F1

Annie Bai	Franklin Chen	Charlie Cheng	Zoey Cheng	Harry Cui	King Deng	Aisling Fu	Ken He	Ivy Jing
Lee Li	Ziheng Li	Lewis Liu	Sam Liu	Ava Peng	Peter Shen	Sylvia Song	Coco Wang	Eileen Wang
Jason Wang	Luna Wang	Cathy Yan	Melody You	Allen Zhang	Lucas Zhang	Lynette Zhang	Ryder Zhang	Flora Zhou
Cyntina Zuo								

F2

Angelina Wang	Azura Cheng	Bonnie Zhao	Cecilia Jia	Diana Li	Oscar Liu	Zavier Liu	Tom Ou	Mia Peng
Joshua Hui	Julius Lv	Justin Jia	Lucas Li	Lyra Zhang	Iris Shao	Dylan Suo	PeytonAson Wang	Struck Wang
Selene Hou	Serena Feng	Silas Lv	Simon Wang	Stella Sun	Yola Wang	Mark Xiao	Elsa Ye	Emily Zhang
Zinnia Dong	Flora Zhang							

F3

Allen Peng	Aurora Yuan	Brittney Wei	Cynthia Liu	Erya Hu	Eva Gai	Felicity Pan	Fiona Ding	Freya Fan
Gordon Yao	Helios Luo	Honey Ruan	Iris Xie	Jasper Wu	Kaven Zhang	Kevin Gao	Leo Yang	Linger Li
Lydia Wei	Maggie Gao	Micheal Zhao	Ray Meng	Rose Jiang	Ross Ma	Roy Liu	Ryan Wang	Sky Bai
Star Su	Stella Xi							

F4

Aris Cheng	Tracy Dang	Jason Du	Alanna Fan	Iris Gao	Eason Gong	Vardy Hai	Edmund He	Liora Li
Miranda Li	Leo Liu	Francis Lv	Dorothy Qian	Dary Song	Jerry Tu	Belinda Wang	Winnie Wang	Elena Wei
James Xu	Mike Yan	Simon Yang	Coco Zhang	Patrick Zhang	Sophia Zhang	Molly Zheng	Carrie Zhou	Rudy Zhu

F5

Abbie Dong	Alan Chen	Angela An	Annette Zhao	Asher Tian	Jove Bai	Camellia Long	Candice Li	Carmen Du
Ricardo Lian	Emma Zhang	Ethan Yu	Ethen Li	Jack Spark Men	Jarry Wang	Cressen Liang	Leo Liu	Lucas Bai
Lycia Liu	Nina Dang	August Mao	Robin Mi	Vicky Yang	Victor Li	Yolanda Wang	Zephyra Hu	

F6

Lucas Chang	Chloe Fang	Bruce Gao	Wyatt Hou	Jackson Jiang	Ivan Lei	Eric Li	Keira Li	Vivian Li
Jeremy Lin	Orange Liu	Serika Ren	Yumiko Shi	Hannah Si	Eric Song	Eric Tan	Silas Wang	Niki Wei
Kiki Wen	Miyu Wu	Tia Wu	Kumi Yuan	Estelle Zhang	Soren Zhang	Viakey Zhang	Jack Zhao	

Turn and Talk

1. Obtuse Angle in the Second Quadrant

- Find the values of $\sin 120^\circ$, $\cos 120^\circ$, and $\tan 120^\circ$.

2. Obtuse Angle in the Second Quadrant

- Find the values of $\sin 135^\circ$, $\cos 135^\circ$, and $\tan 135^\circ$.

3. Reflex Angle in the Third Quadrant

- Find the values of $\sin 210^\circ$, $\cos 210^\circ$, and $\tan 210^\circ$.

