TURN and TALK

Question 1 – Quadratics

Solve the inequality

$$2x^2 - 5x - 3 > 0$$

Question 2 – Functions

Let $f(x) = 2x - 1$ and $g(x) = x^2 + 3$.
Find:

$$(f \circ g)(x) \quad \text{and} \quad (g \circ f)(x)$$

Question 3 – Transformations

Describe the transformations that map

$$y = x^2 \quad \text{to} \quad y = -2(x - 3)^2 + 4$$

Question 1 – Quadratics (Answer)

$$(2x+1)(x-3) > 0$$

$$x < -\tfrac{1}{2} \text{ or } x > 3$$

Question 2 – Functions (Answer)

$$(f \circ g)(x) = f(x^2+3) = 2x^2+5$$

$$(g \circ f)(x) = g(2x-1) = (2x-1)^2+3 = 4x^2-4x+4$$

Question 3 – Transformations (Answer)

From $y = x^2$:

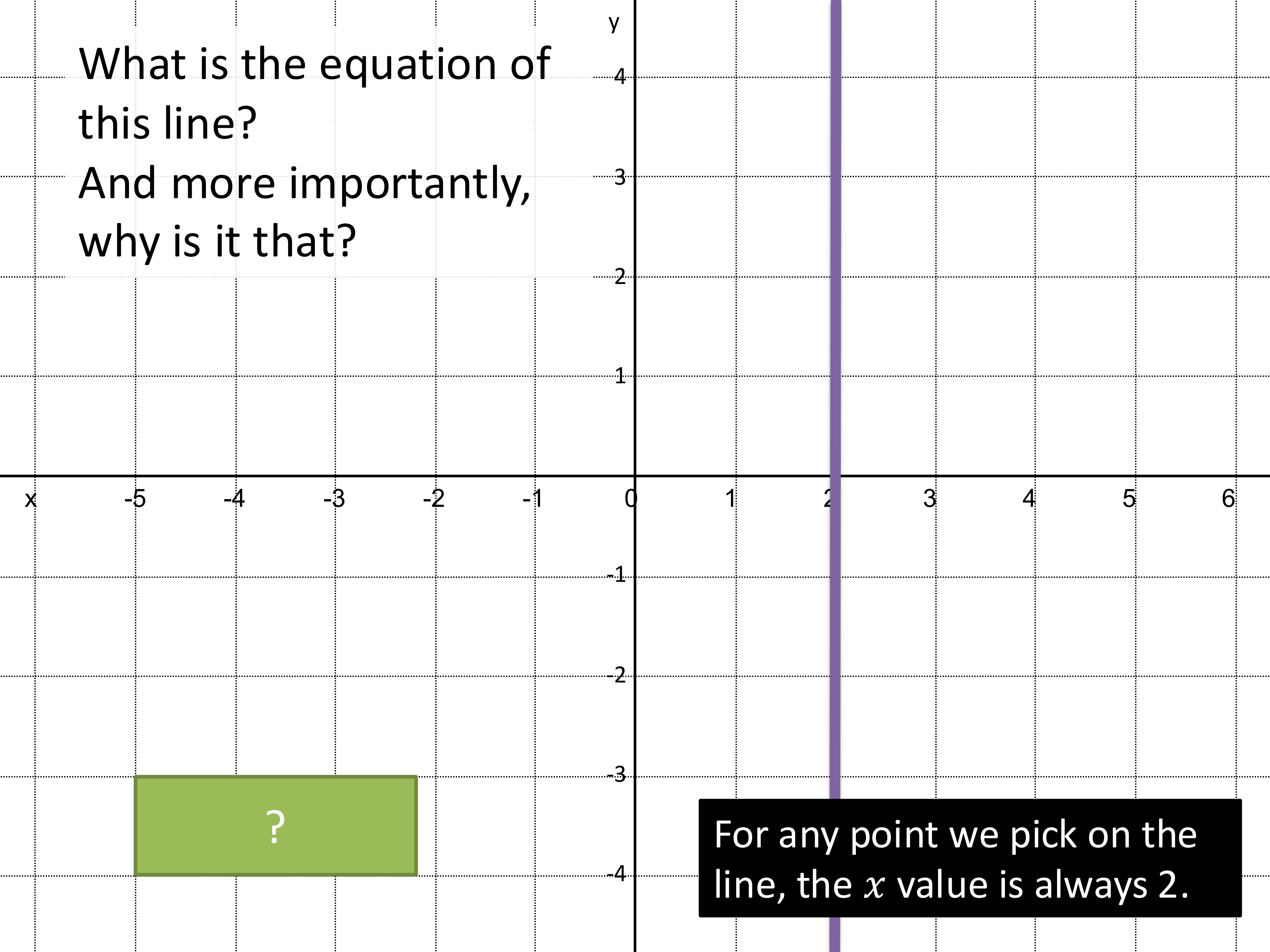
- Translate **right 3**
- Vertical stretch by factor **2**
- Reflect in the **x-axis**
- Translate **up 4**

Part 1

Lines and their Equations

1. Gradient (Slope) — 斜率
2. Intercept — 截距
3. y-intercept — y轴截距
4. x-intercept — x轴截距
5. Straight line — 直线
6. Equation of a line — 直线方程
7. Parallel lines — 平行线
8. Perpendicular lines — 垂直线
9. Midpoint — 中点
10. Distance — 距离

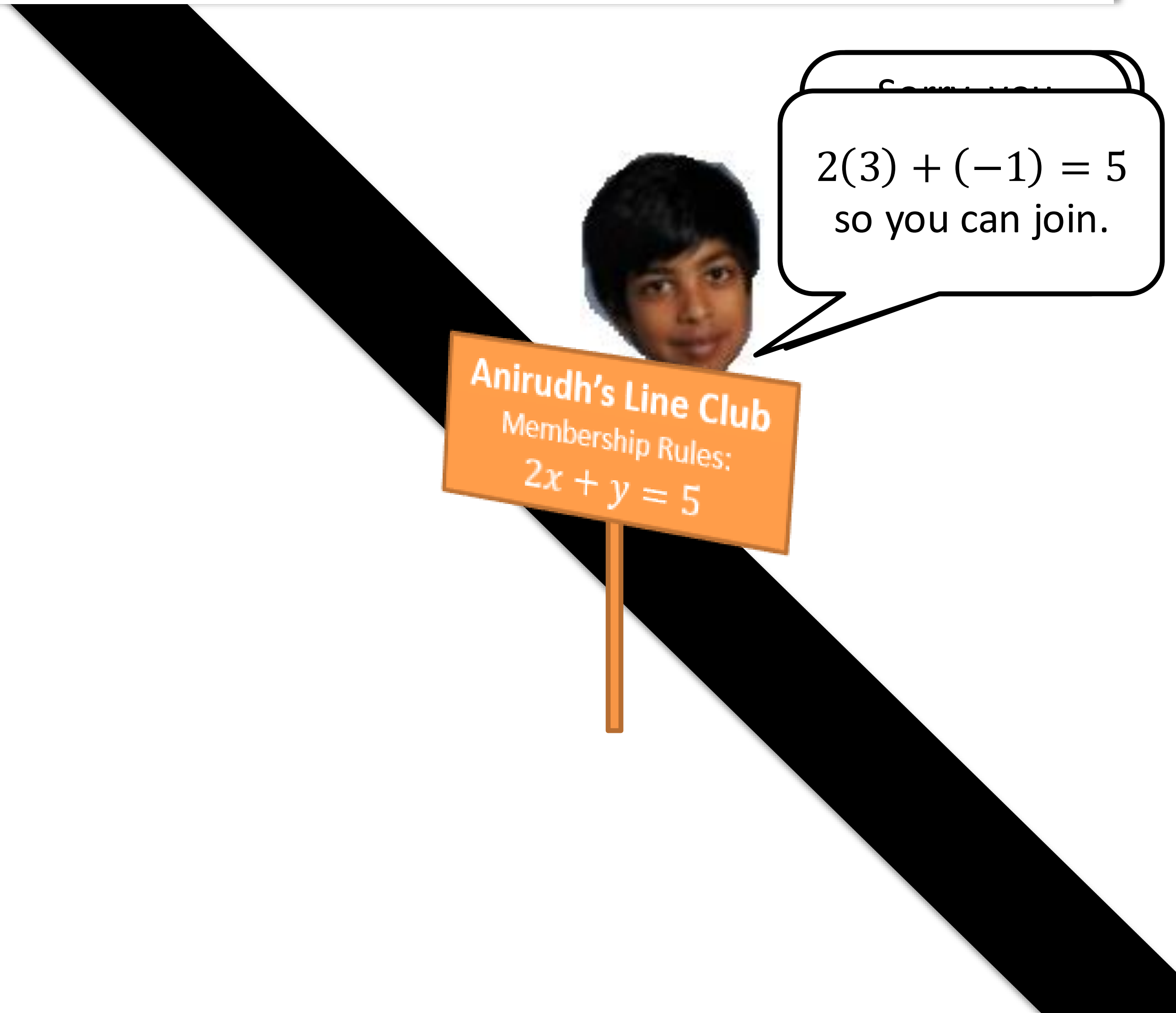
What is the equation of this line?
And more importantly, why is it that?



For any point we pick on the line, the x value is always 2.

Lines and Equations of Lines

✎ A line consists of all points which satisfy some equation in terms of x and/or y.



Examples

This means we can **substitute** the values of a coordinate into our equation whenever we know the point lies on the line.

The point $(5, a)$ lies on the line with equation $y = 3x + 2$. Determine the value of a.

?

Find the coordinate of the point where the line $2x + y = 5$ cuts the x-axis.

?

Another intercept example

Determine where the line $x + 2y = 3$ crosses the:

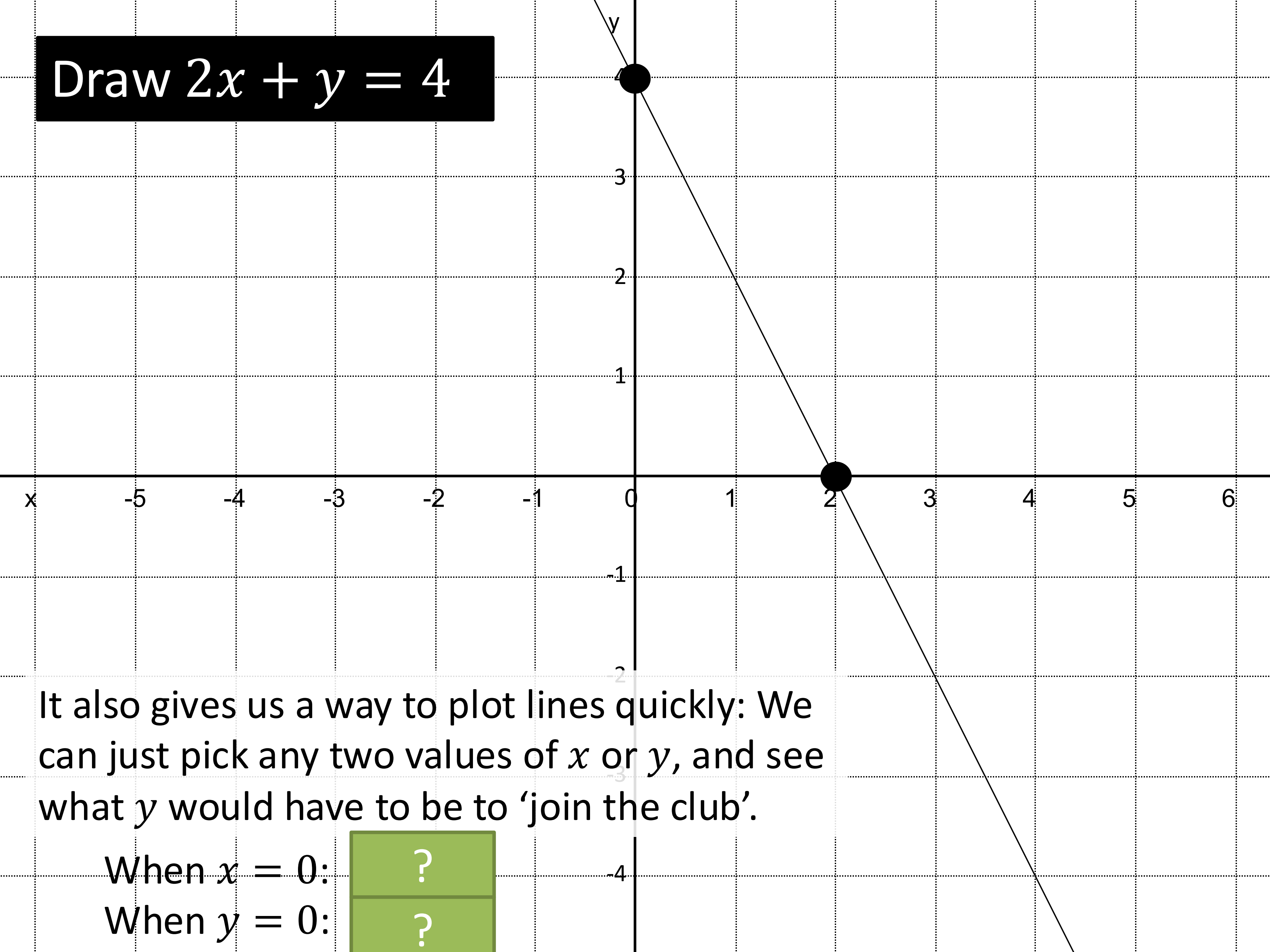
a) y-axis: ?

b) x-axis: ?

