION

- (a) Estimate = 187/200 = 0.935
- (b) Exact = 1 − 0.0625 = 0.9375
- (c) Simulation uses random samples — each run differs; only with more trials does the estimate home in on 0.9375.

TRAP An estimate is not “wrong”. It is an approximation whose accuracy improves as trials accumulate — the Law of Large Numbers.

Question A2

4 marks

Estimating from trials

THE QUESTION

A spinner lands on green with probability 1/4. Each trial is: spin once, then flip a coin once.

A student runs 80 trials; green-then-heads occurred 23 times.

(a) Estimate the probability of green-then-heads.
(b) The exact probability is (1/4)(1/2) = 1/8. How does your estimate compare?
(c) Suggest ONE improvement to make the estimate more reliable.

Question A2 — solution

4 marks

Estimating from trials

CONDENSED SOLUTION

- (a) Estimate = 23/80 = 0.2875
- (b) Exact = 0.125; the estimate is far above — likely just small-sample variation.
- (c) Run many more trials (e.g. 800+); the estimate should move toward 0.125.

TRAP A big gap with only 80 trials is usually SAMPLE SIZE, not design. Check the mechanism before blaming the model.

02

Designing a simulation

Choose the mechanism, assign the digits, match the probabilities

- Every real outcome needs a random stand-in with the right probability
- One digit = 10 chances in 100; two digits = 100 chances in 100

Key points · designing a simulation

The design decides whether the model can be trusted

1	**Define the event**	Say exactly what counts as a success BEFORE running: at least one head, 3 or more made shots, at least one six.
2	**Assign the mechanism**	Give each outcome a share of the random numbers matching its real probability: p = 0.7 → digits 0–6, p = 25% → pairs 00–24.
3	**Match the structure**	If a trial needs 5 free throws, generate 5 digits per trial. One digit per stage, pairs when 10 slices are too coarse.
4	**Run many trials**	Repeat the whole trial many times; count the trials where the event happened.
5	**Compute the estimate**	Estimate = favourable trials ÷ total trials, reported in context.

Worked example · designing a simulation

the question

Question first — a 70% free-throw shooter

THE QUESTION

A player makes 70% of her free throws. A simulation models her next 5 throws using random digits 0–9, each digit equally likely.

(a) Which digits should represent a MADE throw, and why?
(b) How many digits make up one trial?
(c) One trial gave the digits 7, 1, 9, 0, 4. Was it a success (at least 3 made)?

Worked example · designing a simulation — working

the working

Answers only — assign, size the trial, judge it

THE WORKING

(a) Made = digits 0–6 (7 of 10 = 70%); miss = 7–9.

(b) One trial = 5 digits, one per throw.

(c) 7,1,9,0,4 → miss, made, miss, made, made = 3 made → success.

Each digit is one throw, so the count of digits 0–6 in the trial is the number of made throws. 3 made meets "at least 3".

Try it yourself

your turn

Attempt before turning the page — answer on the next slide

YOUR TURN

A genetics model gives a 25% chance of type “aa” offspring.

Design a simulation using two-digit numbers 00–99:

which pairs represent “aa”, and how many digits make one trial?

Question Y2 — solution

3 marks

Try it · designing a simulation

CONDENSED SOLUTION

- “aa” = the 25 pairs 00–24 (25 of 100 = 25%).
- Other pairs 25–99 represent the other types.
- One trial = one two-digit number per offspring.

TRAP Use PAIRS, not single digits: 25% needs 25 slices out of 100, and single digits only give 10. Two-digit numbers 00–99 supply the finer scale.

Question A3

3 marks

Designing a simulation

THE QUESTION

A archer hits the bullseye 60% of the time. A simulation uses single random digits 0–9.

(a) Which digits should represent a hit?
(b) Which digits represent a miss?
(c) One trial is 3 arrows; “success” is at least 2 hits. The digits drawn are 2, 8, 5. Was the trial a success?

Question A3 — solution

3 marks

Designing a simulation

CONDENSED SOLUTION

- (a) Hit = 0–5 (6 of 10 = 60%).
- (b) Miss = 6–9.
- (c) 2 = hit, 8 = miss, 5 = hit → 2 hits → success (at least 2).

TRAP Any block of consecutive digits works as long as it has the right LENGTH: 6 digits out of 10 for 60%. Always say which digits you chose.

Question A4

4 marks

Designing a simulation · two digits

THE QUESTION

A café finds that 8% of customers order oat milk. A simulation will use two-digit random numbers 00–99.

(a) Why are single digits unsuitable here?
(b) Which two-digit numbers should represent “orders oat milk”?
(c) A trial involves 4 customers. How many two-digit numbers make one trial?
(d) One trial drew 11, 90, 07, 43. How many ordered oat milk?

