PRECALCULUS · POLYNOMIALS

Synthetic Division

The fast route: synthetic division, the remainder theorem and the factor theorem

1
starter: algebraic division

1
method: synthetic

11
questions worked

Built from the 2-04 Dividing Polynomials booklet

How this lesson runs

Key concept, worked example, questions to try — then true or false

Starter	Three quick recall questions on the shape of quadratic functions — attempt before the reveal.
Key points	Each section opens with a summary of the ideas you need, main point highlighted first.
Question	Questions appear on their own slides. Attempt each before turning the page.
Solution	The next slide gives the condensed solution with the working set out line by line.
True or false	The lesson ends with eight statements — one per slide, answer on the slide that follows.

Key topic · Synthetic division

What this lesson is about, in one page

The skill	Divide a polynomial by a LINEAR divisor (x − k) using synthetic division — coefficients only, no x's written.
The structure	Bring down, multiply by k, add. The last number is the remainder; the rest build the quotient, one degree lower.
The remainder theorem	The remainder when f(x) is divided by (x − k) is exactly f(k) — synthetic division computes it automatically.
The factor theorem	If the remainder is 0, then (x − k) is a factor of f(x), and k is a root of f(x) = 0.

Vocabulary

Key terms with Chinese translations

dividend	□□□	the polynomial being divided, f(x)
divisor	□□	the polynomial you divide by, d(x)
quotient	□	the result on top, q(x)
remainder	□□	what is left over, r(x)
factor	□□	d(x) is a factor if the remainder is 0
root (zero)	□□□□□	a value of x with f(x) = 0
degree	□□	the highest power of x
coefficient	□□	the number in front of each power
bring down	□□	start the bottom row with the leading coefficient
polynomial	□□□	a sum of terms like ax^n

S

Starter · Algebraic division

Long division you must be able to do before synthetic division is worth anything

- One worked example, then one to try
- Divide, multiply, subtract, bring down

Key points · algebraic division by long division

The four-step cycle you already know

1	**The identity**	$f(x) = d(x)\cdot q(x) + r(x)$. Dividend = Divisor × Quotient + Remainder. It is how you check every answer.
2	**The cycle**	DIVIDE the leading terms, MULTIPLY the whole divisor by that answer, SUBTRACT, bring down the next term. Repeat.
3	**Missing terms**	Insert 0·(any missing power) before you start: $x^4 - 16$ becomes $x^4 + 0x^3 + 0x^2 + 0x - 16$.
4	**Stop when**	The remainder has degree less than the divisor. Then write the answer as $q(x) + r(x)/d(x)$.

Starter · worked example

follow it

$(x^3 + 4x^2 - 3x + 10) \div (x + 2)$ by long division

BY LONG DIVISION

$$x^2 + 2x - 7$$

$$x+2 \;\big\lfloor\; x^3 + 4x^2 - 3x + 10$$

$x^3 + 2x^2$	$x^3 \div x = x^2$
$2x^2 - 3x$	
$2x^2 + 4x$	$2x^2 \div x = 2x$
$-7x + 10$	
$-7x - 14$	$-7x \div x = -7$
24	remainder 24

Answer: $x^2 + 2x - 7 + 24/(x + 2)$

WHY THIS EXAMPLE, TODAY

- This is the SAME division we will do synthetically in the next section — same dividend, same divisor.
- Keep the answer in mind: quotient $x^2 + 2x - 7$, remainder 24.
- The synthetic layout must reproduce exactly this result — that is how you will know it works.
- The whole row of x^2, x and constants is just the COEFFICIENTS 1, 4, −3, 10 — which is all synthetic division writes down.

Try it yourself

your turn

Attempt before turning the page — answer on the next slide

YOUR TURN

Use long division to divide:

$(y^4 + 2y^2 - y + 5) \div (y^2 - y + 1)$

This is the worked example from the 2-04 booklet. Insert any missing terms first, then work the cycle.