What mistakes do you think it's easy to make?

- ?
-

Draw $2x + y = 4$



It also gives us a way to plot lines quickly: We can just pick any two values of x or y, and see what y would have to be to 'join the club'.

When $x = 0$: ?

When $y = 0$: ?

Test Your Understanding

1 A point lies on the line with equation $y = 5x - 2$. The x-value of the point is 6. What would the y value have to be?

?

2 The point $(k, 8)$ lies on the line with equation $y = 20 - 4x$. Determine the value of k.

?

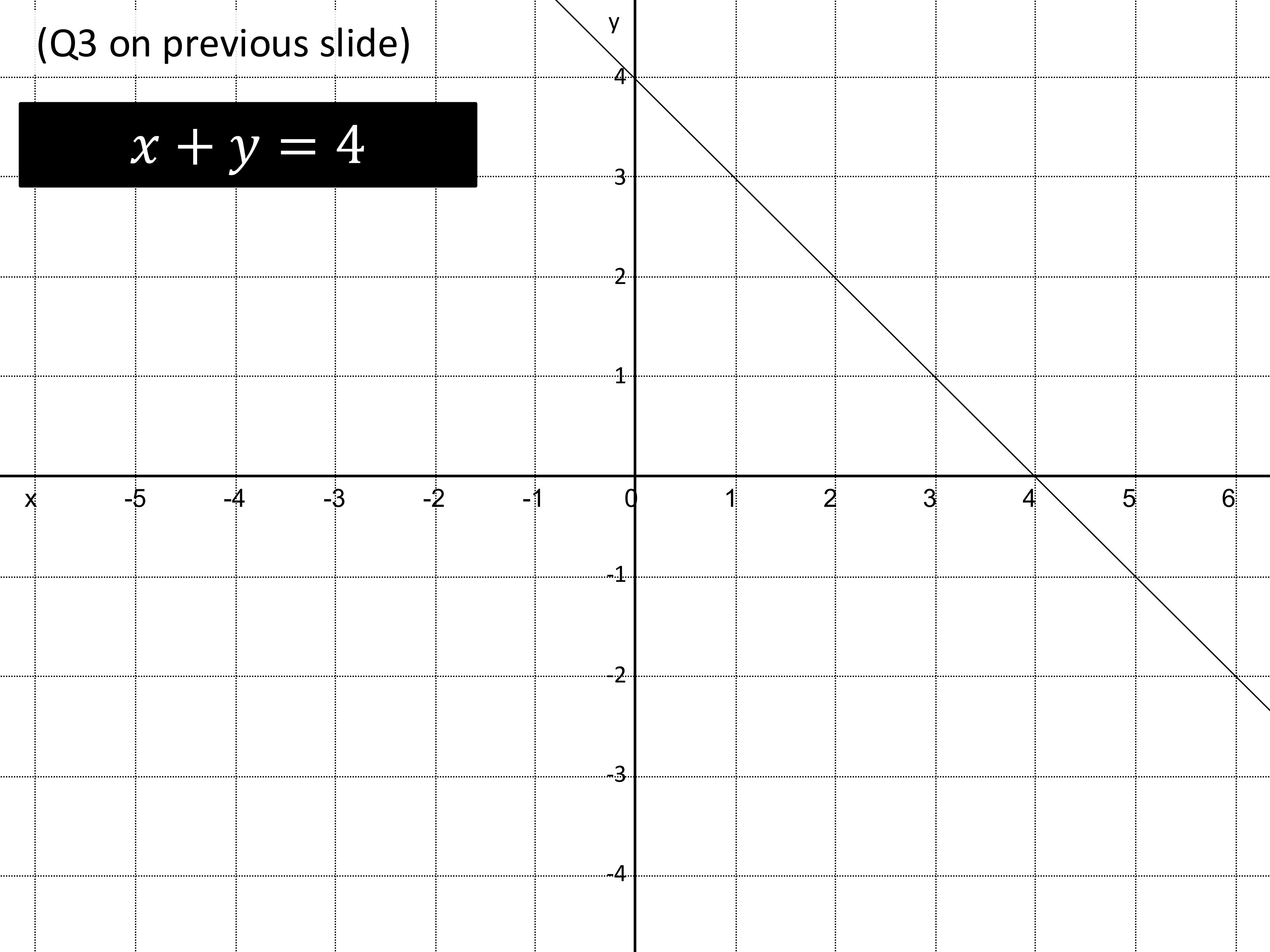
3 Draw coordinate axis in your book with x going from 0 to 6 and y going from 0 to 6. Draw the line with equation $x + y = 4$.

4 A line has equation $3x - 2y + 4 = 0$. Determine the coordinate of the point it intercepts the x-axis.

?

(Q3 on previous slide)