Question A4 — solution

4 marks

Designing a simulation · two digits

CONDENSED SOLUTION

- (a) Single digits give only 10 slices — 8% is impossible to hit exactly; pairs give 100.
- (b) 00–07 (8 of the 100 pairs = 8%).
- (c) 4 pairs, one per customer.
- (d) 11 and 07 are in 00–07? Only 07 → exactly 1 customer.

TRAP Read each pair separately. 07 is inside 00–07; 11 is not. A slip here flips the trial from success to failure — check each pair against your chosen block.

Question A5

3 marks

Designing a simulation · spinner + coin

THE QUESTION

A game show wheel: win the round with probability 1/3. After winning, the contestant flips a coin; heads doubles the prize.

Design a simulation for one complete round using single digits for the wheel and a separate coin mechanism:

(a) Which digits represent “win the round”?
(b) What is the coin mechanism?
(c) List the possible OUTCOMES of one round and their exact probabilities.

Question A5 — solution

3 marks

Designing a simulation · spinner + coin

CONDENSED SOLUTION

- (a) Wheel: digits 0–1 = win (2 of 10 ≈ 1/3, the closest 10-slice split; or use pairs 00–32 for exactly 1/3).
- (b) Coin: flip once — heads or tails, each 1/2.
- (c) Outcomes: win+heads (1/3)(1/2) = 1/6; win+tails 1/6; lose 2/3.

TRAP 1/3 cannot be built from single digits exactly (2/10 is only close). Saying WHICH approximation you made is part of a good design.

03

Trials, accuracy and bias

More trials help a good model — and expose nothing in a bad one

- More trials reduce random variation
- No number of trials fixes a badly designed simulation

Key points · trials, accuracy and bias

What more trials do — and what they never do

1	**More trials, less wobble**	Doubling the trials shrinks the random scatter; 10,000 trials beat 100 for steadiness.
2	**They never fix design**	If the digits are assigned to the wrong probabilities, running longer just approximates the WRONG value with more confidence.
3	**Bias is systematic**	A biased mechanism pushes every estimate the same wrong way — more data will not reveal it.
4	**The generator is honest**	A random number generator is unbiased, but the DESIGNER chooses the assignments — bias can enter there.
5	**Practical balance**	Enough trials to steady the estimate, not so many that recording errors creep in.

Question A6

4 marks

Trials and accuracy

THE QUESTION

Two students estimate the probability that a complex board-game move succeeds.

Amara runs 50 trials and estimates 0.44.
Ben runs 10,000 trials and estimates 0.412.

(a) Whose estimate is likely closer to the true probability?
(b) Explain the statistical reason.
(c) If Amara's model assigned digits incorrectly, could more trials fix her estimate? Explain.

Question A6 — solution

4 marks

Trials and accuracy

CONDENSED SOLUTION

- (a) Ben — 10,000 trials beat 50.
- (b) The Law of Large Numbers: random variation averages out as trials increase, so the estimate steadies near the true value.
- (c) No. A wrong design estimates the wrong probability; more trials just confirm the wrong value more confidently.

TRAP Accuracy has two ingredients: a correct model AND many trials. Extra trials only sharpen the estimate a correct design produces.

Question A7

4 marks

Bias and design flaws

THE QUESTION

A student wants to estimate P(at least one head in 4 flips). She assigns digits so that 0 = heads and 1–9 = tails.

Her simulation of 5,000 trials estimates 0.10.

(a) What is wrong with the design?
(b) Why does running 5,000 trials rather than 50 NOT fix the answer?
(c) State the correct digit assignment and the value the fixed simulation should approach.

Question A7 — solution

4 marks

Bias and design flaws

CONDENSED SOLUTION

- (a) Heads got 1 of 10 digits (10%) instead of 1/2 — the mechanism misrepresents the coin.
- (b) The bias is systematic: every large run converges to the wrong 0.10; sample size cannot repair a design error.
- (c) Heads = 0–4, tails = 5–9 (or evens/odds); the estimate should approach 0.9375.

TRAP More trials increase CONFIDENCE in the result — even when the result is wrong. Fix the design first, then run.