1. 120°
   - $\sin 120^\circ = \frac{\sqrt{3}}{2}$
   - $\cos 120^\circ = -\frac{1}{2}$
   - $\tan 120^\circ = -\sqrt{3}$
2. 135°
   - $\sin 135^\circ = \frac{\sqrt{2}}{2}$
   - $\cos 135^\circ = -\frac{\sqrt{2}}{2}$
   - $\tan 135^\circ = -1$
3. 210°
   - $\sin 210^\circ = -\frac{1}{2}$
   - $\cos 210^\circ = -\frac{\sqrt{3}}{2}$
   - $\tan 210^\circ = \frac{\sqrt{3}}{3}$

Inverse Function

- Inverse function — 反函数
- One-to-one function — 一一对应函数 / 单射函数
- Horizontal line test — 水平线测试
- Inverse notation — 反函数表示法
- Domain — 定义域
- Range — 值域
- Domain–range switch — 定义域和值域互换
- Reflection over $y = x$ — 关于直线 $y = x$ 的对称
- Inverse exists — 反函数存在
- Restrict the domain — 限制定义域

Reminders:

- Q2 Gradebook is closed
- Desmos Project is now due 3rd March

Inverse Functions

DEFINITION

Given a function f with domain D and range R, its **inverse function** (if it exists) is the function f^{-1} with domain R and range D such that $f^{-1}(y) = x$ if $f(x) = y$. In other words, for a function f and its inverse f^{-1},

$$f^{-1}(f(x)) = x \text{ for all } x \text{ in } D, \text{and } f\left(f^{-1}(y)\right) = y \text{ for all } y \text{ in } R. \quad (1.11)$$

https://openstax.org/books/calculus-volume-1/pages/1-1-review-of-functions

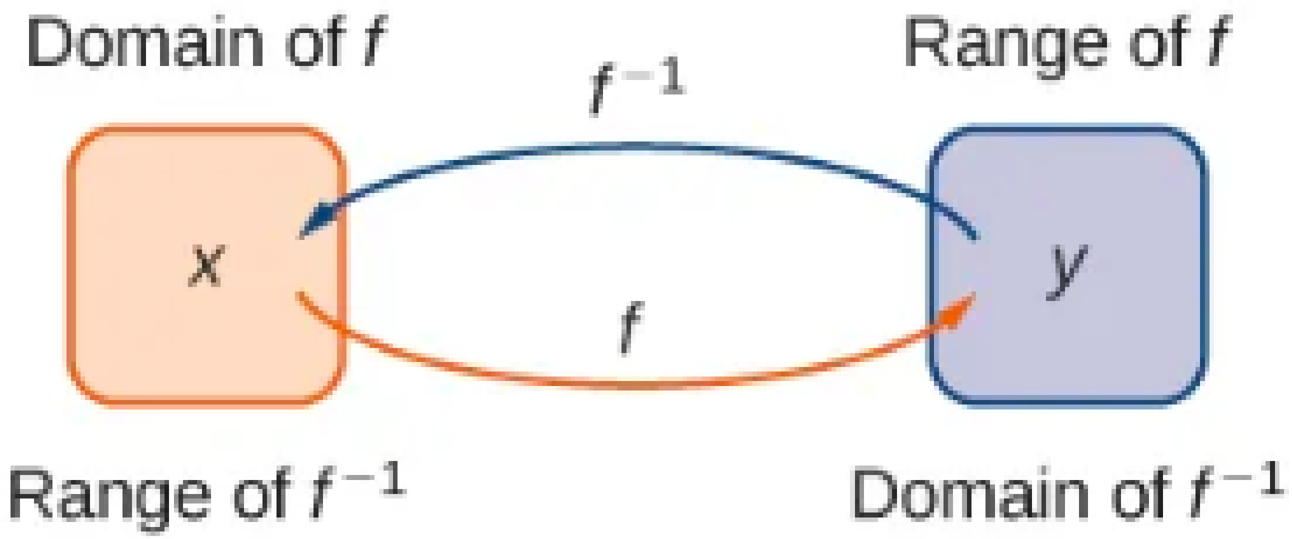


Figure 1.37 Given a function f and its inverse f^{-1}, $f^{-1}(y) = x$ if and only if $f(x) = y$. The range of f becomes the domain of f^{-1} and the domain of f becomes the range of f^{-1}.

DEFINITION

We say a f is a **one-to-one function** if $f(x_1) \neq f(x_2)$ when $x_1 \neq x_2$.