Question S1 — solution

3 marks

Try it · long division

CONDENSED SOLUTION

- Insert the missing term: $y^4 + 0y^3 + 2y^2 - y + 5$.
- $y^4 \div y^2 = y^2$. Multiply: $y^4 - y^3 + y^2$. Subtract: y^3 remains; bring down $2y^2$.
- $y^3 \div y^2 = y$. Multiply: $y^3 - y^2 + y$. Subtract: $2y^2 - 2y$ remains; bring down 5.
- $2y^2 \div y^2 = 2$. Multiply: $2y^2 - 2y + 2$. Subtract: 3.
- Quotient $y^2 + y + 2$, remainder 3.
- Answer: $y^2 + y + 2 + 3/(y^2 - y + 1)$.

TRAP The divisor here is QUADRATIC, so synthetic division does not apply. Long division is the general tool.

01

The Division Algorithm

What dividing polynomials actually produces

- Dividend = Divisor × Quotient + Remainder
- The language used in every mark scheme

Key points · the division algorithm

Four names you must use correctly

1	**The identity**	$f(x) = d(x)\cdot q(x) + r(x)$, where deg $r(x)$ < deg $d(x)$ or $r(x) = 0$.
2	**Dividend**	$f(x)$, the polynomial being divided. It goes inside the division frame.
3	**Divisor**	$d(x)$, the polynomial you divide by. It sits outside the frame.
4	**Quotient and remainder**	$q(x)$ sits on top; $r(x)$ is what is left over at the bottom. Remainder 0 means $d(x)$ divides evenly.

Question A1

2 marks

The division algorithm

THE QUESTION

In the division $(x^3 + 4x^2 - 3x + 10) \div (x + 2)$:

(a) State the dividend and the divisor.
(b) The result is $x^2 + 2x - 7$ with remainder 24. Write the division in the form $f(x) = d(x) \cdot q(x) + r(x)$.
(c) State the degree of the quotient $q(x)$ and explain how you know.

Question A1 — solution

2 marks

The division algorithm

CONDENSED SOLUTION

- (a) Dividend $f(x) = x^3 + 4x^2 - 3x + 10$; divisor $d(x) = x + 2$.
- (b) $x^3 + 4x^2 - 3x + 10 = (x + 2)(x^2 + 2x - 7) + 24$.
- (c) deg q = 2. Degrees subtract: deg q = deg f − deg d = 3 − 1 = 2.

TRAP Always check: divisor × quotient + remainder must rebuild the dividend exactly. Use it to verify your answer.

02

Synthetic division

The fast route when the divisor is x − k

- Coefficients only — no x’s written
- Bring down, multiply, add

Key points · synthetic division

Only works for linear divisors of the form x − k

1	**Rewrite the divisor**	x + 2 must become x − (−2), so k = −2. The number you carry down the left is k, with its sign.
2	**Row of coefficients**	Write the dividend’s coefficients in descending powers, inserting 0 for any missing term.
3	**Bring down, multiply, add**	Bring the leading coefficient down; multiply it by k; ADD under the next coefficient; repeat to the end.
4	**Reading the answer**	The last number is the remainder; the rest are the quotient’s coefficients, one degree lower than the dividend.

Worked example · synthetic division

follow it

$(x^3 + 4x^2 - 3x + 10) \div (x + 2)$ — the same division, faster

SET-UP

- Divisor $x + 2 = x - (-2)$, so $k = -2$.
- Coefficients of the dividend: 1, 4, −3, 10.
- Draw the frame: k on the left, coefficients across the top, a line, and an empty row beneath for the answers.
- No x's are written anywhere — only numbers.

THE WORKING

```
−2  |   1     4     −3     10        coefficients
    |         −2      4      6       multiply by −2
    |   1     2     −7     24        add down the columns
```

Quotient: 1, 2, −7 → $x^2 + 2x - 7$

Remainder: 24

> Answer: $x^2 + 2x - 7 + 24/(x + 2)$

Try it yourself

your turn

Attempt before turning the page — answer on the next slide

YOUR TURN

Use synthetic division to divide:

$(x^3 - 7x - 6) \div (x + 1)$

First decide what k is. Then write the coefficient row — remembering the missing x^2 term — and work: bring down, multiply, add.

Question Y1 — solution

3 marks

Try it · synthetic division

CONDENSED SOLUTION

- $x + 1 = x - (-1)$, so $k = -1$.
- Insert the missing x^2 term: coefficients 1, 0, −7, −6.
- Bring down 1. $1\times(-1) = -1$; $0+(-1) = -1$. $-1\times(-1) = 1$; $-7+1 = -6$. $-6\times(-1) = 6$; $-6+6 = 0$.
- Quotient $x^2 - x - 6$, remainder 0.
- Answer: $x^3 - 7x - 6 = (x + 1)(x^2 - x - 6) = (x + 1)(x - 3)(x + 2)$.

