PRECALCULUS · POLYNOMIAL AND RATIONAL FUNCTIONS

Zeros of Polynomials

Remainder and Factor Theorems · rational zeros · complex zeros · asymptotes · inequalities

19
questions worked

5
sections

8
true/false checks

Built from the OpenStax PreCalculus sections 2-05 to 2-09 booklets

How this lesson runs

Key points, worked examples, questions to try — then true or false

Starter	Two quick recall questions on evaluating polynomials and finding zeros — attempt before the reveal.
Key points	Each section opens with a summary of the ideas you need, main point highlighted first.
Question	Questions appear on their own slides. Attempt each before turning the page.
Solution	The next slide gives the condensed solution: the answer and the key steps only.
True or false	The lesson ends with eight statements — one per slide, answer on the slide that follows.

Key topic · Zeros of polynomials and rational functions

What this lesson is about, in one page

The zeros	Every polynomial of degree n has exactly n zeros (Fundamental Theorem of Algebra). Find them with the Rational Zero Theorem, synthetic division and factoring.
The complex pairs	Complex zeros of real-coefficient polynomials come in conjugate pairs: a + bi brings a − bi with it.
The rationals	Rational functions are ratios of polynomials. Their graphs are ruled by asymptotes: vertical where the denominator is zero, horizontal by the degree comparison.
The inequalities	Solve nonlinear inequalities with a sign chart: critical numbers from zeros and undefined points, then test each region.

Vocabulary

Key terms for this lesson

zero of a polynomial	a value x where f(x) = 0; also called a root
factor theorem	f(k) = 0 exactly when (x − k) is a factor of f(x)
rational zero theorem	possible rational zeros are p/q with p | constant, q | leading coefficient
multiplicity	the number of times a factor repeats; even = touch, odd = cross
conjugate pair	complex zeros a ± bi of a real-coefficient polynomial
linear factorization theorem	$f(x) = a(x - c_1)(x - c_2)\dots(x - c_n)$ over the complex numbers
rational function	a ratio of two polynomials, f(x) = N(x)/D(x), D(x) ≠ 0
vertical asymptote	a line x = c the graph approaches where D(c) = 0 after reduction
horizontal asymptote	a line y = L the graph approaches as $x \to \pm\infty$
sign chart	a number line split at critical numbers to solve an inequality

S

Starter · Evaluating and factoring polynomials

What you must remember before hunting zeros

- Two quick recall questions
- Substituting into polynomials, factoring quadratics and the factor theorem

Key points · the starter ideas

Everything you need before the new content, in five lines

1	**Zero of a polynomial**	x = k is a zero when f(k) = 0. It is where the graph crosses or touches the x-axis.
2	**Remainder Theorem**	Dividing f(x) by (x − k) leaves remainder f(k). Evaluate by substitution, no long division needed.
3	**Factor Theorem**	(x − k) is a factor of f(x) exactly when f(k) = 0. A zero always pairs with a factor.
4	**Synthetic division**	Divide by (x minus k) with the coefficient row only: bring down, multiply by k, add across.
5	**Quadratic factoring**	$x^2 - 5x + 6 = (x - 2)(x - 3)$. After synthetic division you will usually face one of these.

Question S1

4 marks

Starter · Remainder Theorem

THE QUESTION

For the polynomial $f(x) = 3x^3 - 5x^2 - 4x + 4$:

(a) Evaluate $f(1)$.
(b) Use the Remainder Theorem to state the remainder when $f(x)$ is divided by $(x - 1)$.
(c) Use synthetic division to divide $f(x)$ by $(x - 1)$ and write the quotient.
(d) Write $f(x) = (x - 1)\cdot q(x) + r$.

Question S1 — solution

4 marks

Starter · Remainder Theorem

CONDENSED SOLUTION

- (a) $f(1) = 3 - 5 - 4 + 4 = -2$
- (b) Remainder = $f(1) = -2$
- (c) Synthetic row ($k = 1$): $3, -2, -6, -2 \rightarrow q(x) = 3x^2 - 2x - 6$
- (d) $f(x) = (x - 1)(3x^2 - 2x - 6) - 2$

TRAP The Remainder Theorem works in BOTH directions: remainder r means f(k) = r, so you can evaluate a cubic without touching a single power.

Question S2

4 marks

Starter · Factor Theorem

THE QUESTION

The polynomial $p(x) = 2x^3 - x^2 - 13x - 6$ has a zero at $x = 3$.