RULE: HORIZONTAL LINE TEST

A function f is one-to-one if and only if every horizontal line intersects the graph of f no more than once.

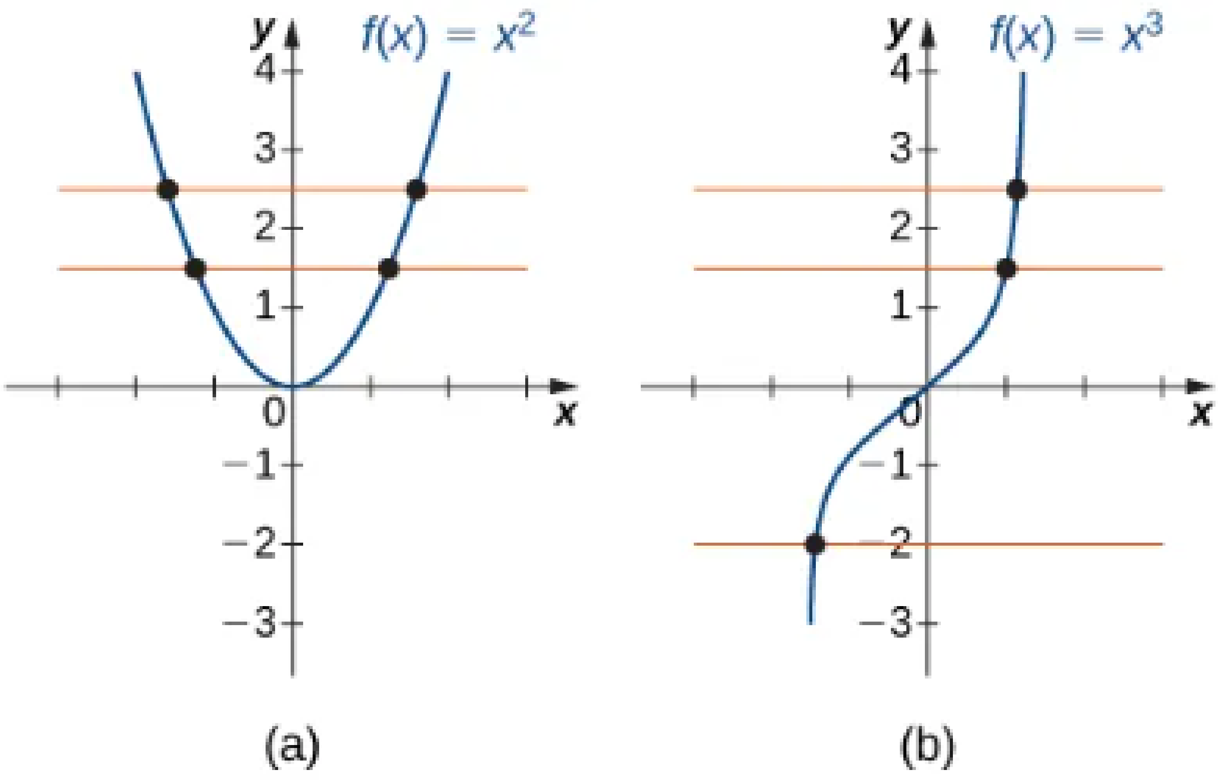
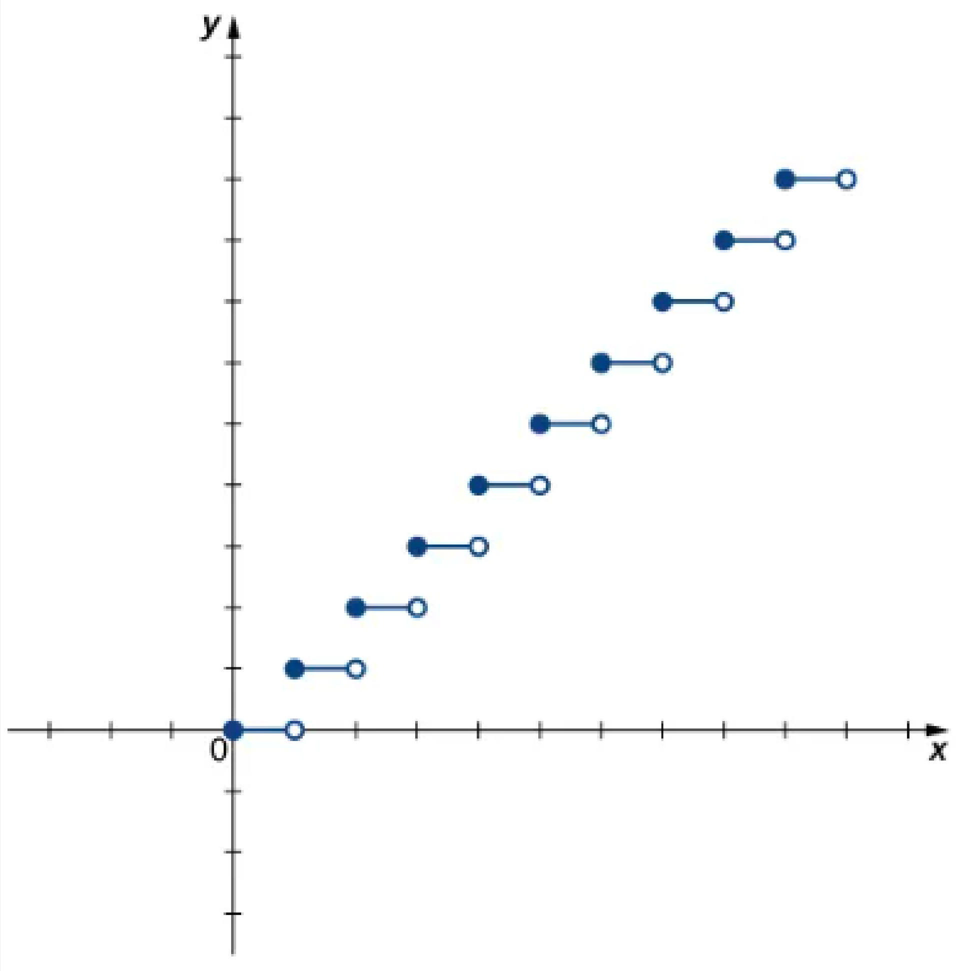


