UNIT 1 KNOWLEDGE – CALCULUS 12 – LIMITS AND CONTINUITY

- 1.1 Introducing Calculus: Can Change Occur at an Instant? ✓
- 1.2 Defining Limits and Using Limit Notation ✓
- 1.3 Estimating Limit Values from Graphs ✓
- 1.4 Estimating Limit Values from Tables ✓
- 1.5 Determining Limits Using Algebraic Properties of Limits ✓
- 1.6 Determining Limits Using Algebraic Manipulation ✓
- 1.7 Selecting Procedures for Determining Limits ✓
- 1.8 Determining Limits Using the Squeeze Theorem
- 1.9 Connecting Multiple Representations of Limits
- 1.10 Exploring Types of Discontinuities
- 1.11 Defining Continuity at a Point
- 1.12 Confirming Continuity over an Interval
- 1.13 Removing Discontinuities
- 1.14 Connecting Infinite Limits and Vertical Asymptotes
- 1.15 Connecting Limits at Infinity and Horizontal Asymptotes
- 1.16 Working with the Intermediate Value Theorem (IVT)

Turn and talk

Find the limit of the following expression as $x \to 3$:

$$\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$$

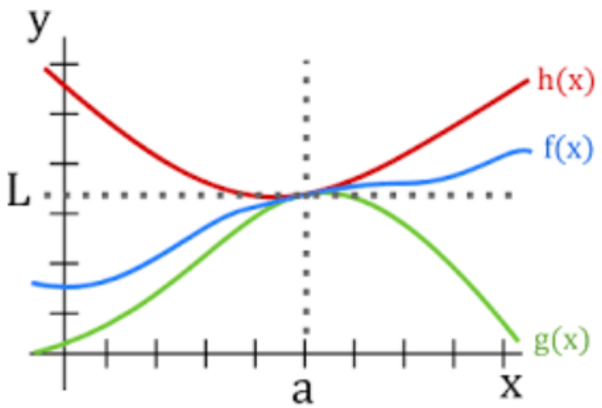
X = 6

1. Squeeze Theorem – 挤压定理
2. Function – 函数
3. Limit – 极限
4. Inequality – 不等式
5. Continuous – 连续
6. Bounds – 界限
7. Sequence – 数列
8. Convergence – 收敛
9. Upper Bound – 上界
10. Lower Bound – 下界

What Will We Learn?

- What’s the Squeeze Theorem /Sandwich Theorem/ Pinching Theorem?

- We’ll focus on the squeeze theorem using an intuitive approach with graphs and connecting to an understandable context.



Understanding the Squeeze Theorem

- So far, we've seen how to use algebraic, graphical and numerical approaches to find limits.

- ***BUT… We have to be given some information to use all three approaches***

- e.g. Find $\lim_{x \to 4} f(x)$

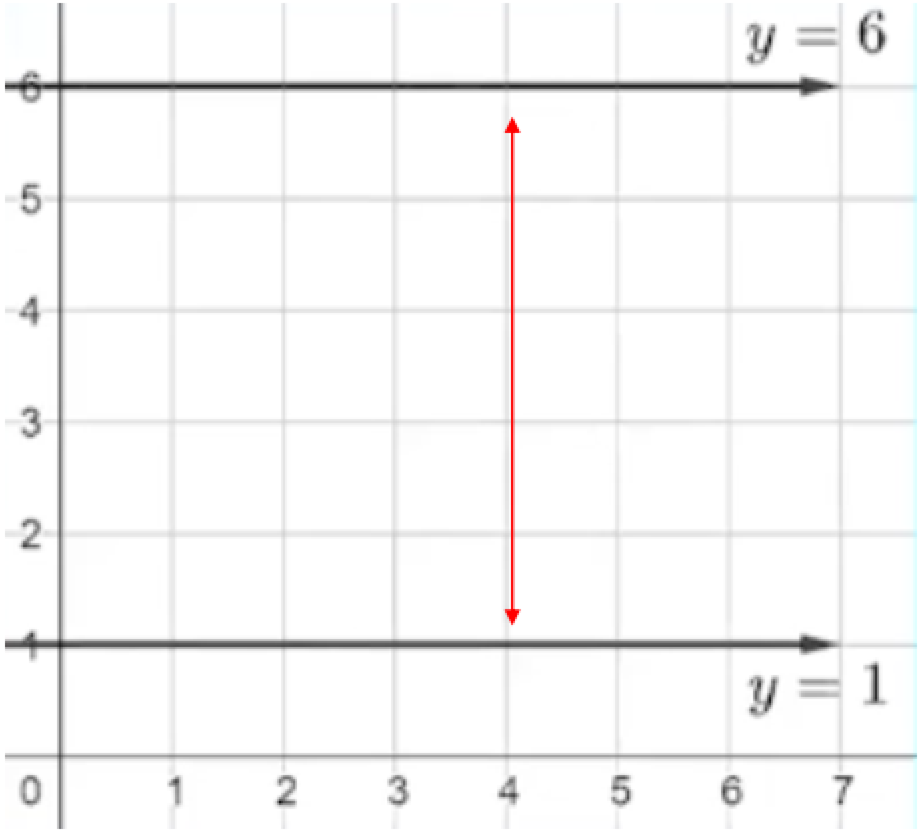
- ***What if we don't know a whole lot of information?***

