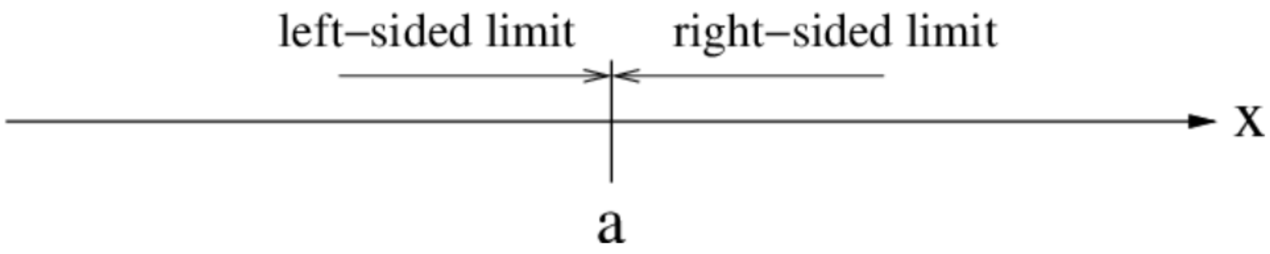
UNIT 1 KNOWLEDGE – CALCULUS 12 – LIMITS AND CONTINUITY

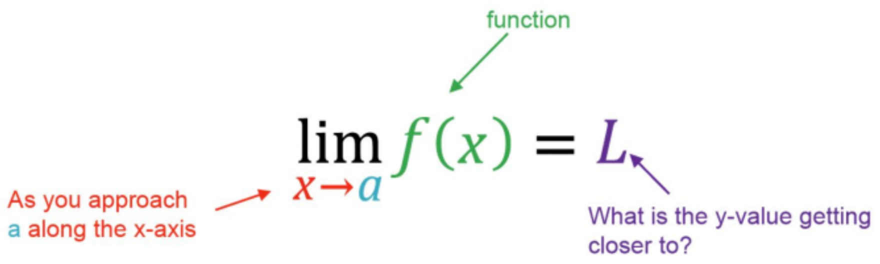
- 1.1 Introducing Calculus: Can Change Occur at an Instant? ✓
- 1.2 Defining Limits and Using Limit Notation ✓
- 1.3 Estimating Limit Values from Graphs ✓
- 1.4 Estimating Limit Values from Tables ✓
- 1.5 Determining Limits Using Algebraic Properties of Limits ✓
- 1.6 Determining Limits Using Algebraic Manipulation
- 1.7 Selecting Procedures for Determining Limits
- 1.8 Determining Limits Using the Squeeze Theorem
- 1.9 Connecting Multiple Representations of Limits
- 1.10 Exploring Types of Discontinuities
- 1.11 Defining Continuity at a Point
- 1.12 Confirming Continuity over an Interval
- 1.13 Removing Discontinuities
- 1.14 Connecting Infinite Limits and Vertical Asymptotes
- 1.15 Connecting Limits at Infinity and Horizontal Asymptotes
- 1.16 Working with the Intermediate Value Theorem (IVT)

What Will We Learn?

- How can we often use algebraic manipulation to help determine limits?

- How can we divide common factors within rational functions when evaluating limits?





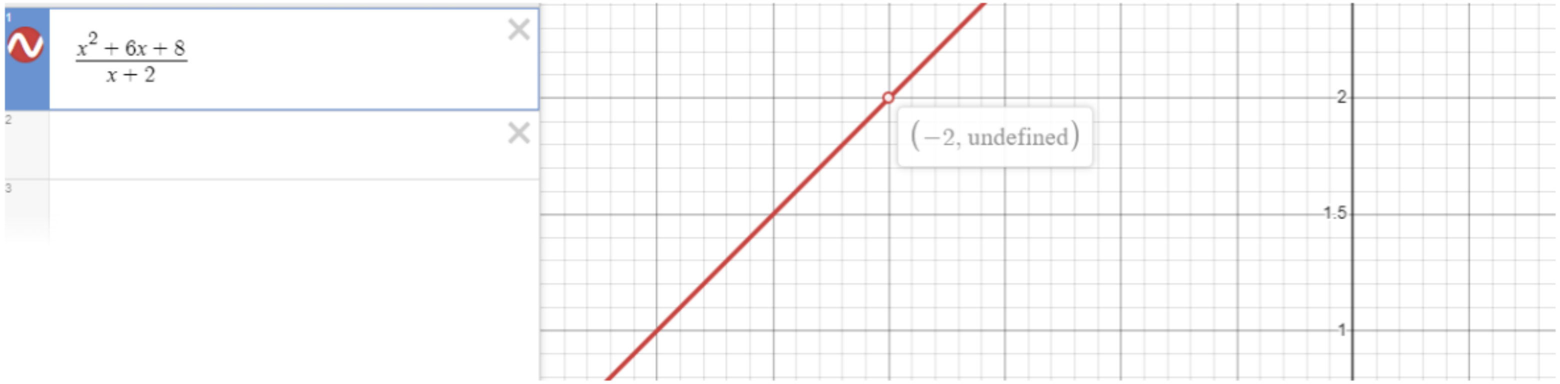
Algebraic manipulation to help determine limits