TRAP Remainder 0 means (x + 1) is a FACTOR — that link is formalised by the factor theorem, coming next.

Question B1 · from the booklet

4 marks

Synthetic division

THE QUESTION

Use synthetic division to divide:

$(-5x^5 - 21x^4 - 3x^3 + 4x^2 + 2x + 2) \div (x + 4)$

State the quotient and the remainder.

Question B1 — solution

4 marks

Synthetic division · booklet

CONDENSED SOLUTION

- $x + 4 = x - (-4)$, so $k = -4$. Coefficients: $-5, -21, -3, 4, 2, 2$.
- Bring down -5. $-5\times(-4) = 20$; $-21+20 = -1$. $-1\times(-4) = 4$; $-3+4 = 1$. $1\times(-4) = -4$; $4+(-4) = 0$. $0\times(-4) = 0$; $2+0 = 2$. $2\times(-4) = -8$; $2+(-8) = -6$.
- Quotient $-5x^4 - x^3 + x^2 + 0x + 2$, remainder -6.
- Answer: $-5x^4 - x^3 + x^2 + 2 - 6/(x + 4)$.

TRAP Five quotient coefficients means degree 4. Writing six is the common error — the remainder takes the last slot.

Question B2 · from the booklet

4 marks

Synthetic division

THE QUESTION

Use synthetic division to divide:

$(y^5 + 32y + 2) \div (y + 1)$

Hint: three powers are missing. Write the full coefficient row before you start.

Question B2 — solution

4 marks

Synthetic division · booklet

CONDENSED SOLUTION

- $y + 1 = y - (-1)$, so $k = -1$.
- Missing powers: $y^5 + 0y^4 + 0y^3 + 0y^2 + 32y + 2$. Coefficients: 1, 0, 0, 0, 32, 2.
- Bring down 1. $1\times(-1) = -1$; $0+(-1) = -1$. $-1\times(-1) = 1$; $0+1 = 1$. $1\times(-1) = -1$; $0+(-1) = -1$. $1\times(-1) = 1$; $32+1 = 33$. $33\times(-1) = -33$; $2+(-33) = -29$.
- Quotient $y^4 - y^3 + y^2 - y + 33$, remainder -29.
- Answer: $y^4 - y^3 + y^2 - y + 33 - 29/(y + 1)$.

TRAP Four missing terms, four zeros in the row. Skip them and every column after the first is wrong.

Question P1

3 marks

Synthetic division · practice

THE QUESTION

Use synthetic division to divide $(x^3 - 6x^2 + 11x - 6)$ by $(x - 1)$.

State the quotient and the remainder, and write the division in the form $f(x) = d(x) \cdot q(x) + r(x)$.

Question P1 — solution

3 marks

Synthetic division · practice

CONDENSED SOLUTION

- k = 1. Coefficients: 1, −6, 11, −6.
- Bring down 1. 1×1 = 1; −6+1 = −5. −5×1 = −5; 11+(−5) = 6. 6×1 = 6; −6+6 = 0.
- Quotient $x^2 - 5x + 6$, remainder 0.
- $f(x) = (x - 1)(x^2 - 5x + 6) + 0 = (x - 1)(x - 2)(x - 3)$.

TRAP Remainder 0: the divisor is a factor. Factorise the quotient fully — it is a quadratic with roots 2 and 3.

Question P2

3 marks

Synthetic division · practice

THE QUESTION

Use synthetic division to divide $(2x^3 + 3x^2 - 4x + 5)$ by $(x + 1)$.

Careful: what is k here?

Question P2 — solution

3 marks

Synthetic division · practice

CONDENSED SOLUTION

- $x + 1 = x - (-1)$, so $k = -1$. Coefficients: 2, 3, −4, 5.
- Bring down 2. $2\times(-1) = -2$; $3+(-2) = 1$. $1\times(-1) = -1$; $-4+(-1) = -5$. $-5\times(-1) = 5$; $5+5 = 10$.
- Quotient $2x^2 + x - 5$, remainder 10.
- Answer: $2x^2 + x - 5 + 10/(x + 1)$.