- ***Can we find the limit?***

Understanding the Squeeze Theorem

What would we need to know to find this limit?

Find $\lim_{x \to 4} f(x)$



❖ What if we say that…

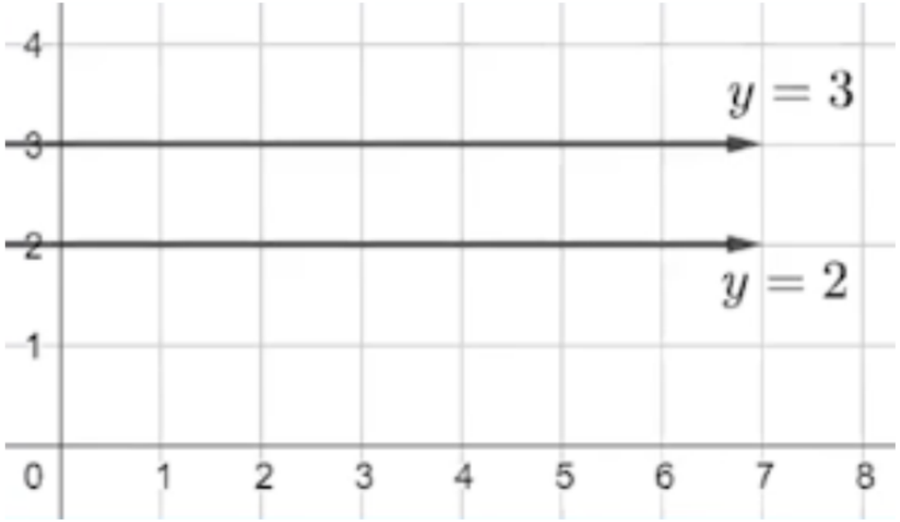
❖ We can't guarantee the limit in this case because there's *a lot of space* $between\ y = 1\ and\ \ y = 6$

Understanding the Squeeze Theorem

What would we need to know to find this limit?