Find $\lim_{x \to -2} \dfrac{x^2 + 6x + 8}{x + 2}$

Algebraic manipulation to help determine limits

$$\lim_{x \to -2} \frac{x^2 + 6x + 8}{x + 2} = \lim_{x \to -2} \frac{(x+4)(x+2)}{(x+2)} = \lim_{x \to -2} (x+4) = -2 + 4 = 2$$

- Using algebraic manipulation, we ***FACTOR THE PROBLEM***

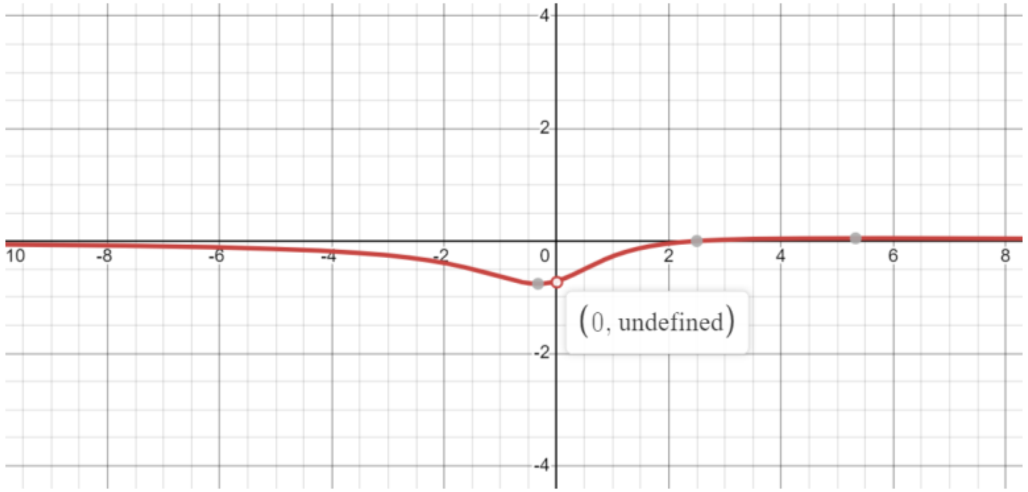


Another example-> Determining Limits: Factoring

$$\lim_{x \to 3} \frac{x^2-9}{x^2+4x-21}$$

Another example-> Determining Limits: Factoring

$$\lim_{x \to 0} \frac{2x^2 - 5x}{4x^3 + 7x}$$



Another example-> Determining Limits: Factoring

$$\lim_{x \to 6} \frac{x^2 - 4x - 12}{x^2 - 12x + 36}$$

Next-> What Will We Learn?

- We'll look at additional cases where algebraic manipulation can be used to find limit values, including functions involving radical or trigonometric expressions.

$$\lim_{x \to -1} \sqrt{x+2}$$

Determining Limits: Problems Involving Radicals

$$\lim_{x \to 5} \frac{\sqrt{(x+4)} - 3}{x - 5}$$

Radicals and Conjugates

For real numbers $a \geq 0$ and $b \geq 0$, where $b \neq 0$ when b is a denominator,

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b} \qquad \sqrt[3]{ab} = \sqrt[3]{a} \cdot \sqrt[3]{b}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \qquad \sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$$

Two binomials of the form $\sqrt{a} + \sqrt{b}$ and $\sqrt{a} - \sqrt{b}$ are called conjugate radicals:

$\sqrt{a} + \sqrt{b}$ is the conjugate of $\sqrt{a} - \sqrt{b}$

$\sqrt{a} - \sqrt{b}$ is the conjugate of $\sqrt{a} + \sqrt{b}$

For example, the conjugate of $2 - \sqrt{3}$ is $2 + \sqrt{3}$

- At first, you may think of plugging in 5 but you get $\frac{0}{0}$ which makes it an indeterminate form.

Determining Limits: Problems Involving Radicals

Funky Form of One (FFOO)

Conjugates: $x + y \leftrightarrow x - y$

MULTIPLY BY THE CONJUGATES!