TRAP k carries the sign: divisor $x + 1$ means $k = -1$. Using +1 here is the single most common synthetic error.

Question P3

3 marks

Synthetic division · practice

THE QUESTION

Use synthetic division to divide $(x^4 - 16)$ by $(x - 2)$.

Watch for missing terms before you write the coefficient row.

Question P3 — solution

3 marks

Synthetic division · practice

CONDENSED SOLUTION

- Insert missing terms: $x^4 + 0x^3 + 0x^2 + 0x - 16$. Coefficients: 1, 0, 0, 0, −16. k = 2.
- Bring down 1. 1×2 = 2; 0+2 = 2. 2×2 = 4; 0+4 = 4. 4×2 = 8; 0+8 = 8. 8×2 = 16; −16+16 = 0.
- Quotient $x^3 + 2x^2 + 4x + 8$, remainder 0.
- Answer: $x^4 - 16 = (x - 2)(x^3 + 2x^2 + 4x + 8)$.

TRAP Missing terms become zeros in the coefficient row. Without them the columns shift and every answer is wrong.

03

The remainder theorem and the factor theorem

What the remainder tells you about factors and roots

- The remainder theorem: remainder = f(k) — no division needed
- The factor theorem: remainder 0 means (x − k) is a factor

Key points · remainders and factors

Two theorems that turn synthetic division into root-finding

1	**Remainder theorem**	The remainder when f(x) is divided by (x − k) is exactly f(k). Synthetic division computes it automatically — it is the last number in the bottom row.
2	**Factor theorem**	If f(k) = 0 — that is, remainder 0 — then (x − k) is a factor of f(x), and k is a root of f(x) = 0.
3	**Reading synthetic output**	The bottom row ends in the remainder. A final 0 means the divisor divides evenly and you can factorise the quotient further.
4	**Watch the sign**	For divisor (x + 3), k = −3. The remainder is f(−3), not f(3).

Worked example · factor and remainder theorems

follow it

Show that $(x + 3)$ is a factor of $x^3 - 19x - 30$, then find the remaining factors

STEP 1 · CHECK THE FACTOR

- $x + 3 = x - (-3)$, so evaluate $f(-3)$.
- $f(-3) = (-3)^3 - 19(-3) - 30$
- $= -27 + 57 - 30 = 0$
- $f(-3) = 0$, so $(x + 3)$ IS a factor ✓

STEP 2 · SYNTHETIC DIVISION BY (x + 3)

```
−3  |   1     0    −19    −30
```

coefficients, note $0x^2$

```
    |         −3     9     30
```

multiply by −3

```
    |   1    −3    −10      0
```

add down: remainder 0

STEP 3 · FIND THE REMAINING FACTORS

- Quotient: $x^2 - 3x - 10$, remainder 0.
- Factorise the quadratic: $(x - 5)(x + 2)$.
- So $x^3 - 19x - 30 = (x + 3)(x - 5)(x + 2)$.
- The three roots are $x = -3$, 5 and −2 — where each factor vanishes.

> Check: remainder 0 at every stage confirms each factor.

Notice the synthetic division did both jobs at once: the 0 remainder PROVED the factor, and the quotient gave the rest of the factorisation.

Question R1

2 marks

Remainders and factors

THE QUESTION

Without dividing, find the remainder when $f(x) = x^3 - 2x^2 + 5x - 7$ is divided by $(x - 3)$.
Then state whether $(x - 3)$ is a factor of $f(x)$.

Question R1 — solution

2 marks

Remainders and factors

CONDENSED SOLUTION

- Remainder theorem: the remainder is f(3).
- f(3) = 27 − 2(9) + 5(3) − 7 = 27 − 18 + 15 − 7 = 17.
- Remainder = 17. Since 17 ≠ 0, (x − 3) is NOT a factor of f(x), and x = 3 is not a root.

TRAP Answer the whole question: the number AND the factor statement. “17” on its own drops a mark.

Question R2

3 marks

Remainders and factors

THE QUESTION

$f(x) = x^3 + ax^2 - 5x + 6$. Given that $(x - 1)$ is a factor of $f(x)$, find the value of a.
Then fully factorise $f(x)$.