(a) Verify that $p(3) = 0$.
(b) Use synthetic division to factor out $(x - 3)$.
(c) Factor the resulting quadratic completely.
(d) Write down all three zeros of p.

Question S2 — solution

4 marks

Starter · Factor Theorem

CONDENSED SOLUTION

- (a) $p(3) = 54 - 9 - 39 - 6 = 0$ ✓
- (b) Synthetic row ($k = 3$): 2, 5, 2, 0 → $p(x) = (x - 3)(2x^2 + 5x + 2)$
- (c) $2x^2 + 5x + 2 = (2x + 1)(x + 2)$
- (d) Zeros: $x = 3$, $x = -1/2$, $x = -2$

TRAP After synthetic division, check the remainder cell is 0 — that cell IS the proof that $(x - 3)$ is a factor.

01

Rational zeros and complex zeros

The Rational Zero Theorem, the Fundamental Theorem of Algebra and conjugate pairs

- Possible rational zeros: p/q with p dividing the constant and q dividing the leading coefficient
- A real polynomial's complex zeros arrive in conjugate pairs

Key points · finding zeros

Four tools, used in order, find every zero

1	**Rational Zero Theorem**	For integer coefficients, every rational zero is ±p/q with p a factor of the constant term and q a factor of the leading coefficient.
2	**The hunt**	Test candidates by synthetic division (fast) until the remainder is 0. Each hit is a factor; keep dividing the depressed polynomial.
3	**Fundamental Theorem of Algebra**	A degree-n polynomial has exactly n zeros over the complex numbers, counted with multiplicity.
4	**Conjugate pairs**	If a + bi is a zero of a real-coefficient polynomial, so is a − bi. Irrational pairs a ± √b behave the same way.

Worked example · finding all zeros

the question

Question first — $f(x) = x^3 - 4x^2 + x + 6$

THE QUESTION

Find all zeros of $f(x) = x^3 - 4x^2 + x + 6$:

(a) List the possible rational zeros.
(b) Find one rational zero by synthetic division.
(c) Solve the remaining quadratic and state all three zeros.

Worked example · finding all zeros — working

the solution

Answers only — list, divide, solve

CONDENSED SOLUTION

(a) ±1, ±2, ±3, ±6 (factors of 6 over factors of 1)

(b) $f(2) = 0$: row 1, −2, −3, 0 → $f(x) = (x - 2)(x^2 - 2x - 3)$

(c) $x^2 - 2x - 3 = (x - 3)(x + 1)$ → zeros 2, 3, −1

Test the easiest candidates first (±1, then ±2 …) and stop at the first zero remainder — the depressed polynomial is one degree lower, so it factors faster.

Question A1

3 marks

Rational zeros

THE QUESTION

The polynomial $f(x) = 2x^3 - 3x^2 - 3x + 2$ has an integer zero.

(a) List all possible rational zeros.
(b) Use synthetic division to find one zero and the depressed polynomial.
(c) Solve the quadratic and write down all three zeros.

Question A1 — solution

3 marks

Rational zeros

CONDENSED SOLUTION

- (a) $\pm 1, \pm 2, \pm 1/2$
- (b) $f(-1) = 0$: row $2, -5, 2, 0 \rightarrow f(x) = (x + 1)(2x^2 - 5x + 2)$
- (c) $2x^2 - 5x + 2 = (2x - 1)(x - 2)$: zeros $-1, 1/2, 2$

TRAP Test negative candidates too. Half the zeros of cubics with positive leading coefficient hide at negative x — like x = −1 here.

Question A2

4 marks

Complex zeros

THE QUESTION

The polynomial $f(x) = x^3 - 5x^2 + 16x - 30$ has a zero at $x = 1 + 3i$.

(a) Write down a second complex zero with no calculation.
(b) Form the quadratic factor with real coefficients from the conjugate pair.
(c) Find the remaining real zero.
(d) Write the complete linear factorization of f.

Question A2 — solution

4 marks

Complex zeros

CONDENSED SOLUTION

- (a) $1 - 3i$ (conjugate pair)
- (b) $(x - (1+3i))(x - (1-3i)) = x^2 - 2x + 10$
- (c) $f(x) = (x - 3)(x^2 - 2x + 10)$, so $x = 3$
- (d) $f(x) = (x - 3)(x - (1+3i))(x - (1-3i))$

TRAP The product of the conjugate pair is $x^2 - 2(a)x + (a^2 + b^2)$ — here $a = 1$, $b = 3$, giving -2 and $1 + 9 = 10$.