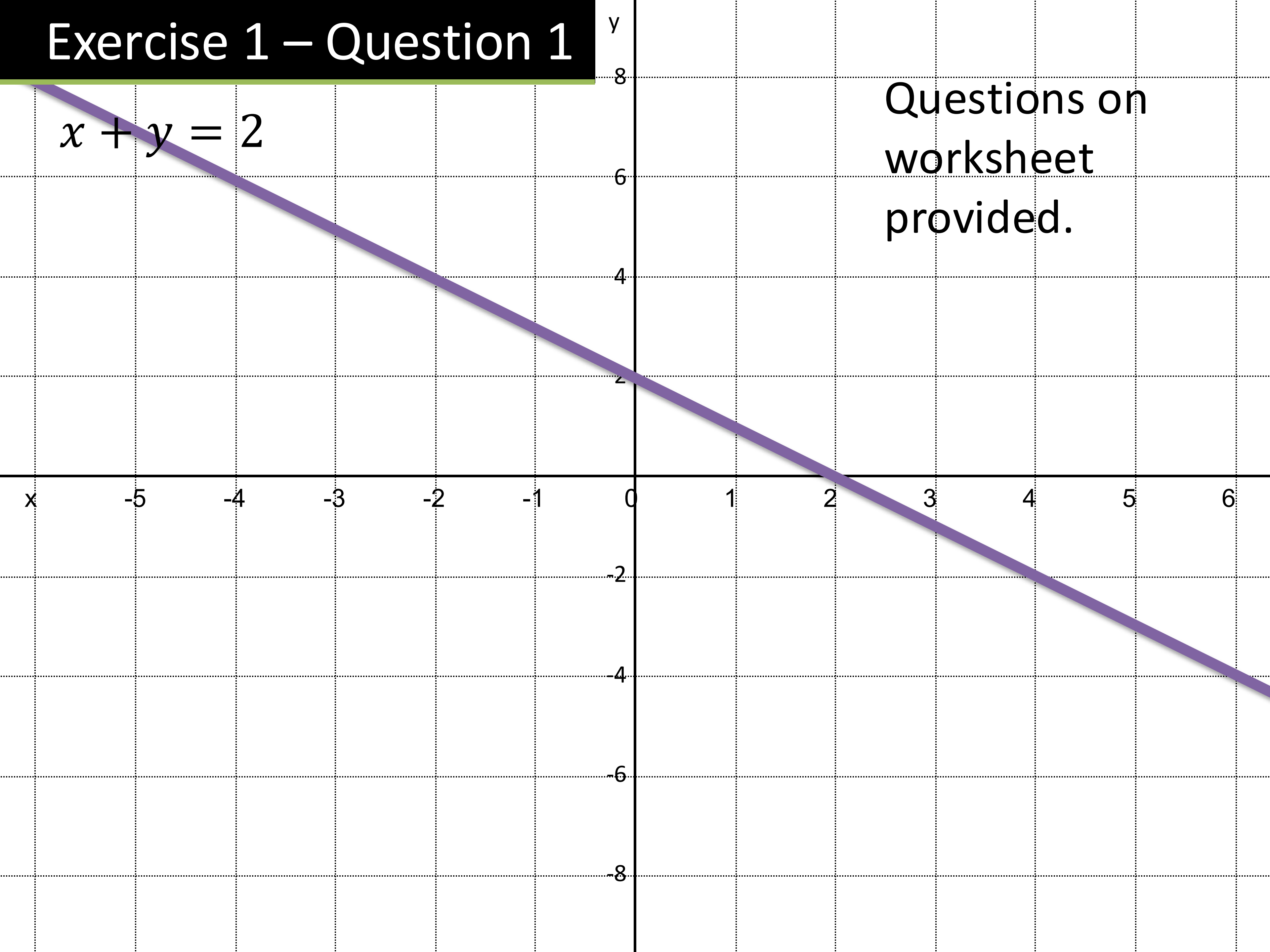
$$x + y = 4$$



Exercise 1 – Question 1

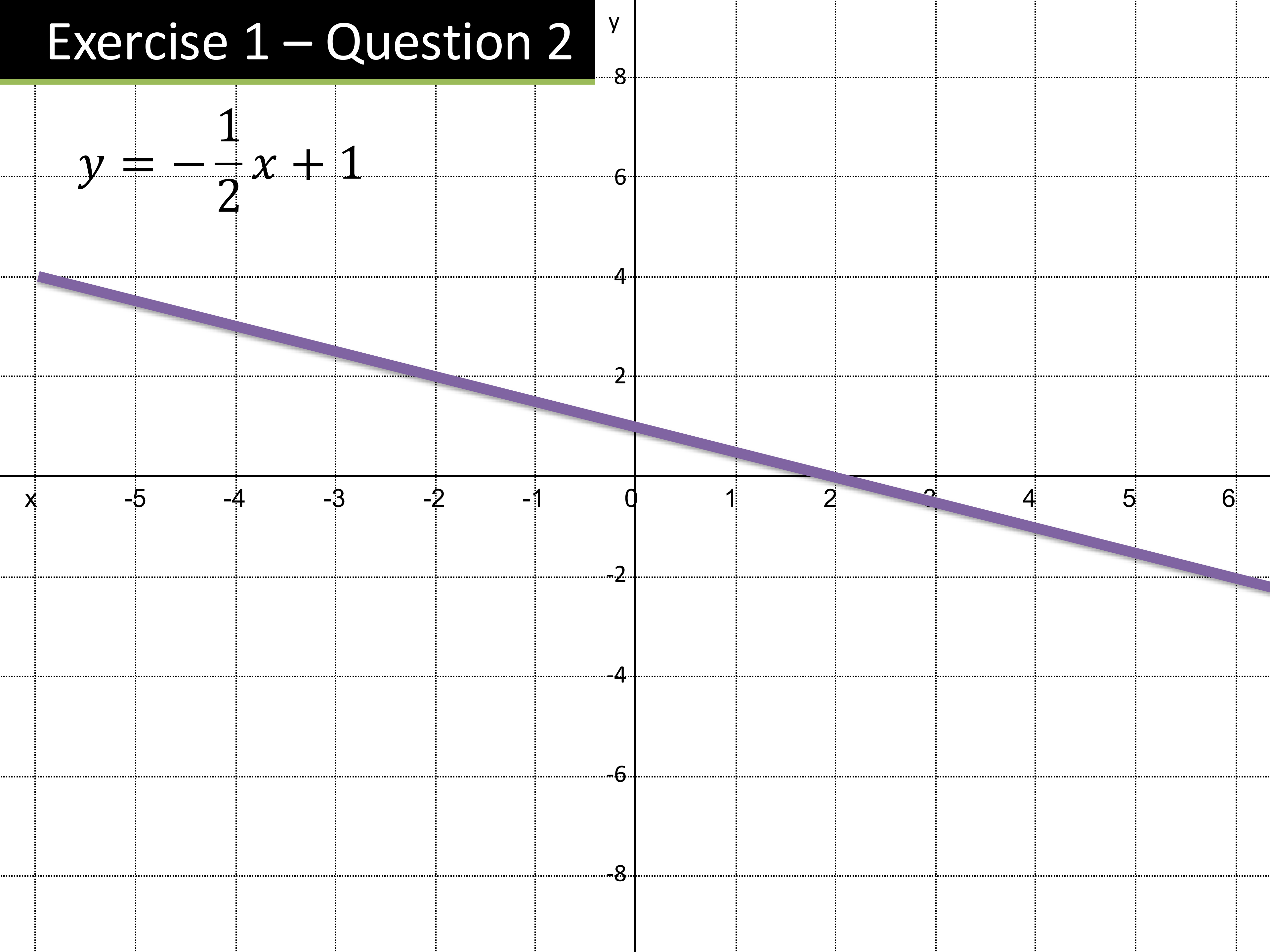
$x + y = 2$

Questions on worksheet provided.



Exercise 1 – Question 2

$$y = -\frac{1}{2}x + 1$$



Exercise 1 – Question 3

When the point $(3, k)$ lies on each of these lines, find the value of k.

a	$y = 3x + 2$	?
b	$y = 4x - 2$	?
c	$y = 3 - 2x$	?
d	$x + y = 7$	?
e	$x - 2y = 1$	?

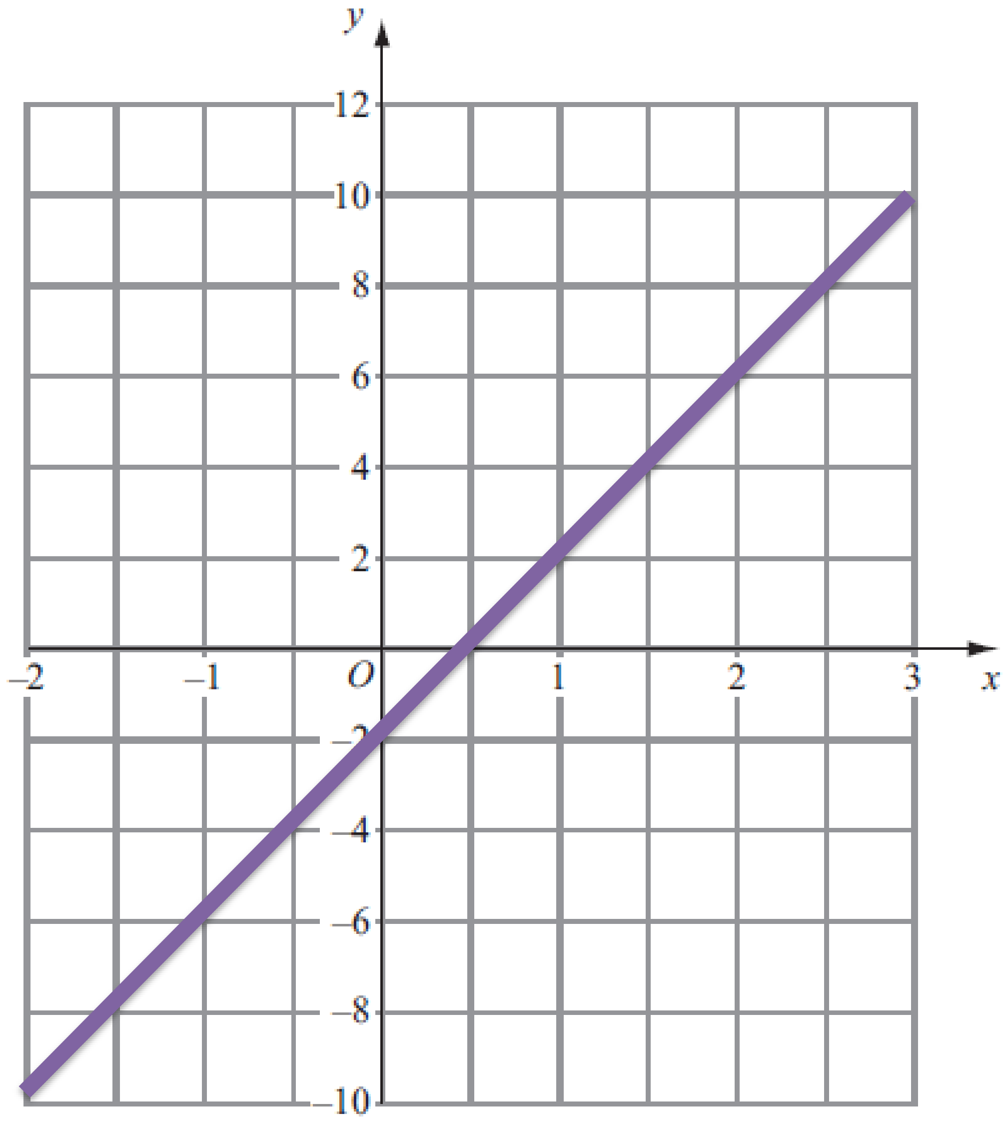
Exercise 1 – Question 4

Copy and complete this table.

Equation	The point where the line crosses the: y-axis	x-axis
$y = 3x + 1$	?	?
$y = 4x - 2$	?	?
$y = \frac{1}{2}x - 1$	?	?
$2x + 3y = 4$	?	?

Exercise 1 – Question 5

$y = 4x - 2$



Exercise 1 – Question 6

When the point $(k, 3)$ lies on each of these lines, find the value of k.

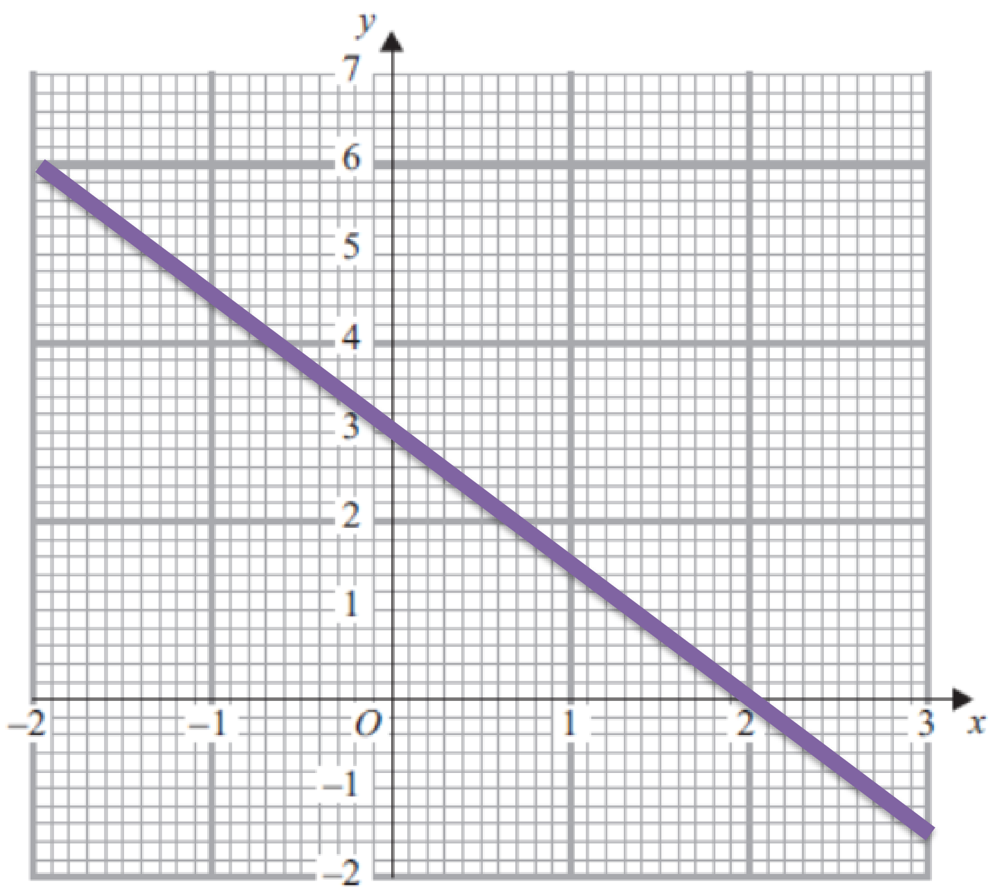
$y = 2x + 1$	?
$y = 2x - 1$	?
$y = 8 - 2x$	?
$2x + 3y = 4$	?

Exercise 1 – Question 7

Complete the table of values for $3x + 2y = 6$

x	–2	–1	0	1	2	3
y	?	4.5	3	?	?	–1.5

(b) On the grid, draw the graph of $3x + 2y = 6$



Click to Reveal

Exercise 1 – Question 8

Complete the table of values for $x + 2y = 1$.

$\boldsymbol{x}$	-2	-1	0	1	2
$\boldsymbol{y}$	?	1	?	?	?

If $x = -2$ just sub it into the equation:

$$-2 + 2y = 1$$
$$2y = 3$$
$$y = \frac{3}{2}$$

Exercise 1 – Question 9

Put a tick or cross to determine whether each of the following points are on the line with the given equation.

	$y = 1 - x$	$x + 2y = 3$
$(3, -2)$	?	?
$(1, 2)$	?	?
$\left(2, \frac{1}{2}\right)$	?	?
$(-1, 2)$	?	?

Exercise 1 – Question 10

For the given equation of a line and point, indicate whether the point is above the line, on the line or below the line. (Hint: Find out what y is on the line for the given x)

		Below the line	On the line	Above the line
$y = 3x + 4$	$(3,11)$	?		
$x + y = 5$	$(7,-2)$	?		
$y = 3 - 2x$	$(-3,10)$	?		
$2x + 3y = 4$	$\left(\frac{3}{4},\frac{4}{5}\right)$	?		

Exercise 1 – Question ☠$_1$

The equation of a line is $ax + by = c$. If the x value of some point on the line is d, what is the full coordinate of the point, in terms of a, b, c, d?

?