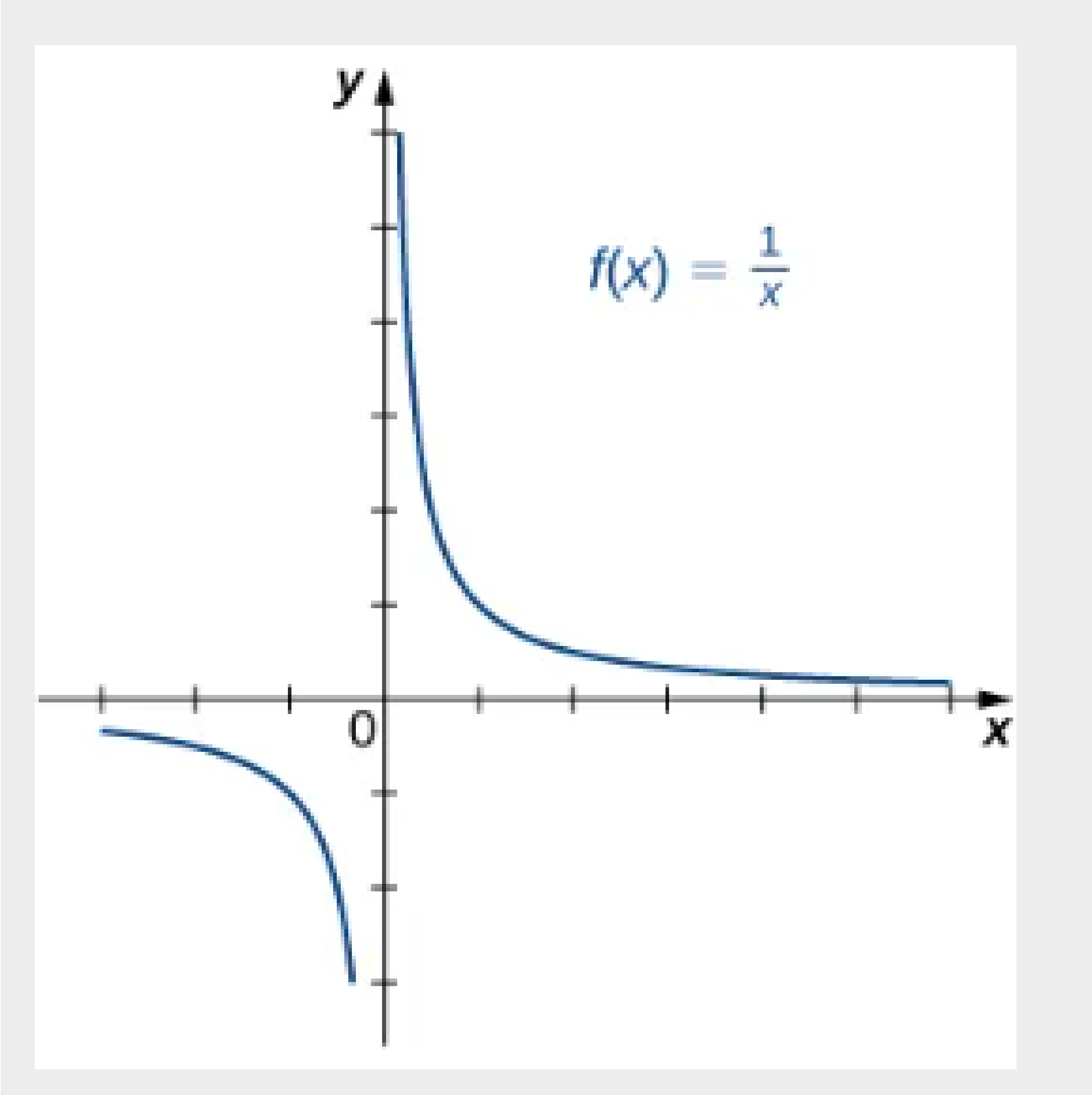
Figure 1.38 (a) The function $f(x) = x^2$ is not one-to-one because it fails the horizontal line test. (b) The function $f(x) = x^3$ is one-to-one because it passes the horizontal line test.