Key points · multiplicity and graph behaviour

What a repeated zero does to the graph

$f(x) = (x - c)^m \cdot g(x)$ has a zero at c with multiplicity m

m odd → graph CROSSES the x-axis at c · m even → graph TOUCHES and turns

1. **Odd multiplicity** — m = 1 or 3: the sign of f changes across c, so the curve passes straight through the axis.
2. **Even multiplicity** — m = 2 or 4: the sign of f stays the same, so the curve bounces off the axis at (c, 0).
3. **Reading factors** — $(x - 4)^2(x + 1)$ has a double zero at 4 (touch) and a simple zero at −1 (cross).
4. **Counting zeros** — Multiplicities add up to the degree: degree 4 could be 2 + 1 + 1, or 2 + 2, or 4 at one point.

Question A3

3 marks

Multiplicity

THE QUESTION

Let $g(x) = x^4 - x^3 - 7x^2 + 13x - 6 = (x - 1)^2(x + 3)(x - 2)$.

(a) State the zeros of g and the multiplicity of each.
(b) At which zeros does the graph cross the x-axis?
(c) At which zero does the graph touch the x-axis and turn?

Question A3 — solution

3 marks

Multiplicity

CONDENSED SOLUTION

- (a) x = 1 (mult 2), x = −3 (mult 1), x = 2 (mult 1)
- (b) Crosses at x = −3 and x = 2 (odd multiplicity)
- (c) Touches and turns at x = 1 (even multiplicity)

TRAP Multiplicity is the POWER on the factor, not how many times x appears in the equation. $(x - 1)^2$ is multiplicity 2 even though 1 shows once.

Question A4

4 marks

Writing polynomials from zeros

THE QUESTION

A degree-3 polynomial f has zeros −2, 2 and 5, and f(3) = 40.

(a) Write f(x) in the form a(x + 2)(x − 2)(x − 5).
(b) Use f(3) = 40 to find a.
(c) Write f(x) in standard form.

Question A4 — solution

4 marks

Writing polynomials from zeros

CONDENSED SOLUTION

- (b) $f(3) = a(5)(1)(-2) = -10a = 40$, so $a = -4$
- (c) $f(x) = -4(x + 2)(x - 2)(x - 5) = -4x^3 + 20x^2 + 16x - 80$
- Check: $f(3) = -108 + 180 + 48 - 80 = 40$ ✓

TRAP Zeros alone never fix a polynomial — the stretch factor a comes from ONE extra point. Always substitute it back to check.

Question A5

3 marks

Polynomial with a complex zero

THE QUESTION

A degree-3 polynomial with real coefficients has zeros 4 and i, and constant term −4 times its leading coefficient squared. Given $f(x) = 2x^3 - 8x^2 + 2x - 8$:

(a) Verify that $x = 4$ is a zero.
(b) Explain why i must be a zero.
(c) Factor f completely.

Question A5 — solution

3 marks

Polynomial with a complex zero

CONDENSED SOLUTION

- (a) $f(4) = 128 - 128 + 8 - 8 = 0$ ✓
- (b) Real coefficients: complex zeros come in pairs, so i brings $-i$
- (c) $f(x) = 2(x - 4)(x - i)(x + i) = 2(x - 4)(x^2 + 1)$

TRAP If the leading coefficient is real, $(x^2 + 1) = (x - i)(x + i)$ is the irreducible quadratic hiding behind the pair $\pm i$.

Formulae · rational functions

Domain, vertical asymptotes and horizontal asymptotes

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f(x) = N(x)/D(x)   ·   Domain: all x with D(x) ≠ 0
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VA: solve D(x)=0 AFTER cancelling common factors (those become holes)  ·  HA by degrees: N<D → y=0, N=D → y=leading ratio, N>D → none
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1. **Cancel first** — Common factors of N and D become holes: $(x^2-9)/(x^2+4x+3) = (x-3)/(x+1)$ with a hole at $x = -3$.
2. **Vertical asymptotes** — After cancelling, each zero of D gives one VA: the graph climbs to $\pm\infty$ beside it.
3. **Horizontal asymptotes** — Compare degrees: N<D gives y = 0; N = D gives y = ratio of leading coefficients; N>D gives none.
4. **Intercepts** — x-intercepts: solve N(x) = 0 (after cancelling). y-intercept: evaluate f(0), if 0 is in the domain.