Find $\lim_{x \to 4} f(x)$

What if we say that…



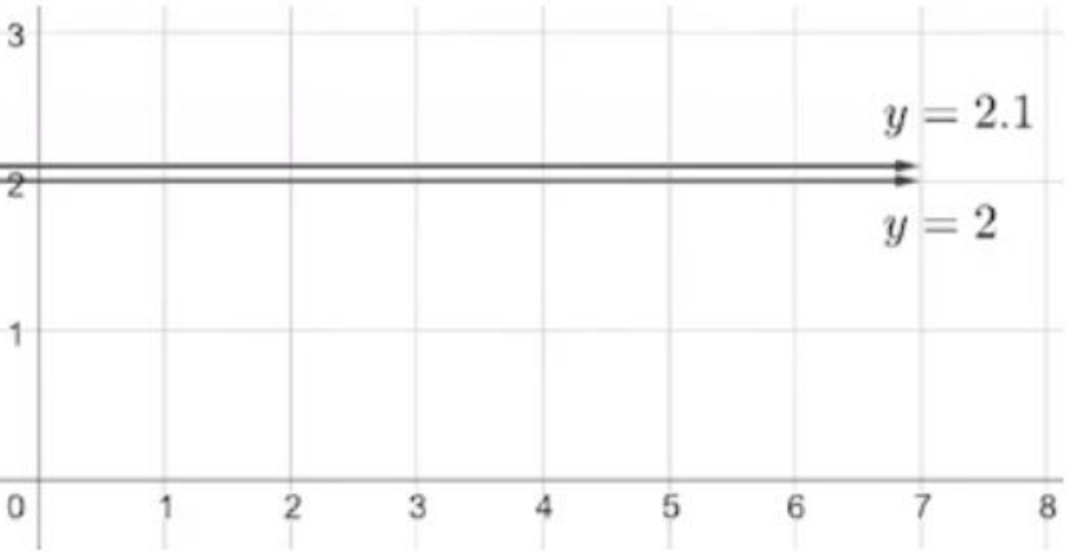
❖Still, there's a lot of gap, so we have no way of knowing what the function is approaching -> $between\ y = 2\ and\ \ y = 3$

Understanding the Squeeze Theorem

What would we need to know to find this limit?

Find $\lim_{x \to 4} f(x)$

What if we say that… now very close!



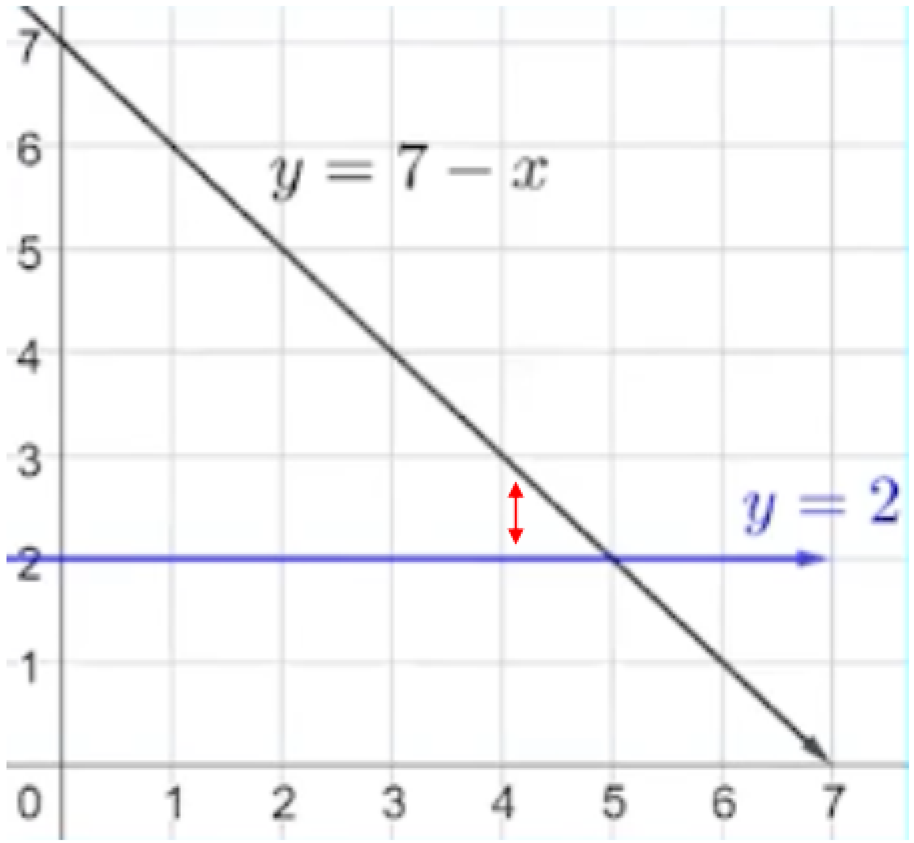
Now, it looks tiny **BUT** there's still an infinite amount of points $between\ y = 2\ and\ \ y = 2.1$

We still can't guarantee the limit!