Mark scheme

Marks by question, with what earns each mark

S1	3	1 per situation: formula / simulation / simulation, each with a valid reason
Y1	3	1 estimate 0.62 · 1 comparison with 0.518 · 1 more-trials comment
A1	3	1 estimate 0.935 · 1 exact 0.9375 · 1 explanation of the difference
A2	4	1 estimate 0.2875 · 1 exact 0.125 · 1 comparison · 1 valid improvement
Y2	3	1 pairs 00–24 · 1 remainder 25–99 · 1 one pair per offspring
A3	3	1 hit digits 0–5 · 1 miss digits 6–9 · 1 trial judged success
A4	4	1 why single digits fail · 1 pairs 00–07 · 1 four pairs per trial · 1 count = 1
A5	3	1 wheel assignment (≈1/3 stated) · 1 coin mechanism · 1 outcomes with 1/6, 1/6, 2/3
A6	4	1 Ben · 1 Law of Large Numbers reason · 1 “no” · 1 wrong-design explanation
A7	4	1 design flaw identified · 1 systematic-bias explanation · 1 correct assignment · 1 value 0.9375

Main lesson total: 34 marks. Starter: 3 practice marks (not assessed).

Answer key

One line per question

ANSWERS

- S1 (a) formula (b) simulation (c) simulation
- Y1 0.62 vs ≈0.518; more trials steady it
- A1 0.935 · 0.9375 · random variation explains the gap
- A2 0.2875 · 0.125 · estimate high; run more trials
- Y2 aa = 00–24; one pair per offspring
- A3 hit 0–5 · miss 6–9 · success (2 hits)

QUICK VERIFICATION

- A4 pairs needed · 00–07 · 4 pairs · 1 ordered
- A5 win ≈ digits 0–1 (or 00–32) · coin flip · 1/6, 1/6, 2/3
- A6 Ben · Law of Large Numbers · no · wrong design stays wrong
- A7 heads got 10% · systematic bias · 0–4 / 5–9 · 0.9375
- Quick check: estimate = favourable trials
- ÷ total trials — and the design must
- match the real probabilities first.

True or false · 1

T / F

Decide, then justify your decision — answer on the next slide

In a simulation, one trial means generating one random digit.

True or false · 1 — answer

FALSE

FALSE

A trial is the WHOLE simulated event. For at least one head in 3 flips, one trial is 3 digits — a single digit is only one stage of it.

True or false · 2

T / F

Decide, then justify your decision — answer on the next slide

If a simulation uses digits 0–4 for success when the true chance is 60%, running 100,000 trials will correct the estimate.

True or false · 2 — answer

FALSE

FALSE

The design gives success a 50% chance, so the estimate converges to 0.50, not 0.60. Sample size sharpens the WRONG answer when the design is wrong.

True or false · 3

T / F

Decide, then justify your decision — answer on the next slide

Estimated probability = favourable trials ÷ total trials.

True or false · 3 — answer

TRUE

TRUE

That ratio is the whole method: count the trials where the event happened and divide by every trial run.

True or false · 4

T / F

Decide, then justify your decision — answer on the next slide

To simulate a 25% event with two-digit numbers 00–99, assign the pairs 00–24.

True or false · 4 — answer

TRUE

TRUE

00 to 24 is 25 pairs out of 100 equally likely pairs — exactly 25%. Single digits could not split 100 into that share.

True or false · 5

T / F

Decide, then justify your decision — answer on the next slide

Using a random number generator guarantees the simulation results will be free of bias.

True or false · 5 — answer

FALSE

FALSE

The generator supplies fair digits, but the DESIGNER chooses what each digit means. Wrong assignments inject bias the generator cannot remove.

True or false · 6

T / F

Decide, then justify your decision — answer on the next slide

A spinner simulation for a 1/3 probability can use digits 0–1 for the event, and that matches 1/3 exactly.

True or false · 6 — answer

FALSE

FALSE

2 of 10 digits is 0.20, not 1/3 ≈ 0.333. It is only an approximation — say so, or switch to pairs 00–32 (33 of 100 ≈ 1/3).

True or false · 7

T / F

Decide, then justify your decision — answer on the next slide

Increasing the number of trials reduces the random variation in the estimated probability.

True or false · 7 — answer

TRUE

TRUE

The Law of Large Numbers: the estimate wanders less and settles closer to the true probability as trials accumulate.

True or false · 8

T / F

Decide, then justify your decision — answer on the next slide

A simulation can estimate a probability even when no formula for it is known.

True or false · 8 — answer

TRUE

TRUE

That is the point of the method: model the real process, run many trials, and the ratio of favourable trials estimates the probability.

FIVE THINGS TO DO IN THE EXAM

Do these and simulation questions will not cost you marks

- Define the event of interest precisely — at least one head, not just “some heads”.
- Assign the chance mechanism so each outcome gets a share matching its real probability.
- Give the estimate as favourable trials ÷ total trials, stated in context.
- Say why more trials improve accuracy: the Law of Large Numbers reduces random variation.
- Remember: extra trials never repair a badly designed simulation — fix the assignments first.

Built from the Estimating Probabilities Using Simulation source deck.