Worked example · rational functions

the question

Question first — $f(x) = (x^2 - 9)/(x^2 + 4x + 3)$

THE QUESTION

For $f(x) = (x^2 - 9)/(x^2 + 4x + 3)$:

(a) State the domain.
(b) Factor and cancel: identify any hole with its coordinates.
(c) Find the vertical and horizontal asymptotes.
(d) Find the intercepts.

Worked example · rational functions — working

the solution

Answers only — factor, cancel, compare degrees

CONDENSED SOLUTION

(a) D(x) = (x+1)(x+3): domain $x \neq -1$, $x \neq -3$

(b) $(x-3)(x+3)/((x+1)(x+3)) = (x-3)/(x+1)$ · hole at $(-3, 3)$

(c) VA: $x = -1$ · HA: $y = 1$ (equal degrees, 1/1)

(d) x-int $(3, 0)$ · y-int: $f(0) = -3/1 = (0, -3)$

The hole lives at the CANCELLED factor $x = -3$; its y-value comes from the reduced function. The surviving denominator zero $x = -1$ is the real asymptote.

Try it yourself

your turn

Attempt before turning the page — answer on the next slide

YOUR TURN

For $f(x) = (3x^2 + 2)/(x^2 - 9)$:

State the domain, the vertical asymptotes,
the horizontal asymptote and the intercepts.

Question Y2 — solution

4 marks

Try it · rational functions

CONDENSED SOLUTION

- Domain: $x \neq 3, x \neq -3$
- VA: $x = 3$ and $x = -3$
- HA: $y = 3$ (equal degrees, 3/1)
- y-intercept $(0, -2/9)$; no x-intercepts ($3x^2 + 2 > 0$)

TRAP No x-intercept is a valid answer: when N(x) = 0 has no real solutions, the graph simply never touches the x-axis.

Question F1

4 marks

Rational functions · domain and asymptotes

THE QUESTION

For $f(x) = (x^2 - 4)/(x^2 + x - 2)$:

(a) Factor the numerator and the denominator.
(b) State the domain.
(c) Identify any hole and give its coordinates.
(d) State the vertical and horizontal asymptotes and the intercepts.

Question F1 — solution

4 marks

Rational functions · domain and asymptotes

CONDENSED SOLUTION

- (a) $(x-2)(x+2)$ over $(x-1)(x+2)$
- (b) Domain: $x \neq 1$, $x \neq -2$
- (c) Hole at $x = -2$: on the reduced graph, $y = (-4)/(-3) = 4/3 \rightarrow (-2, 4/3)$
- (d) VA $x = 1$; HA $y = 1$; x-int $(2, 0)$; y-int $(0, 2)$

TRAP A hole needs coordinates: substitute into the REDUCED function. The original form 0/0 tells you only where it is, not how high.

Question F2

3 marks

Rational functions · no cancellation

THE QUESTION

For $f(x) = (2x^2 - 8)/(x^2 - 2x)$:

(a) Factor top and bottom completely.
(b) State the domain, the hole, the vertical asymptote and the horizontal asymptote.
(c) Explain why f has no y-intercept.

Question F2 — solution

3 marks

Rational functions · no cancellation

CONDENSED SOLUTION

- (a) $2(x-2)(x+2)$ over $x(x-2)$
- (b) Domain $x \neq 0$, $x \neq 2$; hole $(2, 4)$; VA $x = 0$; HA $y = 2$; x-int $(-2, 0)$
- (c) $x = 0$ is not in the domain (denominator zero), so $f(0)$ does not exist

TRAP The hole sits at $x = 2$ with $y = 2(4)/2 = 4$ on the reduced function $2(x+2)/x$. The VA takes the OTHER denominator zero: $x = 0$.

Question F3

4 marks

Graph to equation

THE QUESTION

A rational function has x-intercepts at $x = 4$ (the graph touches and turns) and $x = -1$ (the graph crosses).
Its vertical asymptotes are $x = 3$ (both ends up) and $x = -2$ (ends in opposite directions). It passes through $(0, 8)$.

(a) Write the numerator's factors with multiplicities.
(b) Write the denominator's factors with multiplicities.
(c) Use $(0, 8)$ to find the stretch factor a and write the full function.