Understanding the Squeeze Theorem

What would we need to know to find this limit?

Find $\lim_{x \to 4} f(x)$



What if we say that…

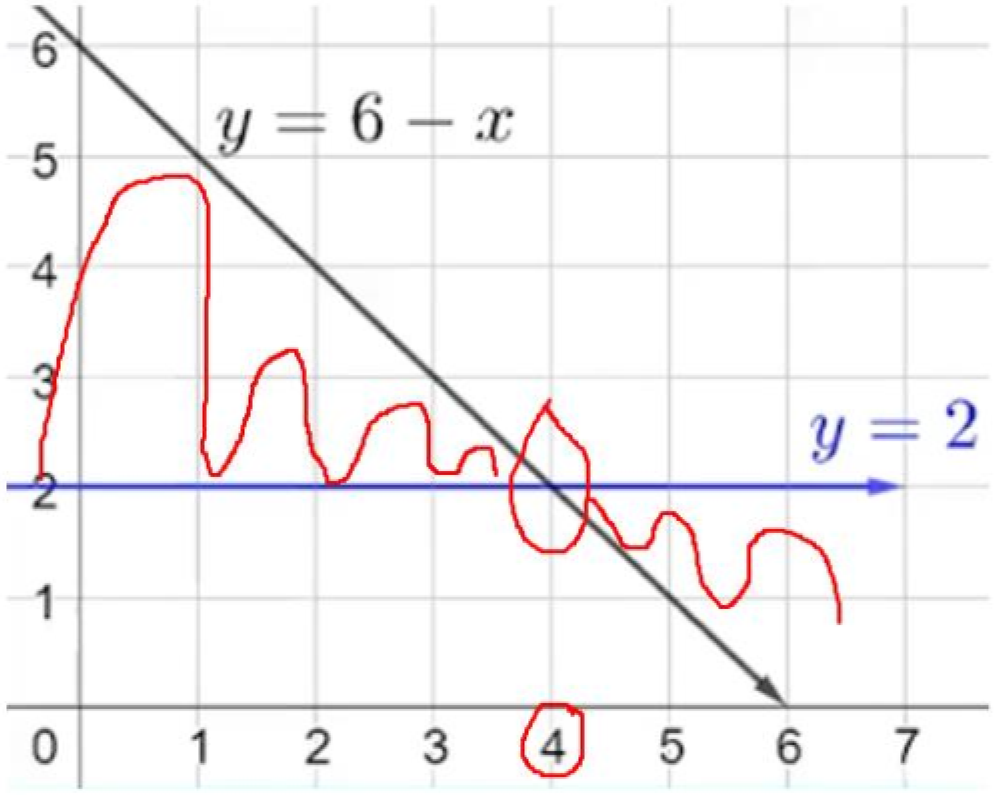
As we get closer to 4, the 2 lines are converging on each other…BUT as we get closer to 4, we still have a lot of space.

We still can't guarantee what the limit was of $f(x) as\ x \to 4$

Understanding the Squeeze Theorem

What would we need to know to find this limit?

Find $\lim_{x \to 4} f(x)$



What if we say that…

If x is wedged between there at all times, we don't know what $f(x)$ will look like exactly because of variations…

BUT no matter what happens, as we get closer to 4, ***since the 2 functions are converging on each other, our function $f(x)$ is going to be forced to the same point as there is no space to move***, so $\lim_{x \to 4} f(x) = 2$

The Squeeze Theorem and its conditions

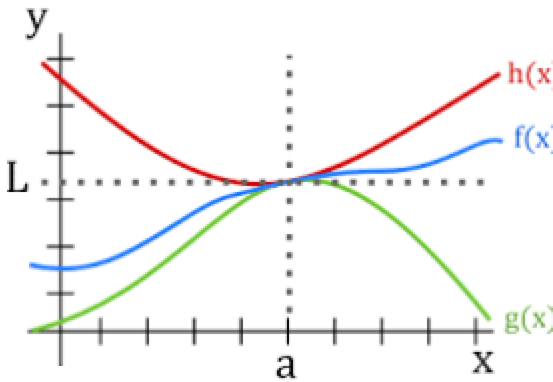
Squeezed between

Condition 1: If $g(x) \leq f(x) \leq h(x)$ for all values of x in an open interval containing c, except possibly at c itself, and….