Question R2 — solution

3 marks

Remainders and factors

CONDENSED SOLUTION

- $(x - 1)$ a factor means $f(1) = 0$: $1 + a - 5 + 6 = 0$, so $a + 2 = 0$ and $a = -2$.
- So $f(x) = x^3 - 2x^2 - 5x + 6$. Divide by $(x - 1)$: coefficients 1, −2, −5, 6.
- Long division: $x^3 \div x = x^2$; subtract → $-x^2$; $-x$; subtract → $-6x$; -6; remainder 0.
- Quotient $x^2 - x - 6 = (x - 3)(x + 2)$.
- $f(x) = (x - 1)(x - 3)(x + 2)$.

TRAP After finding a, rebuild f(x) before dividing — students who skip this step divide the wrong polynomial.

Question R3

3 marks

Remainders and factors

THE QUESTION

$f(x) = 2x^3 - 3x^2 + px + 1$ leaves a remainder of 4 when divided by $(x - 1)$.

(a) Find the value of p.
(b) Write f(x) in the form $(x - 1)\cdot q(x) + r$.

Question R3 — solution

3 marks

Remainders and factors

CONDENSED SOLUTION

- (a) By the remainder theorem the remainder is $f(1) = 2 - 3 + p + 1 = p$. So $p = 4$.
- So $f(x) = 2x^3 - 3x^2 + 4x + 1$.
- (b) Long division by $(x - 1)$: $2x^3 \div x = 2x^2$; subtract → $-x^2$; $-x$; subtract → $3x$; $+3$; subtract → 4.
- $f(x) = (x - 1)(2x^2 - x + 3) + 4$.
- Check: $(x - 1)(2x^2 - x + 3) = 2x^3 - 3x^2 + 4x - 3$, and $+ 4$ gives $2x^3 - 3x^2 + 4x + 1$ ✓

TRAP Set up the equation from the theorem first: remainder = $f(1) = 4$. That single line earns the first mark.

Question R4

3 marks

Remainders and factors

THE QUESTION

Show that $(x + 2)$ is a factor of $f(x) = x^3 + 3x^2 - 4x - 12$, and hence fully factorise $f(x)$.

Question R4 — solution

3 marks

Remainders and factors

CONDENSED SOLUTION

- $(x + 2)$ is a factor means $f(-2) = 0$.
- $f(-2) = -8 + 3(4) - 4(-2) - 12 = -8 + 12 + 8 - 12 = 0$ ✓
- Divide by $(x + 2)$: $x^3 \div x = x^2$; subtract → x^2; $+x$; subtract → $-6x$; -6; remainder 0.
- Quotient $x^2 + x - 6 = (x + 3)(x - 2)$.
- $f(x) = (x + 2)(x + 3)(x - 2)$.

TRAP "Show that" questions need the evaluation written out — $f(-2) = 0$ with the working shown, not just the conclusion.

Mark scheme

Marks by question, with what earns each mark

S1	**3**	1 set-up in the identity form · 1 long-division method · 1 answer as q(x) + r(x)/d(x)
A1	**2**	1 dividend/divisor named · 1 identity written correctly with degree reasoning
Y1	**3**	1 k = −1 and zero coefficient inserted · 1 synthetic working · 1 quotient with remainder 0 and factors
B1	**4**	1 k = −4 · 1 coefficient row · 1 quotient $-5x^4 - x^3 + x^2 + 2$ · 1 remainder −6
B2	**4**	1 missing powers inserted · 1 coefficient row · 1 quotient · 1 remainder −29
P1	**3**	1 working · 1 quotient $x^2 - 5x + 6$ · 1 factor statement $(x - 1)(x - 2)(x - 3)$
P2	**3**	1 k = −1 · 1 synthetic working · 1 answer $2x^2 + x - 5 + 10/(x + 1)$
P3	**3**	1 zeros inserted · 1 synthetic working · 1 factorised answer
R1	**2**	1 remainder f(3) = 17 · 1 factor statement
R2	**3**	1 finding a · 1 synthetic division · 1 full factorisation
R3	**3**	1 finding p · 1 synthetic division · 1 identity written correctly
R4	**3**	1 f(−2) = 0 shown · 1 synthetic division · 1 full factorisation

Main lesson total: 33 marks. Starter: 3 practice marks (not assessed).