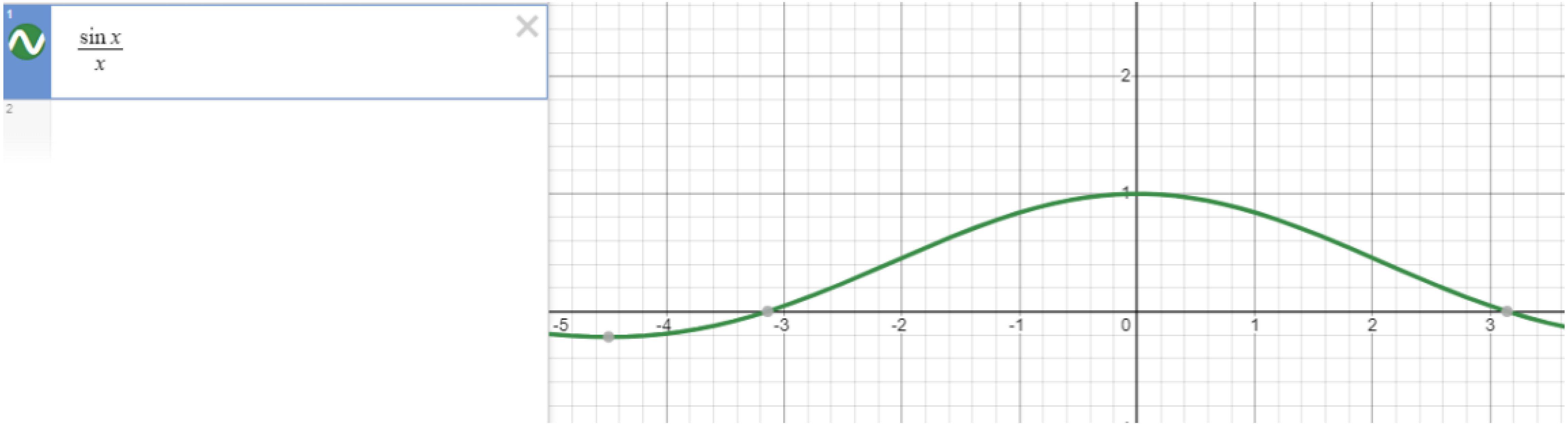
$$\lim_{x \to 5} \frac{\sqrt{(x+4)} - 3}{x-5} = \lim_{x \to 5} \left(\frac{\sqrt{(x+4)} - 3}{x-5} \times \frac{\sqrt{(x+4)} + 3}{\sqrt{(x+4)} + 3} \right)$$

$$= \lim_{x \to 5} \frac{x+4-9}{(x-5)\,(\sqrt{(x+4)}+3)} = \lim_{x \to 5} \frac{(x-5)}{(x-5)\,(\sqrt{(x+4)}+3)} = \lim_{x \to 5} \frac{1}{(\sqrt{(x+4)}+3)}$$

Plug in 5: $\frac{1}{3+3} = \mathbf{\frac{1}{6}}$

Determining Limits: Using Known Limits

It is known that $\lim_{x \to 0} \frac{\sin(x)}{x} = 1.$ Find $\lim_{x \to 0} \frac{\sin(x)}{3x}$



Determining Limits: Using Known Limits

It is known that $\lim\limits_{x\to 0} \dfrac{\sin(x)}{x} = 1$.

Find $\lim\limits_{x\to 0} \dfrac{\sin(x)}{3x} = \lim\limits_{x\to 0} \left(\dfrac{\sin(x)}{x} \times \dfrac{1}{3}\right) = 1 \times \dfrac{1}{3} = \dfrac{1}{3}$

We do some manipulation:

1. Rewrite first
2. Use our limit laws since both limits exist

Recall: Theorems About Limits	
If $\lim\limits_{x\to c} f(x) = L$ *and* $\lim\limits_{x\to c} g(x) = M$, *then* ...	
$\lim\limits_{x\to c}[bf(x)] = b\cdot L$ (where b is a constant)	$\lim\limits_{x\to c}[f(x)g(x)] = L\cdot M$

Determining Limits: Using Known Limits

It is known that $\lim_{x \to 1} \frac{f(x)}{x-1} = 4$. Find $\lim_{x \to 1} \frac{2xf(x)}{x^2+5x-6}$

Past AP Exam Question

$$\lim_{x \to 5^-} \frac{|x-5|}{x-5} ?$$

A. -1 B. 1 C. 5 D. nonexistent

Key Takeaways

Limit Notation	Left: $\lim_{x \to c^-} f(x)$	Right: $\lim_{x \to c^+} f(x)$
A limit exists when...	$\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$	
A limit does NOT exist when...	$\lim_{x \to c^-} f(x) \neq \lim_{x \to c^+} f(x)$	