Both approaching the same values

Condition 2: If $\lim_{x \to c} g(x) = L = \lim_{x \to c} h(x)$

then $\lim_{x \to c} f(x)$ exists and equals L

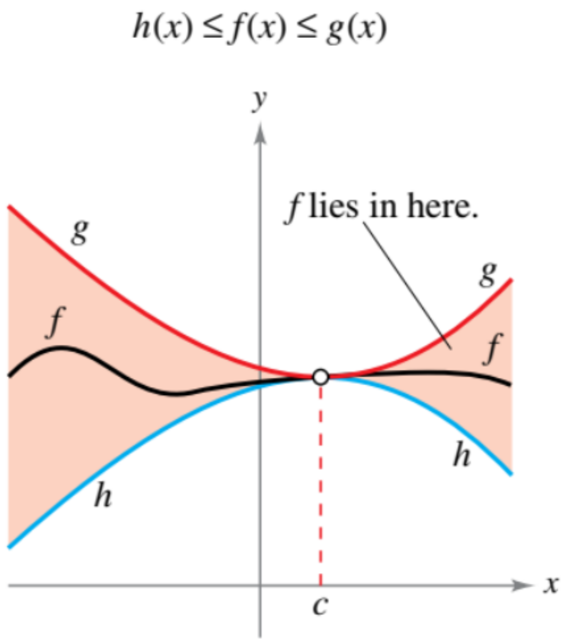


The Squeeze Theorem

If $h(x) \le f(x) \le g(x)$ for all x in an open interval containing c, except possibly at c itself, and if

$$\lim_{x \to c} h(x) = L = \lim_{x \to c} g(x)$$

then $\lim_{x \to c} f(x)$ exists and is equal to L.

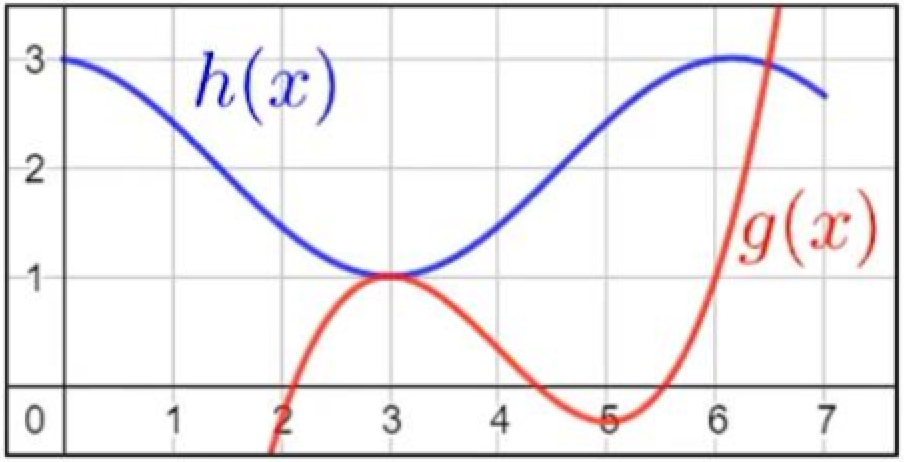


The Squeeze Theorem

The Squeeze Theorem and its conditions

- Consider the graphs of $g\ and\ h$ to the right. It is known that $g(x) \leq f(x) \leq h(x)$ on the open interval (2,4). *.......condition 1*

- The Squeeze Theorem can be applied *(condition 2 met)* to show that $\lim_{x \to c} f(x) = L$. Find c and L.