Question F3 — solution

4 marks

Graph to equation

CONDENSED SOLUTION

- (a) Numerator: $a(x - 4)^2(x + 1)$ (touch = even, cross = odd)
- (b) Denominator: $(x - 3)(x + 2)^2$ (both ends up = even; opposite ends = odd)
- (c) $a(16)(1)/(9 \cdot 4) = a \cdot 16/36 = 8 \rightarrow a = 18$
- $f(x) = 18(x - 4)^2(x + 1)/((x - 3)(x + 2)^2)$

TRAP Match the END BEHAVIOUR at each asymptote, not just its location: even power = both ends same side, odd power = opposite sides.

Key points · nonlinear inequalities

Sign charts turn a hard inequality into four easy tests

1	**Move everything over**	Rewrite so one side is 0: $x^2 - 2x - 15 > 0$, not $x^2 > 2x + 15$.
2	**Critical numbers**	Solve the other side = 0 (zeros) and find where it is undefined (denominator zeros).
3	**Split the number line**	Critical numbers cut the line into regions; the sign cannot change inside a region.
4	**Test and read**	Pick any test point per region; collect the regions matching the inequality. Strict symbols exclude endpoints; undefined points are never included.

Worked example · nonlinear inequality

the question

Question first — $x^3 + 2x^2 - 3x \le 0$

THE QUESTION

Solve $x^3 + 2x^2 - 3x \le 0$.

(a) Factor the left-hand side completely.
(b) Find the critical numbers and the sign chart regions.
(c) Test each region and write the solution set.

Worked example · nonlinear inequality — working

the solution

Answers only — factor, chart, test

CONDENSED SOLUTION

(a) $x(x + 3)(x - 1) \le 0$

(b) Critical numbers -3, 0, 1 → regions $x<-3$, $-3<x<0$, $0<x<1$, $x>1$

(c) Signs: $- / + / - / +$ → solution $x \le -3$ or $0 \le x \le 1$

Test $x = -4, -1, 0.5$ and 2. The $\le$ includes the zeros ($-3, 0, 1$) because they make the expression equal to 0 — write square brackets at those points.

Try it yourself

your turn

Attempt before turning the page — answer on the next slide

YOUR TURN

Solve $(x - 3)(x + 1)/(x - 2) \leq 0$.

Critical numbers first — remember that an undefined point can never belong to the solution.

Question Y3 — solution

4 marks

Try it · nonlinear inequality

CONDENSED SOLUTION

- Critical numbers: −1, 2, 3 (2 is undefined, not included)
- Signs: − / + / − / + across $(-\infty,-1)$, $(-1,2)$, $(2,3)$, $(3,\infty)$
- Solution: $x \le -1$ or $2 < x \le 3$

TRAP Brackets are asymmetric on purpose: [−1 included, (2 open — it makes the fraction undefined, 3] included because ≤ allows zero.

Question F4

3 marks

Nonlinear inequalities

THE QUESTION

Solve $x^2 - 2x - 15 > 0$.

(a) Factor and find the critical numbers.
(b) Draw a sign chart with a test point in each region.
(c) State the solution set in interval notation.

Question F4 — solution

3 marks

Nonlinear inequalities

CONDENSED SOLUTION

- (a) $(x - 5)(x + 3) > 0$: critical -3 and 5
- (b) Test: $x = -4$ gives $+$, $x = 0$ gives $-$, $x = 6$ gives $+$
- (c) $(-\infty, -3) \cup (5, \infty)$

TRAP A positive product needs BOTH factors positive or BOTH negative — the two outer regions. Never multiply across the inequality to “check”.

Question F5

4 marks

Nonlinear inequalities with asymptotes

THE QUESTION

Solve $(x - 1)/(x^2 - x - 6) \geq 0$.

(a) Factor the denominator and list the critical numbers.
(b) State which critical numbers are zeros and which are undefined points.
(c) Build the sign chart and write the solution set.

Question F5 — solution

4 marks

Nonlinear inequalities with asymptotes

CONDENSED SOLUTION

- (a) Denominator $(x - 3)(x + 2)$: critical numbers -2, 1, 3
- (b) Zero: $x = 1$ (included with $\geq$). Undefined: $x = -2, 3$ (always excluded)
- (c) Signs: $- / + / - / +$ across $(-\infty,-2)$, $(-2,1)$, $(1,3)$, $(3,\infty)$
- (d) Solution: $(-2, 1] \cup (3, \infty)$

TRAP Keep two lists: ZEROS may enter the solution when the symbol is $\leq$ or $\geq$; UNDEFINED points never do, whatever the symbol says.