Exercise 1 – Question ☠$_2$

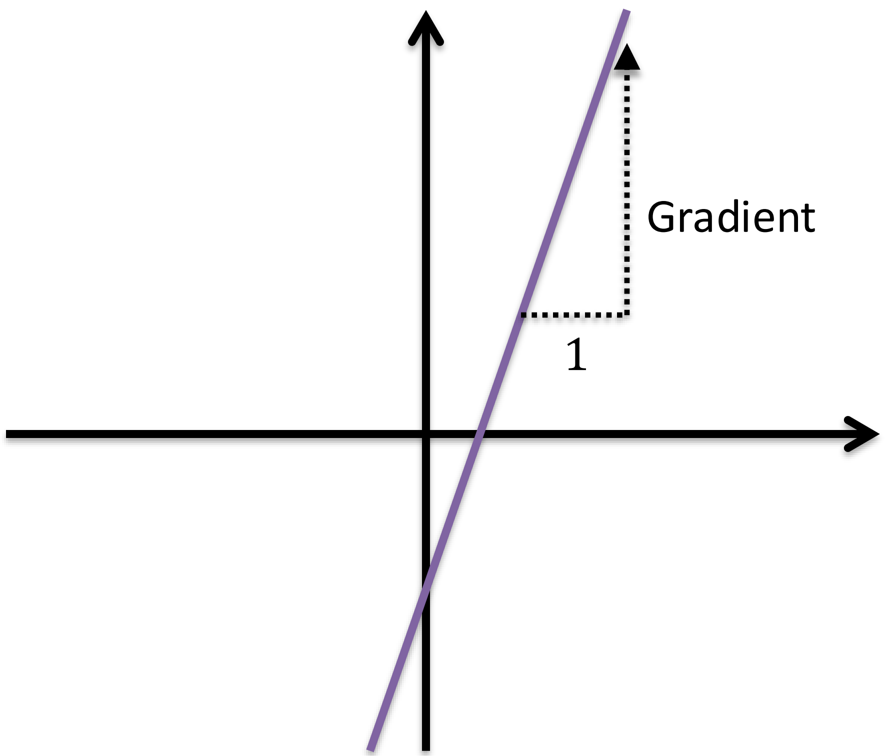
What is the area of the region enclosed between the line with equation $2x + 7y = 3$, the x axis, and the y axis?

?

Recap of gradient

The steepness of a line is known as the **gradient**.

It tells us what y changes by as x increases by 1.



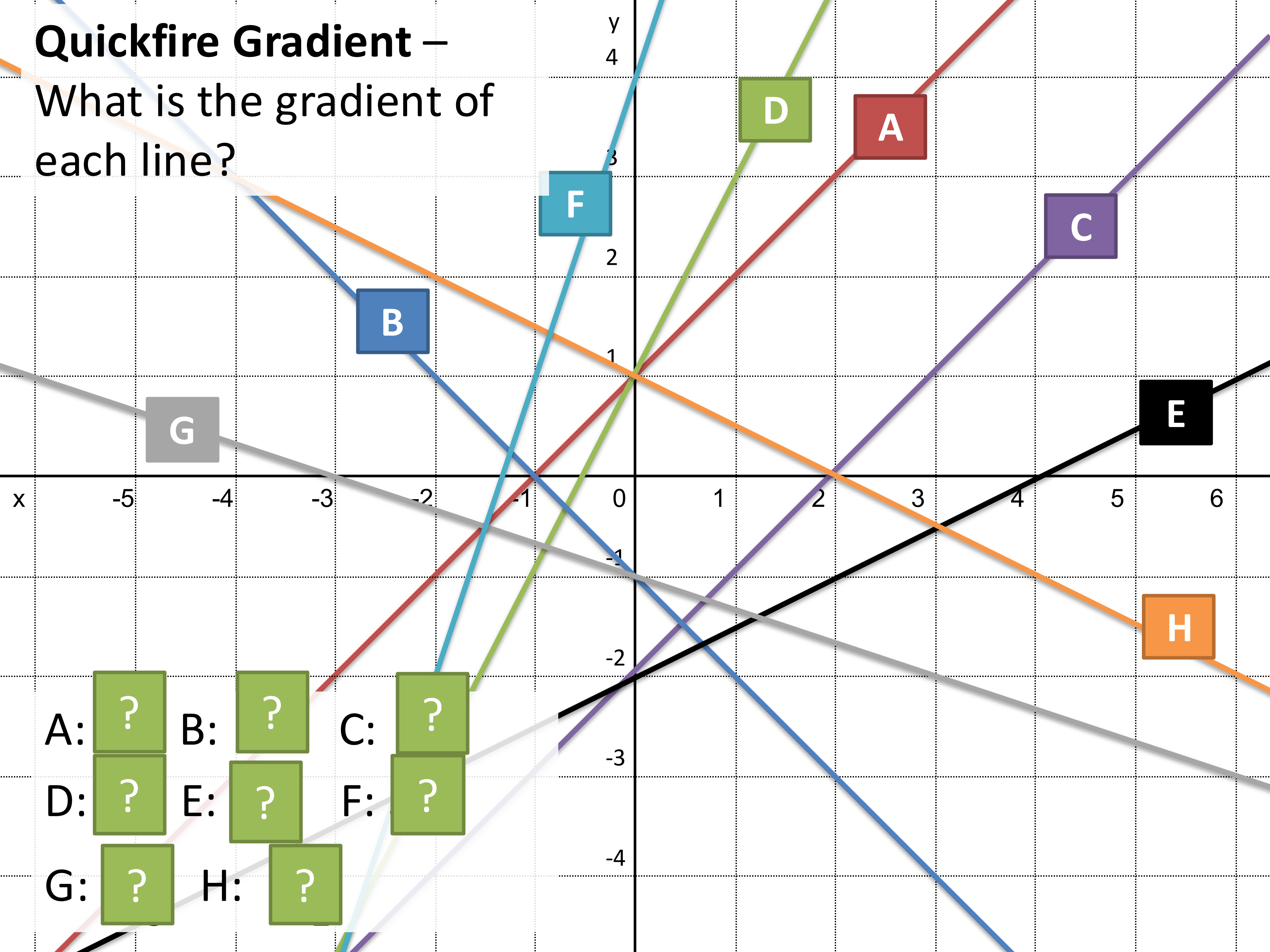
The equation of a straight line is of the form:

$$\boldsymbol{y} = \ ?$$

The gradient is m.

c is the 'y-intercept'.

Quickfire Gradient – What is the gradient of each line?



A: ? B: ? C: ?

D: ? E: ? F: ?

G: ? H: ?

Gradient using the Equation

We can get the gradient of a line using just its equation.
Rearrange into the form $y = mx + c$ (i.e. make y the subject; the gradient is m.

Examples

$$y + 2x = 1$$

?

$$2y = x + 1$$

?

Test Your Understanding

1 $y = 1 + 3x$

?

2 $x - y = 1$

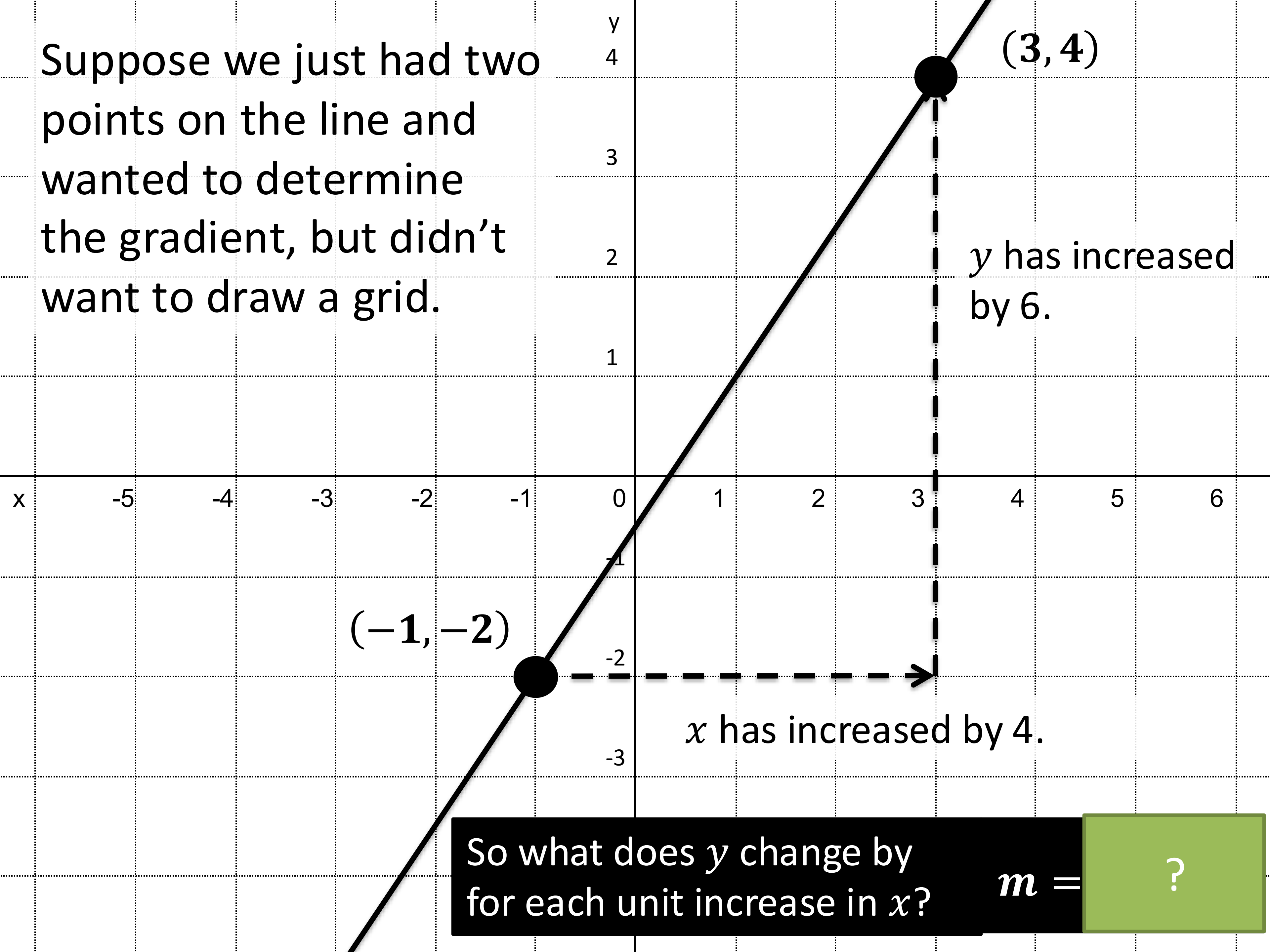
?

3 $2y + 3x = 4$

?

4 $3x - 2y = 1$

?


Suppose we just had two points on the line and wanted to determine the gradient, but didn't want to draw a grid.
y
4
3
2
1
-1
-2
-3
x
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
$(3, 4)$
$(-1, -2)$
y has increased by 6.
x has increased by 4.
So what does y change by for each unit increase in x?
$m =$
?

Gradient using two points

Given two points on a line, the gradient is:

$$m = \frac{change\ in\ y}{change\ in\ x}$$

$(1, 4) \quad (3, 10)$ $\qquad m =$?

$(5, 7) \quad (8, 1)$ $\qquad m =$?

$(2, 2) \quad (-1, 10)$ $\qquad m =$?

Quickfire Gradients

$y = 1 - x$	$\boldsymbol{m} =$	?
$(1,4), (3,12)$	$\boldsymbol{m} =$	?
$3y = 5x + 4$	$\boldsymbol{m} =$	?
$(5,7), (9,9)$	$\boldsymbol{m} =$	?
$2x + y = 1$	$\boldsymbol{m} =$	?
$(-1,0), (4,-10)$	$\boldsymbol{m} =$	?
$5x - 2y = 4$	$\boldsymbol{m} =$	?

Exercise 2

(On provided sheet)

1 By rearranging the equations into the form $y = mx + c$, determine the gradient of each line.

Equation	Gradient
$y = x + 1$	?
$y = 2 - x$	?
$y = 3$	?
$2y = 6x - 4$	?
$4y = 5x + 1$	?
$x + y = 1$	?
$2x + 3y = -4$	?
$x - 3y = 4$	?
$x + 4y = 5$	?
$3x - 4y = 7$	?