Answer key

One line per question

ANSWERS

- S1 $y^2 + y + 2 + 3/(y^2 - y + 1)$
- A1 $f(x) = (x + 2)(x^2 + 2x - 7) + 24$
- Y1 $(x + 1)(x - 3)(x + 2)$, remainder 0
- B1 $-5x^4 - x^3 + x^2 + 2 - 6/(x + 4)$
- B2 $y^4 - y^3 + y^2 - y + 33 - 29/(y + 1)$
- P1 $(x - 1)(x - 2)(x - 3)$, remainder 0

QUICK VERIFICATION

- P2 $2x^2 + x - 5 + 10/(x + 1)$
- P3 $(x - 2)(x^3 + 2x^2 + 4x + 8)$, remainder 0
- R1 remainder 17; not a factor
- R2 $a = -2$; $f(x) = (x - 1)(x - 3)(x + 2)$
- R3 $p = 4$; $f(x) = (x - 1)(2x^2 - x + 3) + 4$
- R4 $f(x) = (x + 2)(x + 3)(x - 2)$

True or false · 1

T / F

Decide, then justify your decision — answer on the next slide

Synthetic division can be used to divide by any divisor, such as $(x^2 - y + 1)$.

True or false · 1 — answer

FALSE

FALSE

Synthetic division only works when the divisor is LINEAR of the form $(x - k)$. For a quadratic divisor like $(x^2 - y + 1)$ you must use long division — as in the starter.

True or false · 2

T / F

Decide, then justify your decision — answer on the next slide

When dividing by (x + 4) using synthetic division, you place −4 in the box.

True or false · 2 — answer

TRUE

TRUE

Rewrite x + 4 as x − (−4). The number carried down the left is k = −4 — the sign flips, exactly as in Question B1.

True or false · 3

T / F

Decide, then justify your decision — answer on the next slide

The last number in the bottom row of a synthetic division is the quotient.

True or false · 3 — answer

FALSE

FALSE

It is the REMAINDER. The quotient comes from the other numbers in the bottom row, one degree lower than the dividend.

True or false · 4

T / F

Decide, then justify your decision — answer on the next slide

A zero remainder means (x − k) is a factor of f(x), and k is a root of f(x) = 0.

True or false · 4 — answer

TRUE

TRUE

This is the factor theorem. The zero remainder in Y1 and P1 is what let us write each dividend as a product of factors.

True or false · 5

T / F

Decide, then justify your decision — answer on the next slide

A polynomial remainder can be valid, as long as its degree is less than the degree of the divisor.

True or false · 5 — answer

TRUE

TRUE

You stop when deg r < deg d. For a linear divisor the remainder must be a CONSTANT — like the −6 in B1 or the 24 in the starter.

True or false · 6

T / F

Decide, then justify your decision — answer on the next slide

Missing terms in the dividend can be skipped as long as you are careful.

True or false · 6 — answer

FALSE

FALSE

Missing terms must be written in with zero coefficients: $y^5 + 32y + 2$ needed three zeros in B2, and $x^4 - 16$ needed three in P3.

True or false · 7

T / F

Decide, then justify your decision — answer on the next slide

By the remainder theorem, the remainder when f(x) is divided by (x + 3) is f(3).

True or false · 7 — answer

FALSE

FALSE

$x + 3 = x - (-3)$, so $k = -3$. The remainder is $f(-3)$ — and in the worked example $f(-3) = 0$, which proved $(x + 3)$ is a factor.

True or false · 8

T / F

Decide, then justify your decision — answer on the next slide

If $(x - 1)$ is a factor of $f(x)$, then $f(1) = 0$.

True or false · 8 — answer

TRUE

TRUE

By the factor theorem: a factor $(x - k)$ always satisfies $f(k) = 0$. That is exactly how R2 found a: $f(1) = 0$ gave $a = -2$.

FIVE THINGS TO REMEMBER

Do these and algebraic division will not cost you marks

- Write missing terms with zero coefficients before you start.
- The cycle never changes: divide, multiply, subtract, bring down.
- Subtraction flips every sign in the row — check twice.
- The remainder when dividing by (x − k) is f(k) — no division needed.
- Zero remainder means a factor: f(k) = 0 and k is a root.

Built from the 2-04 Dividing Polynomials booklet.