Determining Whether a Function Is One-to-One

For each of the following functions, use the horizontal line test to determine whether it is one-to-one.



FALSE



TRUE

PROBLEM-SOLVING STRATEGY

Finding an Inverse Function

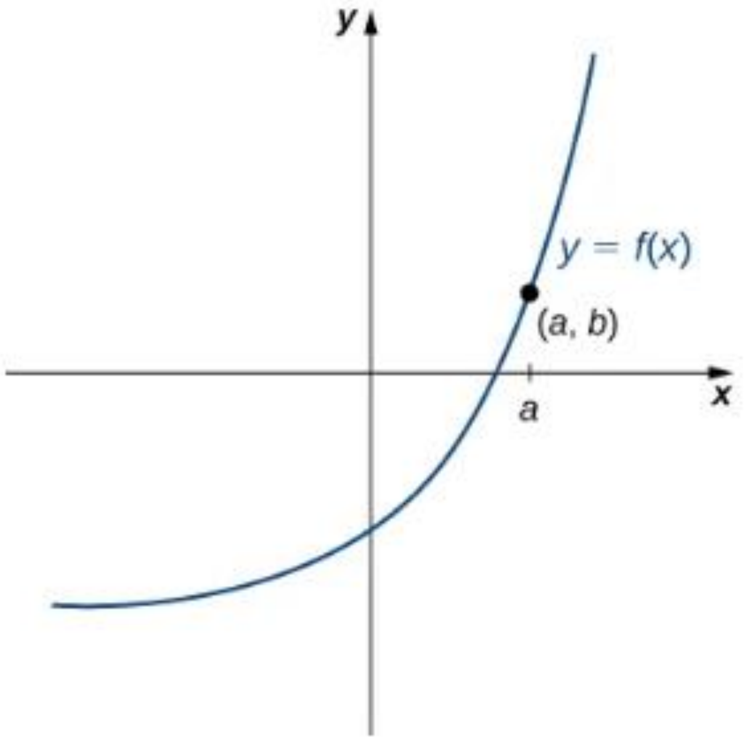
1. Solve the equation $y = f(x)$ for x.
2. Interchange the variables x and y and write $y = f^{-1}(x)$.