2 Determine the gradient of the line which goes through the following points.

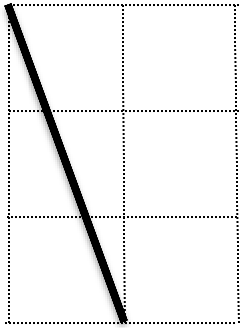
Point 1	Point 2	Gradient
$(0,0)$	$(2,2)$	?
$(1,3)$	$(3,7)$	?
$(0,5)$	$(4,25)$	?
$(2,2)$	$(-1,5)$	?
$(4,3)$	$(10,6)$	?
$(7,8)$	$(-4,-3)$	?
$(7,1)$	$(-1,5)$	?
$(6,5)$	$(8,1)$	?
$(1,3)$	$(5,10)$	?
$(-1,4)$	$(9,-5)$	?
$(1,0)$	$(-2,-4)$	?

Summary

The gradient of a line is the **steepness**: how much y changes as x increases by 1.
We've seen 3 ways in which we can calculate the gradient:

a. Counting Squares

(but only works if scales on x and y axis are the same)



$m =$?

b. Using the equation

$$2y = 4 - 5x$$

$m =$?

c. Using two points

$$(1, 4), (4, 13)$$

$m =$?

Part 3

Equations given gradients/points

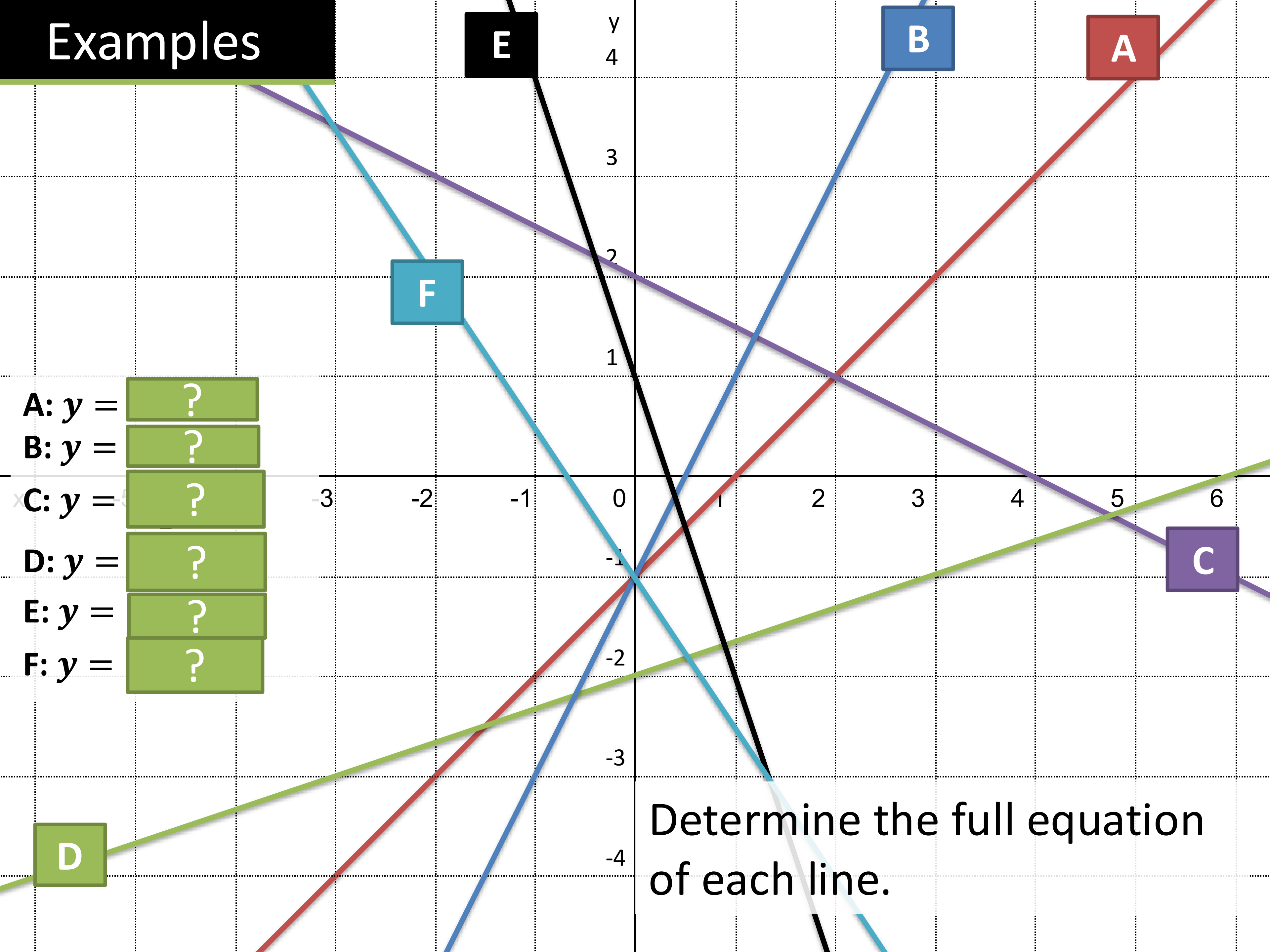
RECAP: Equation of a line

From earlier:

The equation of a straight line is $y = \boldsymbol{m}x + \boldsymbol{c}$

gradient **y-intercept**

Examples



A: $y =$?

B: $y =$?

C: $y =$?

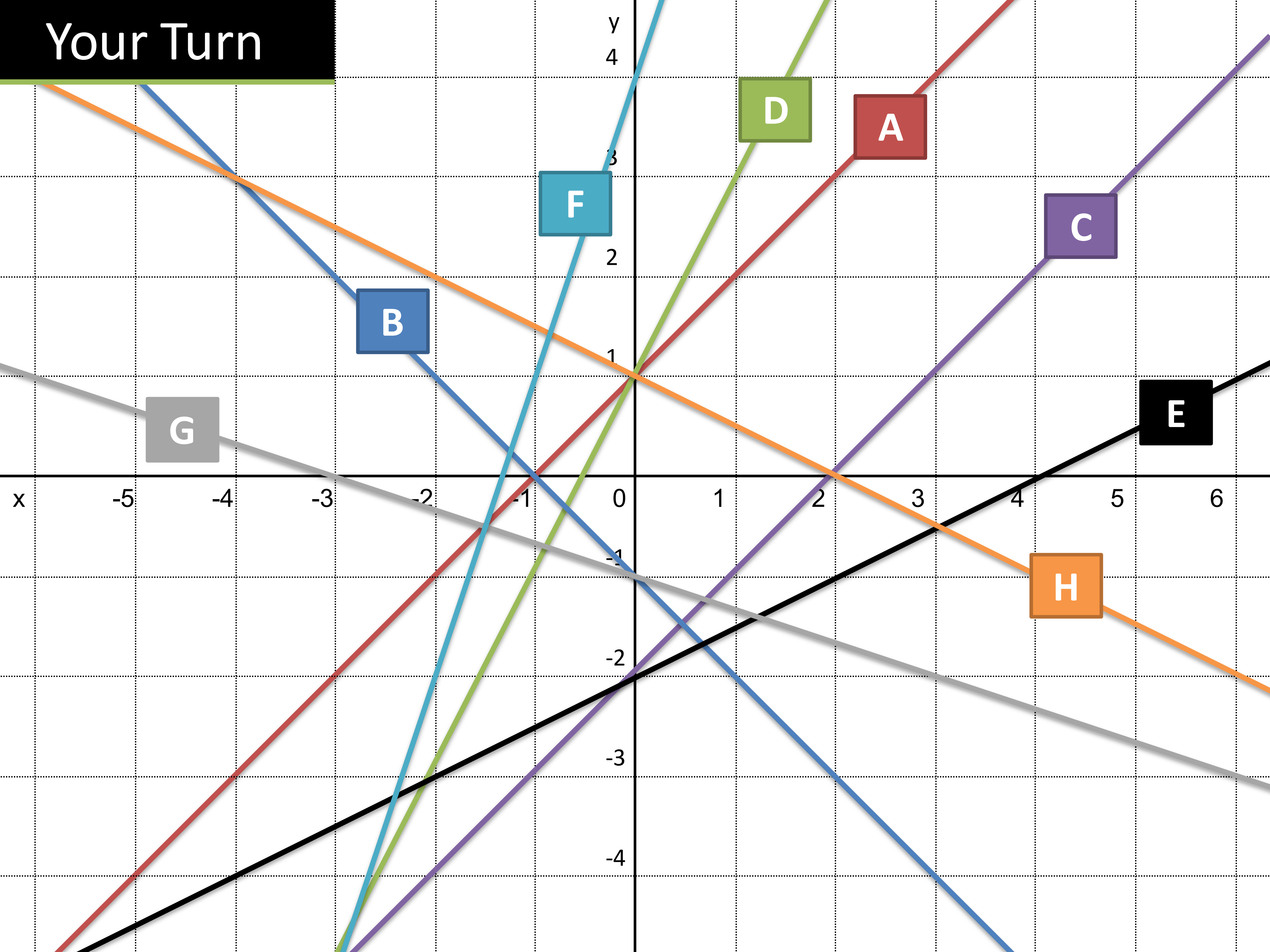
D: $y =$?

E: $y =$?

F: $y =$?

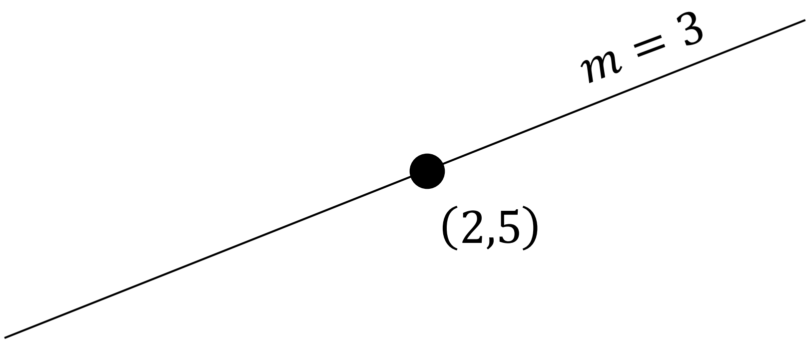
Determine the full equation of each line.

Your Turn



Getting equation of line using gradient and point

A line with gradient 3 goes through the point $(2,5)$. Determine the equation of the line in the form
$y = mx + c$.



'GCSE' method

$y =$?

Using gradient, write the formula so far. y-intercept is not yet known.

?

?

Point $(2,5)$ is on the line, so it must satisfy the equation. Use to find c.

A few more examples

A line has gradient $(4,9)$ and has gradient 2. What is the equation of the line in the form $y = mx + c$?

?

Mental Tip: If you know the formula starts with $2x$, find what this is for your x value, then think what you have to 'adjust' by to get this to y. In this case, $2x$ is 8, so we can see we have to +1 to get to 9.

Quickfire Questions:
(using the mental trick)

Gradient of 2. Goes through $(7,11)$ → $y =$?
Gradient of 4. Goes through $(7,30)$ → $y =$?
Gradient of 1. Goes through $(7,11)$ → $y =$?
Gradient of -3. Goes through $(2,5)$ → $y =$?
Gradient of $-\frac{1}{2}$. Goes through $(4,5)$ → $y =$?

A few more examples

A line l_1 passes through $(3,5)$ and $(7,7)$. What is the equation of the line in the form $ax + by + c = 0$, where a, b, c are integers?

?

We want a form where there are no fractions and 0 is on one side:

?