Question F6

4 marks

Rational inequality

THE QUESTION

Solve $\frac{2}{(x + 3)} \geq 1$.

(a) Rearrange so one side is 0 — do NOT multiply both sides by (x + 3).
(b) Combine into a single fraction.
(c) Find the critical numbers and build the sign chart.
(d) State the solution set.

Question F6 — solution

4 marks

Nonlinear inequality · rational

CONDENSED SOLUTION

- (a) $2/(x+3) - 1 \geq 0$
- (b) $(2 - (x+3))/(x+3) = -(x + 1)/(x + 3) \geq 0$
- (c) Critical -3 (undefined), -1 (zero); signs: + / − across $(-\infty,-3)$, $(-3,-1)$, $(-1,\infty)$
- (d) Solution: $-3 < x \leq -1$

TRAP Multiplying by $(x + 3)$ flips the inequality only when $x + 3$ is negative — which you do not know yet. Move everything to one side FIRST.

Question F7

4 marks

Mixed · evaluate and solve

THE QUESTION

Let $f(x) = 2x^3 - x^2 + 4x - 9$.

(a) Find the remainder when f(x) is divided by (x − 1).
(b) Is (x − 1) a factor? Justify with the Factor Theorem.
(c) Given f has exactly one real zero, what can you say about its other two zeros?
(d) What does the Fundamental Theorem of Algebra tell you about the total number of zeros?

Question F7 — solution

4 marks

Mixed · evaluate and solve

CONDENSED SOLUTION

- (a) $f(1) = 2 - 1 + 4 - 9 = -4$
- (b) Not a factor: remainder $\neq 0$
- (c) The other two zeros form a conjugate pair $a \pm bi$
- (d) Degree 3 $\rightarrow$ exactly 3 zeros counted with multiplicity

TRAP Odd degree guarantees at least one real zero; the remaining pair, when complex, must be conjugates because the coefficients are real.

Mark scheme

Marks by question, with what earns each mark

S1	4	1 per part: f(1) = −2, remainder, synthetic quotient, rearranged form
S2	4	1 verification · 1 synthetic division · 1 quadratic factored · 1 all zeros
A1	3	1 candidates · 1 synthetic hit at x = −1 · 1 quadratic solved: −1, 1/2, 2
A2	4	1 conjugate 1 − 3i · 1 quadratic $x^2 - 2x + 10$ · 1 zero x = 3 · 1 factorization
A3	3	1 zeros+multiplicities · 1 crossing zeros · 1 touching zero at x = 1
A4	4	1 set-up a(x+2)(x−2)(x−5) · 1 a = −4 · 1 standard form · 1 verification
A5	3	1 verification f(4)=0 · 1 conjugate-pair reason · 1 complete factorization
F1	4	1 factorization · 1 domain · 1 hole (−2, 4/3) · 1 VA/HA + intercepts
F2	3	1 complete factorization · 1 domain/hole/VA/HA · 1 no y-intercept reason
F3	4	1 numerator factors · 1 denominator factors · 1 a = 18 · 1 final function
F4	3	1 factoring+critical numbers · 1 sign chart · 1 interval solution
F5	4	1 critical numbers · 1 zero vs undefined classification · 1 chart · 1 solution
F6	4	1 rearrangement · 1 single fraction −(x+1)/(x+3) · 1 chart · 1 $-3<x\leq-1$
F7	4	1 remainder −4 · 1 factor verdict · 1 conjugate-pair statement · 1 FTA count

Main lesson total: 50 marks. Starter: 8 practice marks (not assessed).

Answer key

One line per question

ANSWERS

- S1 −2 · remainder −2 · q = $3x^2 - 2x - 6$ · f = (x−1)q − 2
- S2 (x−3)(2x+1)(x+2) · zeros 3, −1/2, −2
- WE ±1,±2,±3,±6 · $(x-2)(x^2-2x-3)$ · zeros 2, 3, −1
- A1 ±1,±2,±1/2 · $(x+1)(2x^2-5x+2)$ · zeros −1, 1/2, 2
- A2 conj 1−3i · $x^2-2x+10$ · real zero x = 3
- A3 1 (mult 2) touches · −3, 2 cross
- A4 a = −4 · f(x) = $-4x^3+20x^2+16x-80$
- A5 f(4)=0 · i brings −i · $2(x-4)(x^2+1)$
- Y2 x ≠ ±3 · VA ±3 · HA y=3 · (0,−2/9), no x-int