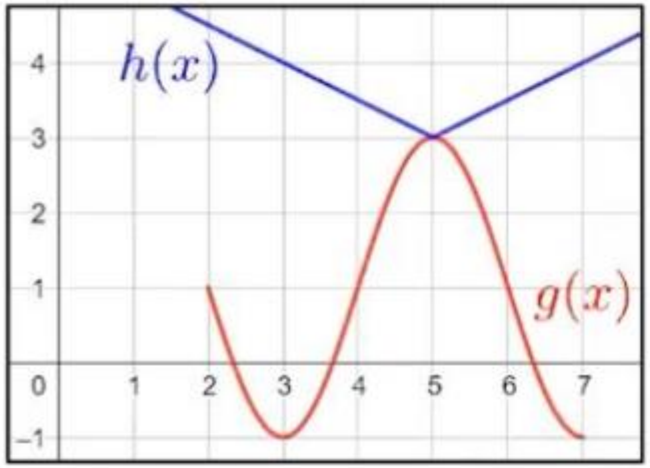
$x \rightarrow \boldsymbol{c} \ = x \rightarrow \mathbf{3}$

$\boldsymbol{L}\ will\ be\ the\ limit\ value = \mathbf{1}$

The Squeeze Theorem: Two Truths and a Lie

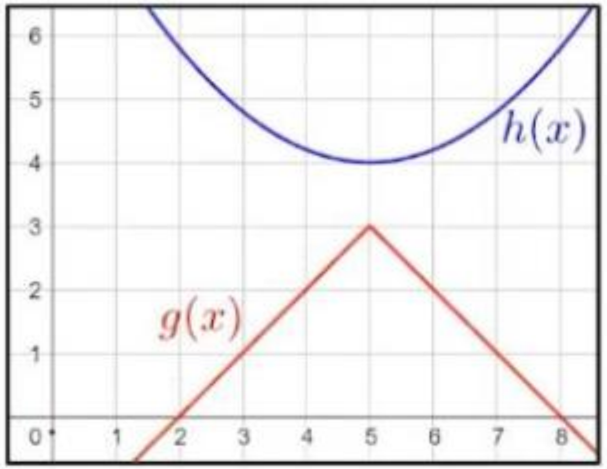
- Find $\lim_{x \to 5} f(x)$ given that $g(x) \leq f(x) \leq h(x)$ for all values of x.

- Consider the 3 cases below. Each case claims that it could be used to determine the limit above. However, only two of the cases are true and the other is a LIE! Determine which case is the lie!



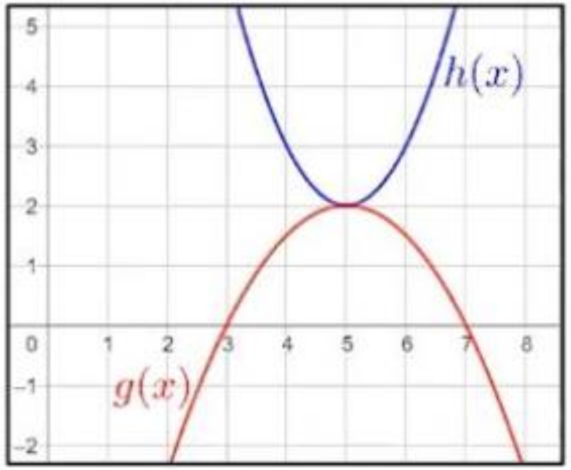
CASE 1

$\lim_{x \to 5} h(x) = 3 = \lim_{x \to 5} g(x)$



CASE 2

$\lim_{x \to 5} h(x) = 4 \ \lim_{x \to 5} g(x) = 3$



CASE 3

$\lim_{x \to 5} h(x) = 2 = \lim_{x \to 5} g(x)$

Next-> What Will We Learn?

- We'll focus on implementing and reinforcing the conditions required to apply the squeeze theorem.

The Squeeze Theorem

If $g(x) \leq f(x) \leq h(x)$ for all values of x in an open interval containing c, except possibly at c itself, and if

$$\lim_{x \to c} g(x) = \mathrm{L} = \lim_{x \to c} h(x)$$

then $\lim_{x \to c} f(x)$ exists and equals L

The Squeeze Theorem

- It is known that $g(x) \leq f(x) \leq h(x)$ for all values of x in the open interval (3,5).