Test Your Understanding

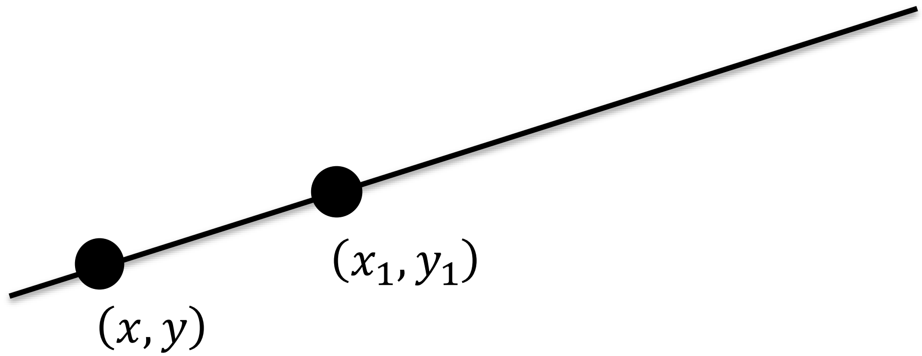
The gradient of a line is $\frac{1}{3}$, and passes through $(12,2)$. What is the equation of the line, in the form $y = mx + c$.

?

A line l_1 passes through $(6,3)$ and $(9,2)$. Determine the equation of the line in the form $ax + by + c = 0$, where a, b, c are integers, and hence determine the coordinate of the point where the line crosses the x-axis.

?

Another way (IGCSEFM/C1)



For the previous questions, we've fixed a particular point on the line (let's call this (x_1, y_1). But we'd still get an equation in terms of x and y, representing all the points (x, y) that lie on the line.*

Can we use our approach before to find a more general equation for the line?

$m =$?

✏ The equation of a straight line with gradient m and goes through (x_1, y_1) is

$$y - y_1 = m(x - x_1)$$

* **Note**: For this reason, we say that x_1 and y_1 are constants, because they are fixed, but x and y are variables because they can vary for a particular straight line as we consider different points on the line.

Quickfire Questions

In a nutshell: You can use this formula whenever you have (a) a gradient and (b) any point on the line.

Gradient	Point	(Unsimplified) Equation
3	$(1,2)$	?
5	$(3,0)$	?
2	$(-3,4)$	?
$\frac{1}{2}$	$(1,-5)$	?
9	$(-4,-4)$	?

Side Note: I've found that many students shun this formula and just use the GCSE method. But you'll find later on in school it's common to have algebraic gradients or points, where this new method would be much easier. It's worth getting used to.

Test Your Understanding

A line l_1 passes through $(7,5)$ and $(13,2)$. Determine the equation of the line in the form $ax + by + c = 0$, where a, b, c are integers.

?

Exercise 3

(On provided sheet)

1 Find the equation of the line with the specified gradient which goes through the specified point, leaving your answer in the form $y = mx + c$.

- a $(4,3), m = 2$ → ?
- b $(5,20), m = 3$ → ?
- c $(4,0), m = 5$ → ?
- d $(4,3), m = \frac{1}{2}$ → ?
- e $(-4,3), m = -1$ → ?
- f $(6,4), m = -\frac{1}{3}$ → ?

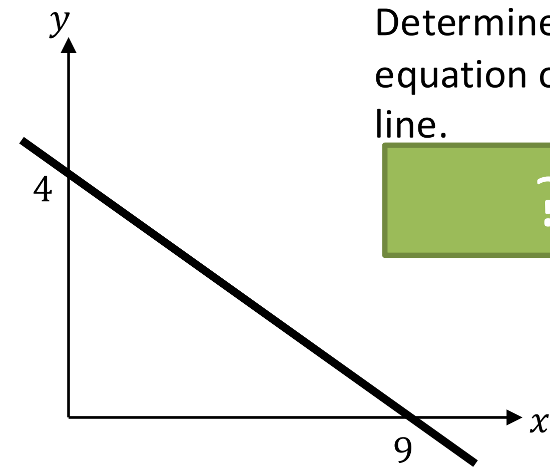
2 Do the same, but leave your equations in the form $ax + by + c = 0$ where a, b, c are integers. (I advise using the formula)

- a $(2,3), m = 4$ → ?
- b $(5,11), m = \frac{1}{2}$ → ?
- c $(7,-2), m = \frac{1}{3}$ → ?
- d $(-2,5), m = \frac{2}{3}$ → ?
- e $(4,-1), m = \frac{3}{4}$ → ?

3 Find the equation of the line that goes through the following points, leaving your equation in the form $y = mx + c$.

- a $(2,3), (6,7)$ → ?
- b $(-1,3), (4,-7)$ → ?
- c $(4,5), (-2,2)$ → ?
- d $(3,7), (9,5)$ → ?

4



Determine the equation of this line.

?

Exercise 3

(On provided sheet)

5 A line passes through the points $(2,5)$ and $(9,10)$.
a) Find the equation of the line in the form $ax + by + c = 0$, where a, b, c are integers. ?
b) Hence determine the coordinate of the point where the line crosses the x-axis. ?

6 The line l_1 passes through the points $A(15,11)$ and $B(21,9)$ and intercepts the y-axis at the point C. The line l_2 passes through C and $D(5,17)$. Determine the equation of the line l_2 in the form $y = mx + c$.

?

7 A line passes through $(4, a + 13), (a, 4a + 1)$ for some constant a. Determine the gradient of the line.

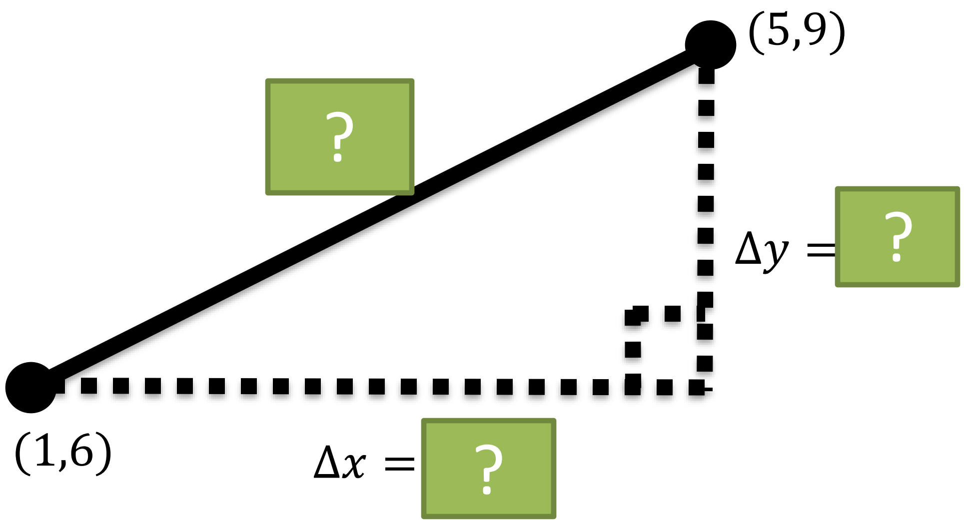
?

Part 4

Distances between points and points of intersection

Distances between points

Δ (Greek letter 'delta') means "change in"



How could we find the **distance** between these two points?

Hint: ?

?

Distance between two points:

$$\sqrt{(\Delta x)^2 + (\Delta y)^2}$$

Examples

Distance between:

$(3,4)$ and $(5,7)$?
$(5,1)$ and $(6,-3)$?
$(0,-2)$ and $(-1,3)$?

Note: Note that unlike with gradient, we don't care if the difference is positive or negative (it's being squared to make it positive anyway!)

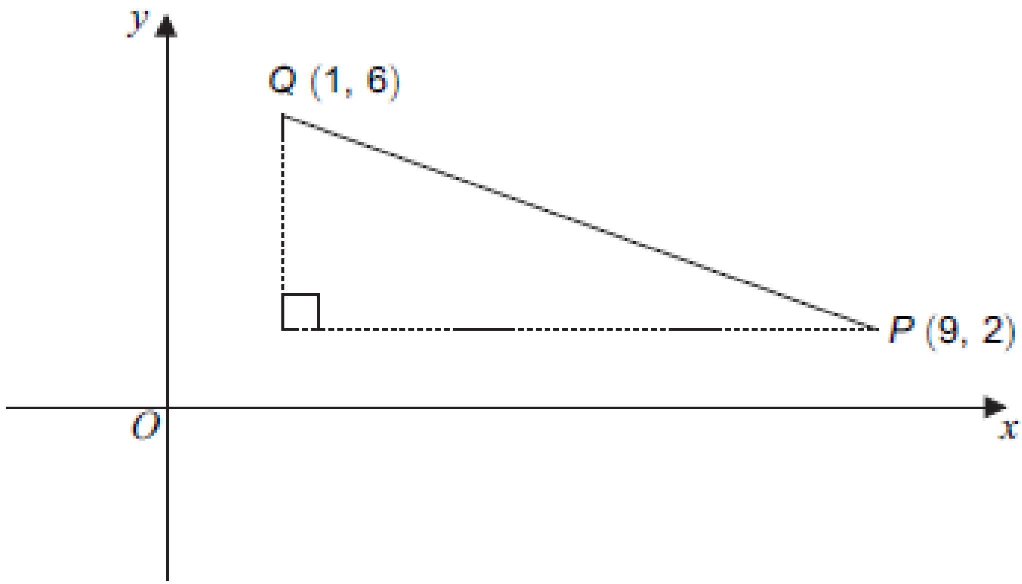
Quickfire Questions:

Distance between:

$(1,10)$ and $(4,14)$?
$(3,-1)$ and $(0,1)$?
$(-4,-2)$ and $(-12,4)$?

Test Your Understanding So Far...

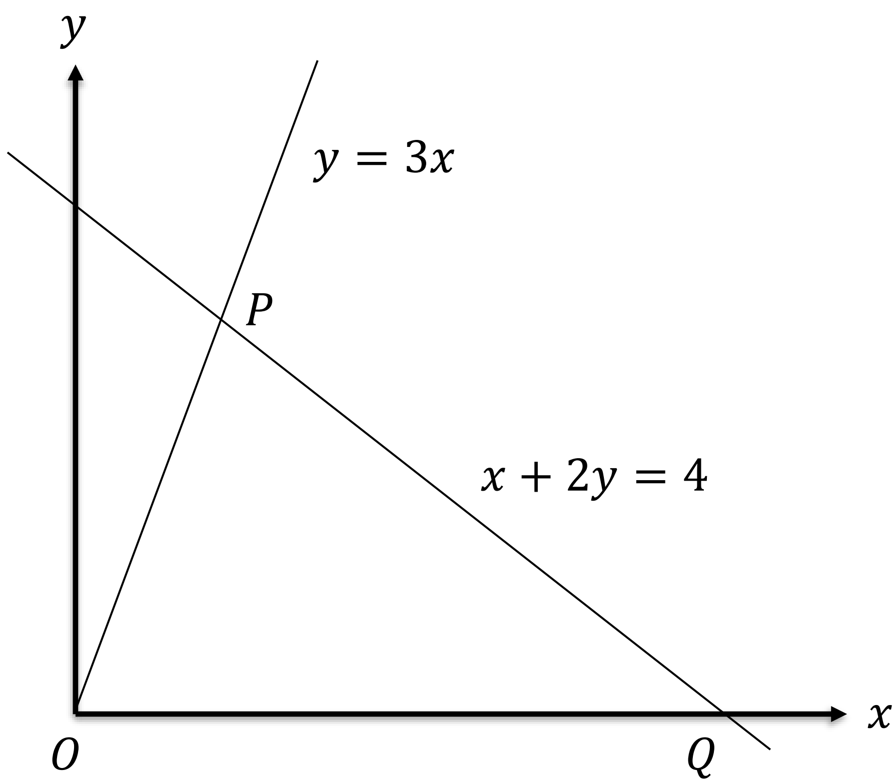
AQA IGCSEFM June 2012 Paper 2 Q3



Work out the length of *PQ*.
Give your answer to 3 significant figures.

$PQ =$?

Intersection of lines



The diagram shows two lines with equations $y = 3x$ and $x + 2y = 4$, which intersect at the point P. The line OP passes through the origin.