Find the inverse for the function $f(x) = 3x - 4$. State the domain and range of the inverse function. Verify that $f^{-1}(f(x)) = x$.

Therefore, $f^{-1}(x) = \frac{1}{3}x + \frac{4}{3}$.

Since the domain of f is $(-\infty, \infty)$, the range of f^{-1} is $(-\infty, \infty)$. Since the range of f is $(-\infty, \infty)$, the domain of f^{-1} is $(-\infty, \infty)$.

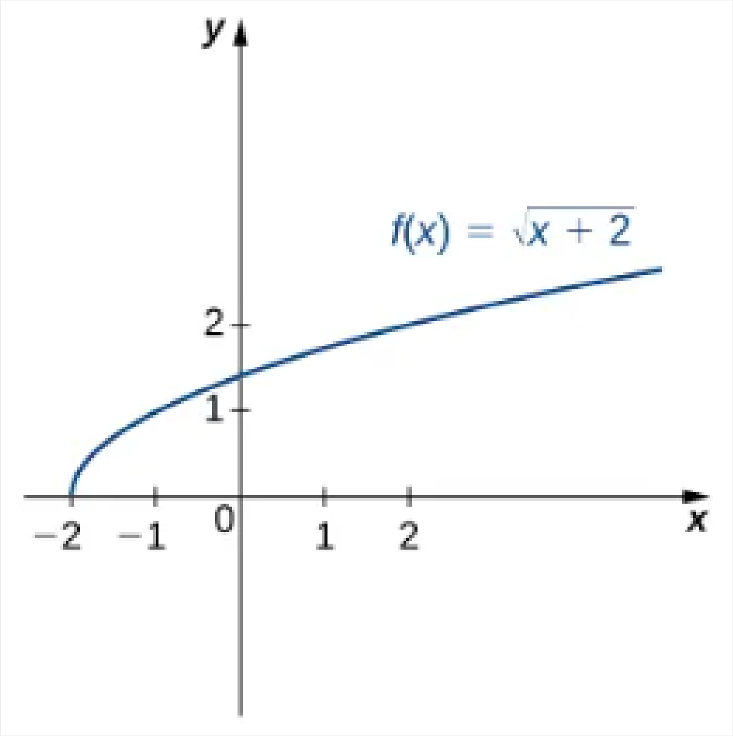
Graphing Inverse Functions



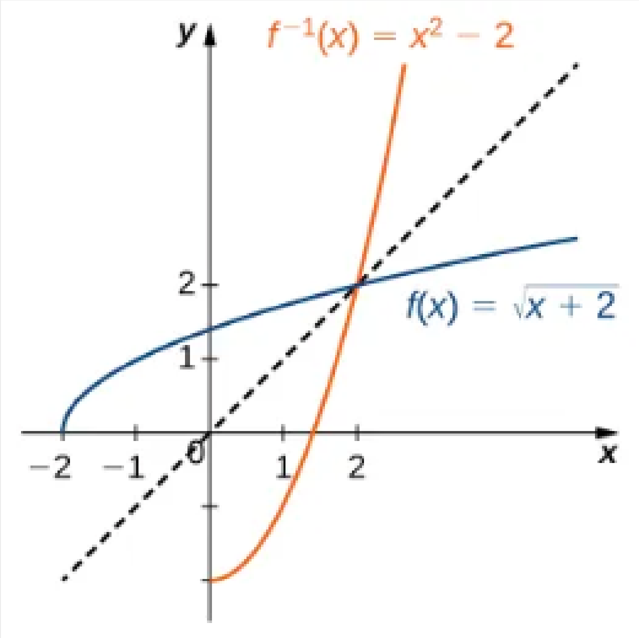
(a)

Sketching Graphs of Inverse Functions

For the graph of f in the following image, sketch a graph of f^{-1} by sketching the line $y = x$ and using symmetry. Identify the domain and range of f^{-1}.