SOLUTION SETS

- F1 (−2, 4/3) hole · VA x=1 · HA y=1 · (2,0), (0,2)
- F2 hole (2,4) · VA x=0 · HA y=2 · x-int (−2,0), no y-int
- F3 $18(x-4)^2(x+1)/((x-3)(x+2)^2)$
- F4 $(x-5)(x+3) > 0 \rightarrow (-\infty,-3) \cup (5,\infty)$
- F5 $(-2, 1] \cup (3, \infty)$
- F6 $-3 < x \leq -1$
- F7 remainder −4 · not a factor · pair a±bi · 3 zeros
- Y3 $x \leq -1$ or $2 < x \leq 3$
- Quick check: every solution set must exclude
- undefined points, whatever the symbol says.

True or false · 1

T / F

Decide, then justify your decision — answer on the next slide

If f(k) = 0, then (x − k) is a factor of f(x).

True or false · 1 — answer

TRUE

TRUE

That is the Factor Theorem: a zero always pairs with a linear factor. Example: $f(4) = 0$ for $2x^3-7x^2+5x-12$ would mean $(x - 4)$ divides it.

True or false · 2

T / F

Decide, then justify your decision — answer on the next slide

A degree-7 polynomial with real coefficients must have at least one real zero.

True or false · 2 — answer

TRUE

TRUE

Complex zeros come in pairs, so at most 6 can be complex — an odd count forces at least one real zero. Degree 7 → 1, 3, 5 or 7 real zeros.

True or false · 3

T / F

Decide, then justify your decision — answer on the next slide

A polynomial of degree n has exactly n real zeros.

True or false · 3 — answer

FALSE

FALSE

The Fundamental Theorem of Algebra counts COMPLEX zeros with multiplicity. $x^2 + 1$ has degree 2 but no real zeros at all.

True or false · 4

T / F

Decide, then justify your decision — answer on the next slide

If 2 + i is a zero of a real-coefficient polynomial, then 2 − i is also a zero.

True or false · 4 — answer

TRUE

TRUE

Conjugate pair theorem: real coefficients force complex zeros to arrive in pairs a ± bi. One without the other would make a coefficient complex.

True or false · 5

T / F

Decide, then justify your decision — answer on the next slide

The graph of $(x - 4)^2(x + 3)$ crosses the x-axis at $x = 4$.

True or false · 5 — answer

FALSE

FALSE

Multiplicity 2 is even, so the graph touches $x = 4$ and turns. It crosses at $x = -3$, where the multiplicity is odd.

True or false · 6

T / F

Decide, then justify your decision — answer on the next slide

The rational zero theorem guarantees that every listed candidate is a zero.

True or false · 6 — answer

FALSE

FALSE

The list contains every POSSIBLE rational zero; most candidates fail the synthetic division test. It bounds the search, it does not solve it.

True or false · 7

T / F

Decide, then justify your decision — answer on the next slide

The horizontal asymptote of $(2x^2 + 1)/(x^2 - 4)$ is $y = 2$.

True or false · 7 — answer

TRUE

TRUE

Degrees are equal, so the HA is the ratio of leading coefficients: 2/1 = 2.

The VAs are the denominator zeros x = ±2.

True or false · 8

T / F

Decide, then justify your decision — answer on the next slide

When solving $(x - 1)/(x + 2) > 0$, you may multiply both sides by $(x + 2)$.

True or false · 8 — answer

FALSE

FALSE

Multiplying by an expression whose sign is unknown is illegal: it can flip the inequality. Move everything to one side and use a sign chart: $x < -2$ or $x > -1$.

FIVE THINGS TO DO IN THE EXAM

Do these and zeros of polynomials will not cost you marks

- List rational zeros as ±p/q FIRST, then test by synthetic division — easiest candidates first.
- Real coefficients mean complex zeros travel in pairs: write the conjugate before factoring.
- Check multiplicities: odd crosses the axis, even touches and turns.
- For rational functions, cancel common factors before naming VAs — cancelled factors become holes with coordinates.
- Solve inequalities with sign charts: zeros may join the solution, undefined points never do.

Built from the OpenStax PreCalculus sections 2-05 to 2-09 booklets.