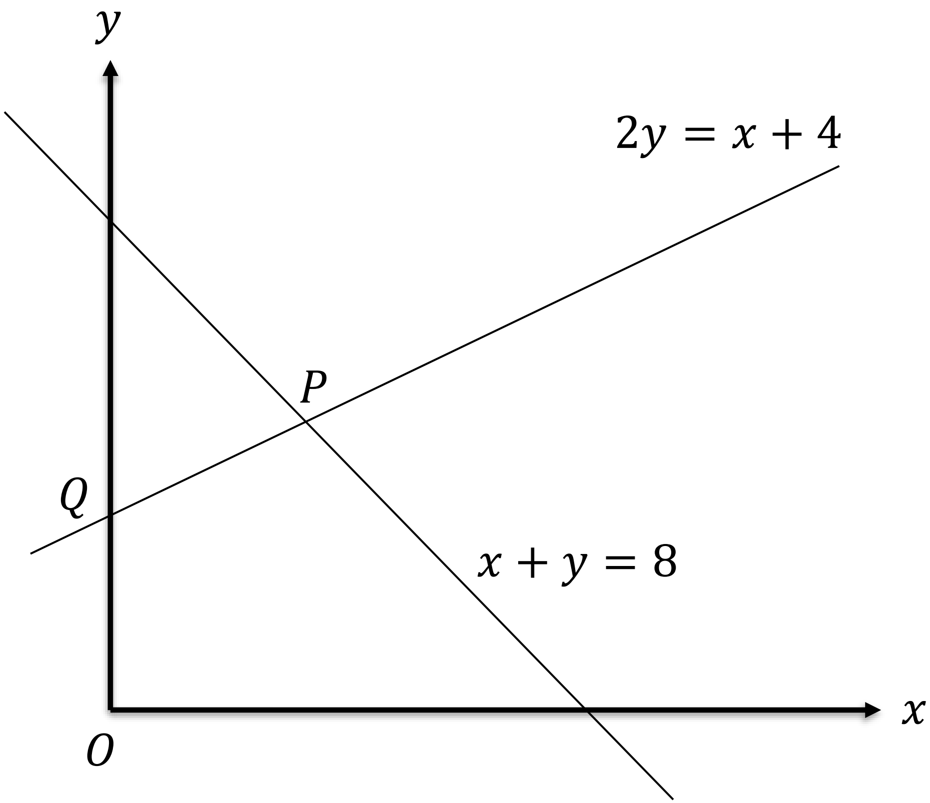
a) Determine the coordinates of P.

?

b) The line $x + 2y = 4$ intersects the x-axis at the point Q. Determine the area of the triangle OPQ.

?

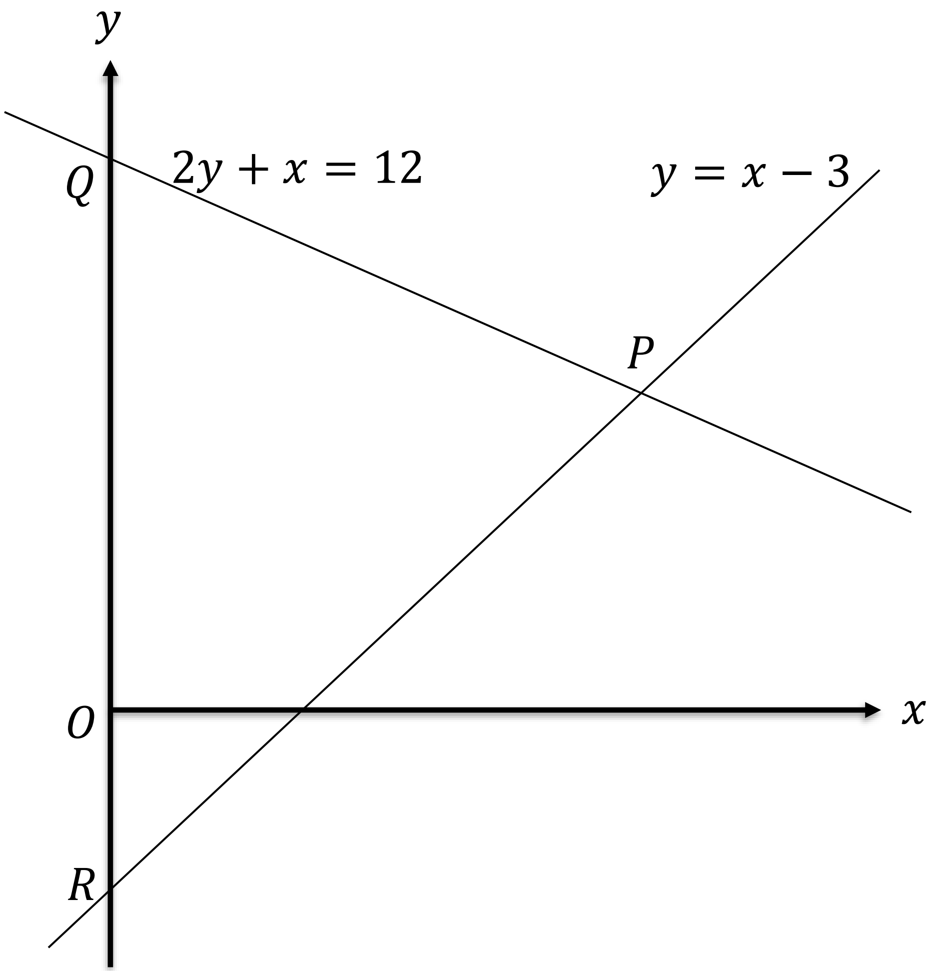
Further Example



Determine the length of PQ.

?

Test Your Understanding



a) Determine the coordinate of P.

?

b) Determine the area of PQR.

?

c) Determine the length PQ.

?

Exercise 4

(On provided sheet)

1 Find the coordinate of the point of intersection between these lines:

a $y = x + 5, \quad y = 2x \quad \rightarrow$?

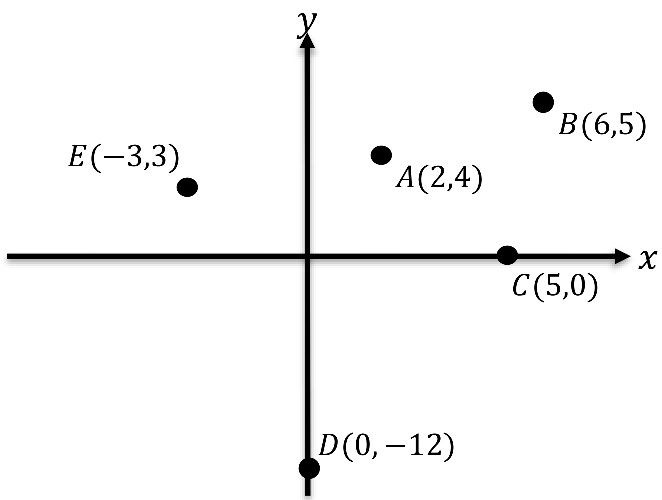
b $y = 2x - 5, \quad y = x + 5 \quad \rightarrow$?

c $x + y = 5, \quad y = 2x - 4 \quad \rightarrow$?

d $2x + y = 7, \quad x - 2y = 6 \quad \rightarrow$?

e $4x + 3y = 1, \quad y = 1 - x \quad \rightarrow$?

2 Find the distance: (giving exact values)



a $AB =$?

b $AC =$?

c $CD =$?

d $DE =$?

e $CE =$?

3 Find the distance between the two points where $y = 3x + 12$ crosses the coordinate axes.

?

4 Line l_1 passes through $(-1,1)$ and $(6,15)$. Another line l_2 passes through $(0,-12)$ and $(3,3)$. Determine the coordinate of the point at which they intersect.

?

5 Line l_1 has the equation $y = x$ and l_2 has the equation $y = -2x + 12$. The two lines intersect at point A and line l_2 intersects the x and y-axis at B and C respectively, as indicated. Find the area of:

a) OAB (where O is the origin) ?

b) OAC ?

Exercise 4

(On provided sheet)

6 [AQA IGCSEFM Jan 2013 Paper 1 Q16]

A, B and C are points on the line

$2x + y = 8.$

DCE is a straight line.

$$AB : BC = 2 : 1$$
$$EC : CD = 1 : 2$$

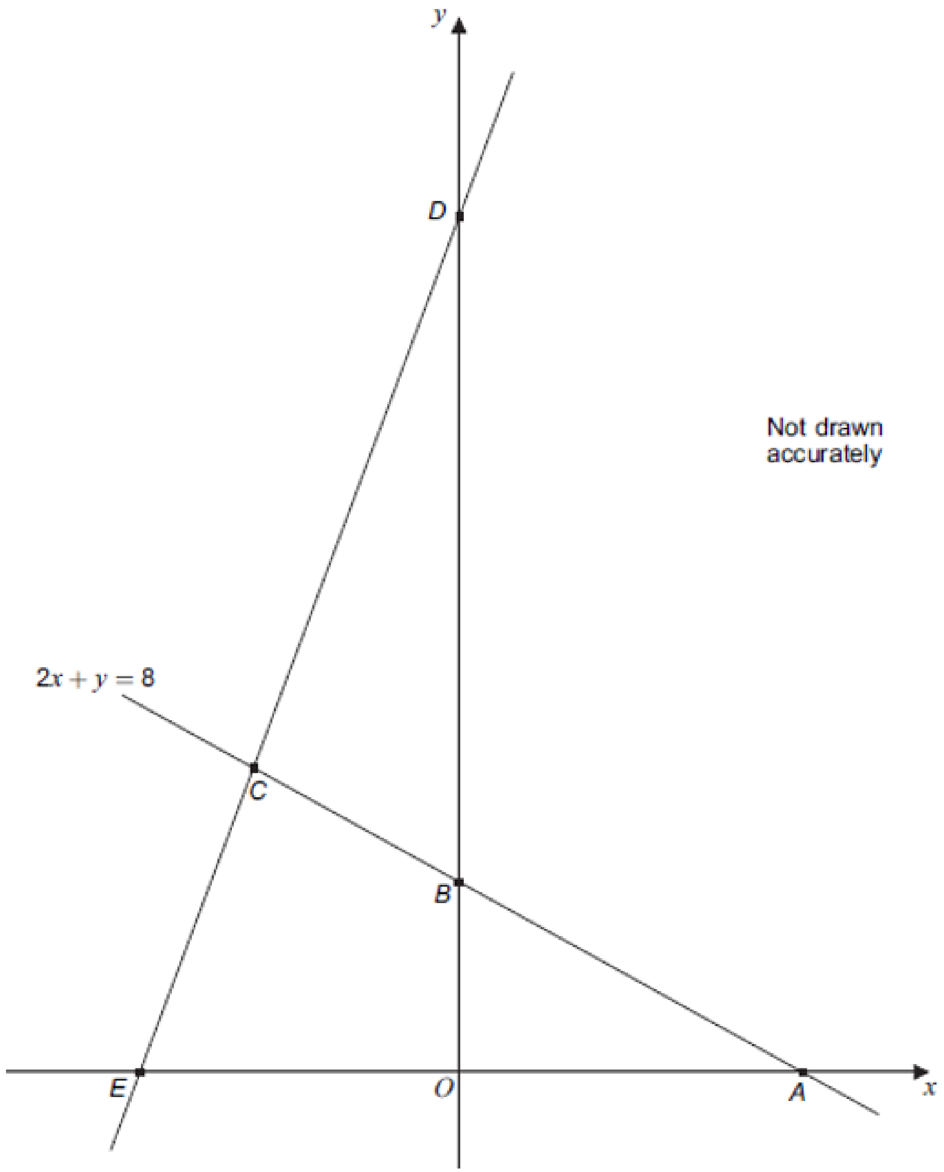
Work out the ratio:

$Area\ of\ triangle\ AEC$

$: Area\ of\ triangle\ BCD$

Give your answer in its simplest form.

?



Putting in the form $ax + by = c$ or $ax + by + c = 0$

Sometimes a question might ask you to put your equation in a particular form.

…Leave your answer in the form $ax + by = c$, where a, b, c are integers.

$$y = \frac{1}{2}x - 2$$

?

?