- Can the Squeeze Theorem be used with the functions g and h below to find $\lim_{x \to 4} f(x)$?

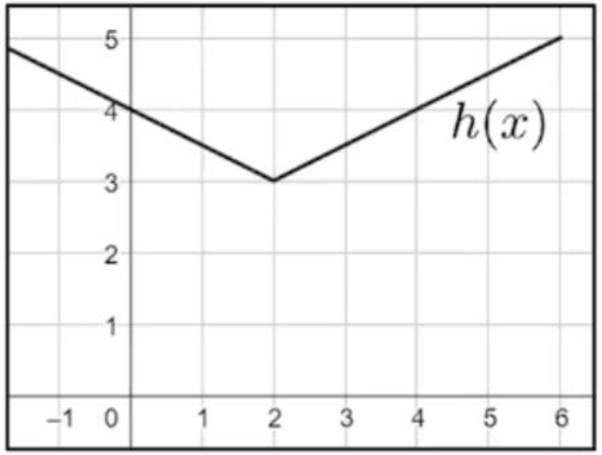
$$g(x) = -(x-4)^2 + 5 \text{ and } h(x) = \frac{x^2-2x-8}{x^2-7x+12}$$

The Squeeze Theorem

It is known that $g(x) \leq f(x) \leq h(x)$ for all values of x in the open interval (0,2).
Find the value of c such that the Squeeze Theorem be used with the functions g and h below to find $\lim_{x \to 1} f(x)$?

$$g(x) = \begin{cases} 3e^{x-1}, & x < 1 \\ 6 - 3x, & x > 1 \end{cases} \quad \text{and } h(x) = x^2 - 4x + c$$

The Squeeze Theorem

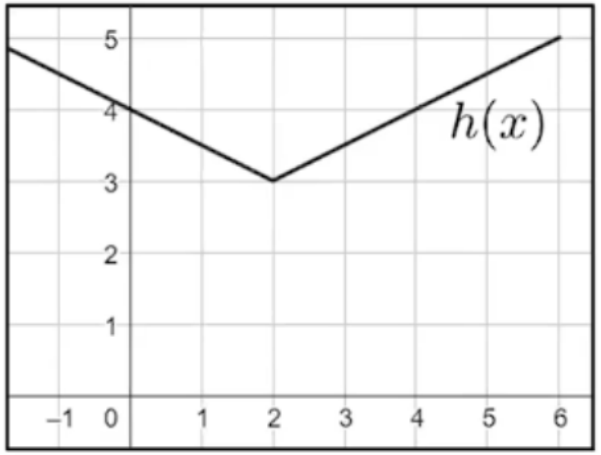


A portion of the graph of $h(x)$ is given above. Which of the following functions below can be used to find $\lim_{x \to 2} f(x)$ by using the Squeeze Theorem where $f(x)$ is between $h(x)$ and the function below.

x	1.97	1.98	1.99	2.01	2.02	2.03
$g(x)$	2.96	2.97	2.99	4.01	4.04	4.05

$$k(x) = \begin{cases} x + 1, & x < 2 \\ 7 - 2x, & x > 2 \end{cases}$$

The Squeeze Theorem



A portion of the graph of $h(x)$ is given above. Which of the following functions below can be used to find $\lim_{x \to 2} f(x)$ by using the Squeeze Theorem where $f(x)$ is between $h(x)$ and the function below.

$$\lim_{x \to 2} h(x) = 3$$

x	1.97	1.98	1.99	2.01	2.02	2.03
$g(x)$	2.96	2.97	2.99	4.01	4.04	4.05

$$k(x) = \begin{cases} x+1, & x<2 \\ 7-2x, & x>2 \end{cases}$$

Summary

The Squeeze Theorem

If $g(x) \le f(x) \le h(x)$ for all values of x in an open interval containing c, except possibly at c itself, and if

$$\lim_{x \to c} g(x) = \mathrm{L} = \lim_{x \to c} h(x)$$

then $\lim_{x \to c} f(x)$ exists and equals L