We want whole numbers not fractions, so what should we do to both sides of the equation?

I tend to put everything on the side that makes x positive, but it doesn't hugely matter.

…Leave your answer in the form $ax + by + c = 0$, where a, b, c are integers.

$$y = \frac{2}{3}x + 4$$

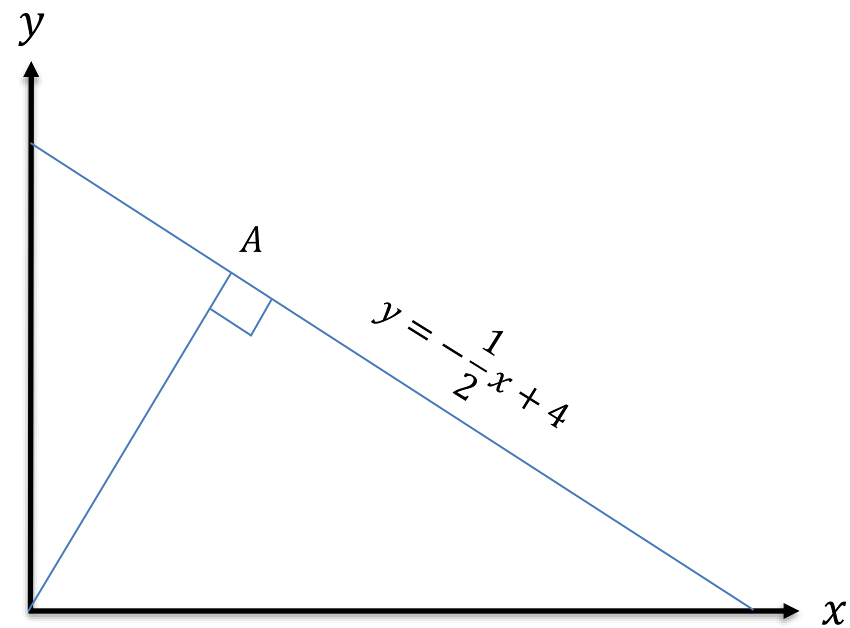
?

Test Your Understanding

1 A line is goes through the point (4,7) and is perpendicular to another line with equation $y = 2x + 2$. What is the equation of the line? Put your answer in the form $ax + by + c = 0$, where a, b, c are integers.

?

2 Determine the point A.



?

Exercise 5

(On provided sheet)

1 Are the following lines parallel, perpendicular or neither?

$y = 2x + 3, \quad y = 2x$?

$y = 3x - 4, \quad y = -3x + 1$?

$y = \frac{1}{2}x + 1, \quad y = -2x$?

2 A line is parallel to $y = 2x + 3$ and goes through the point $(4,3)$. What is its equation?

?

3 A line l_1 goes through the indicated point and is perpendicular to another line l_2. Determine the equation of l_1 in each case.

$(2,5) \quad l_2: y = 2x + 1$?

$(-6,3) \quad l_2: y = 3x$?

$(0,6) \quad l_2: y = -\frac{1}{2}x - 1$?

$(-9,0) \quad l_2: y = -\frac{1}{3}x + 1$?

$(10,10) \quad l_2: y = -5x + 5$?

4 $A(2,5)\ B(4,9)$

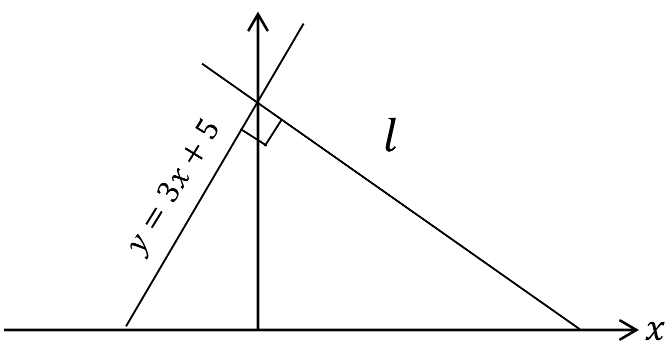
Find the equation of the line which passes through B, and is perpendicular to the line passing through both A and B.

?

5 Line l_1 has the equation $2y + 3x = 4$. Line l_2 goes through the points $(2,5)$ and $(5,7)$. Are the lines parallel, perpendicular, or neither?

?

6



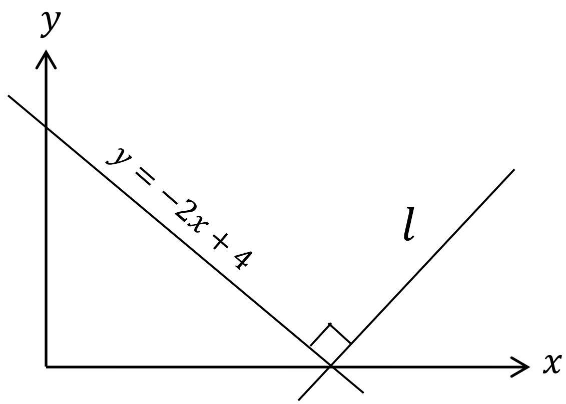
Determine the equation of the line l.

?

Exercise 5

(On provided sheet)

7



Determine the equation of the line l.

?

8

$A(3,7), B(5,13)$

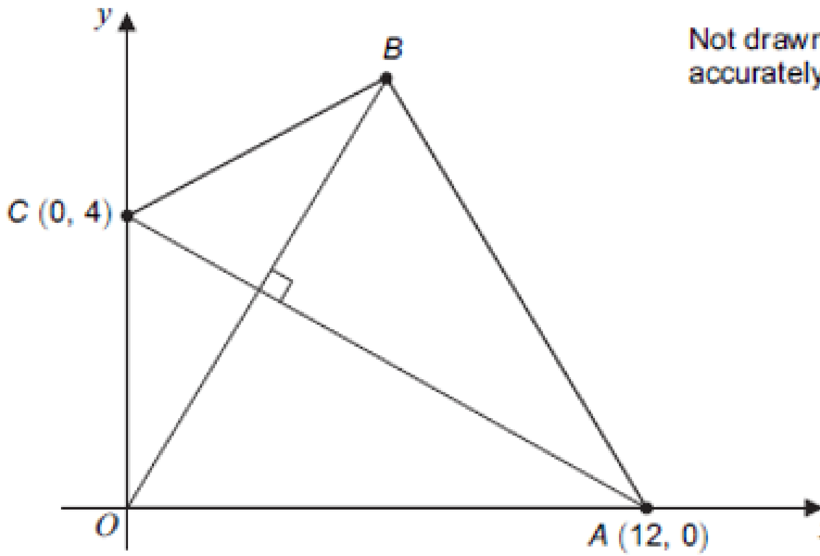
Find the equation of the line passing through B and is perpendicular to the line passing through A and B, giving your answer in the form $ax + by + c$, where a, b, c are integers.

?

9

[AQA IGCSEFM June 2012 Paper 1 Q11]

$OABC$ is a kite.



a) Work out the equation of AC.

?

b) Work out the coordinates of B.

?

Exercise 5

(On provided sheet)

Suppose O is the origin, and $A(1,2), B(4,2), C(2.2,-0.4)$.
Prove that $OABC$ is a kite.
(Hint: you need to prove **two** things as part of this.)

?

Exercise 6 – Mixed Exercises

(On provided sheet)

1 Line l_1 passes through the points $(4,5)$ and $(7,11)$. Line l_2 has the equation $2y = 3x - 1$. Do the lines intersect?

?

2 A is the point $(4, -1)$ and B is the point $(7,7)$.

a) Find the coordinates of the midpoint of AB.

?

b) Find the distance AB to 2 dp.

?

3 Line l_1 has the equation $y = 2x + 1$ and line l_2 the equation $y = 4x - 3$. Find the coordinates of the point at which they intersect.

?

4 a) Find the gradient of the line with equation $3x - 4y = 12$.

?

b) Prove that $3x - 4y = 12$ and $3y = 12 - 4x$ are perpendicular.

?

5 A line passes through the points $(0,4)$ and $(6,1)$. Find the equation of the line in the form:

a) $y = mx + c$

?

b) $ax + by = c$ where a, b, c are integers.

?

6 Find the coordinates of the points where $2x - 3y = 6$ crosses:

a) The x-axis.

?

b) The y-axis.

?

Exercise 6 – Mixed Exercises

(On provided sheet)

7 [Edexcel]

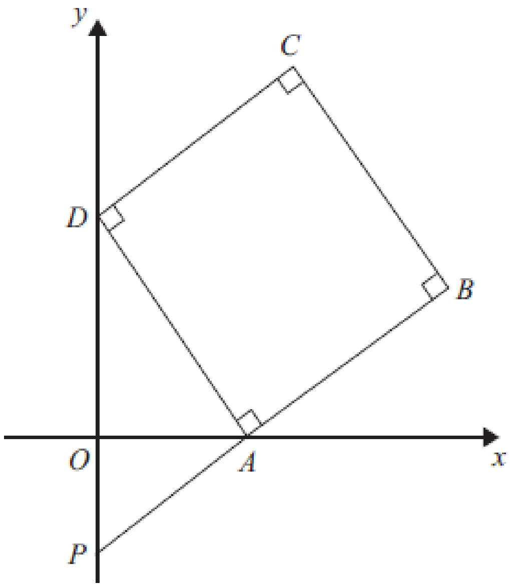
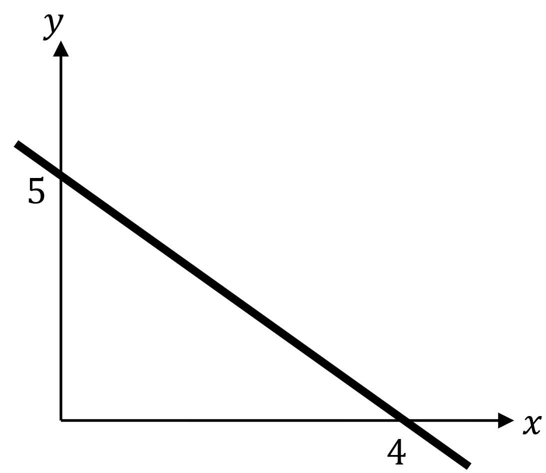


Diagram **NOT** accurately drawn

$ABCD$ is a square. P and D are points on the y-axis. A is a point on the x-axis. PAB is a straight line. The equation of the line that passes through the points A and D is $y = -2x + 6$. Find the length of PD.

?

8



Determine the equation of this line, putting your answer in the form $ax + by + c = 0$, where a, b, c are integers.

?

Exercise 6 – Mixed Exercises

(On provided sheet)

9 A triangle consists of the points $P(3, k)$, $Q(6,8)$ and $R(10,10)$. PQR is a right angle.
Determine the equation of the line passing through P and R, leaving your answer in the form $ax + by = c$, where a, b, c are integers.

?

CHALLENGE QUESTIONS

Question 1 – Locus & Perpendicular Condition

Find the equation of the locus of points $P(x, y)$ such that the line joining P to $A(2, 3)$ is **perpendicular** to the line joining P to $B(-4, 1)$.

Question 2 – Parameter & Intersection

A line L passes through the point $(1, -2)$ and has gradient m. It intersects the line

$$2x - y + 5 = 0$$

at point R, and also intersects the x-axis at point S.

Find the value(s) of m such that the **distance** $RS = 5$.

Question 1 – Locus (Answer)

Condition for perpendicular lines:

$$m_{PA} \cdot m_{PB} = -1$$

$$\frac{y-3}{x-2} \cdot \frac{y-1}{x+4} = -1$$

Simplifying:

$$(y-3)(y-1) = -(x-2)(x+4)$$

$$y^2 - 4y + 3 = -x^2 - 2x + 8$$

$$x^2 + y^2 + 2x - 4y - 5 = 0$$

Question 2 – Parameter (Answer)

Final values of m:

$$\boxed{m = 1 \quad \text{or} \quad m = -\frac{5}{3}}$$