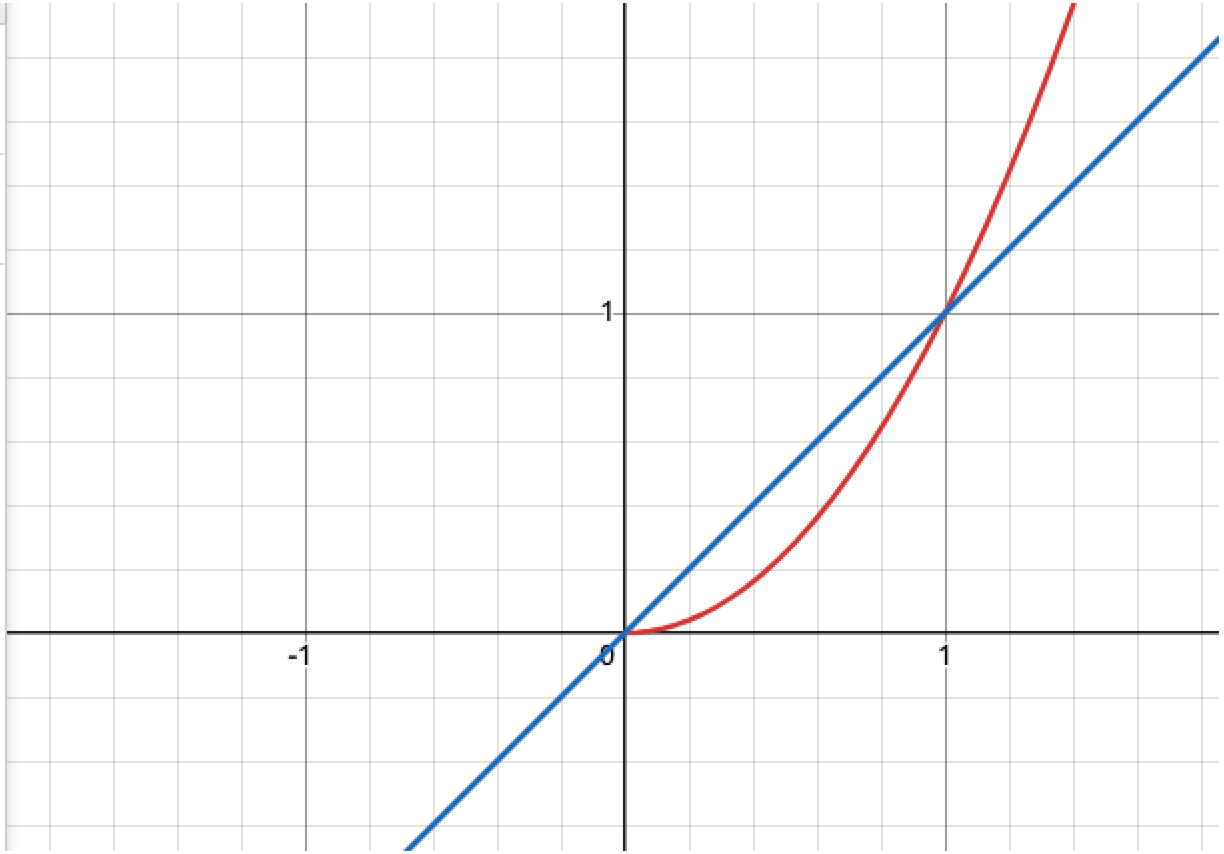
Reflect the graph about the line $y = x$. The domain of f^{-1} is $[0, \infty)$. The range of f^{-1} is $[-2, \infty)$. By using the preceding strategy for finding inverse functions, we can verify that the inverse function is $f^{-1}(x) = x^2 - 2$, as shown in the graph.



Restricting Domains. - I DO

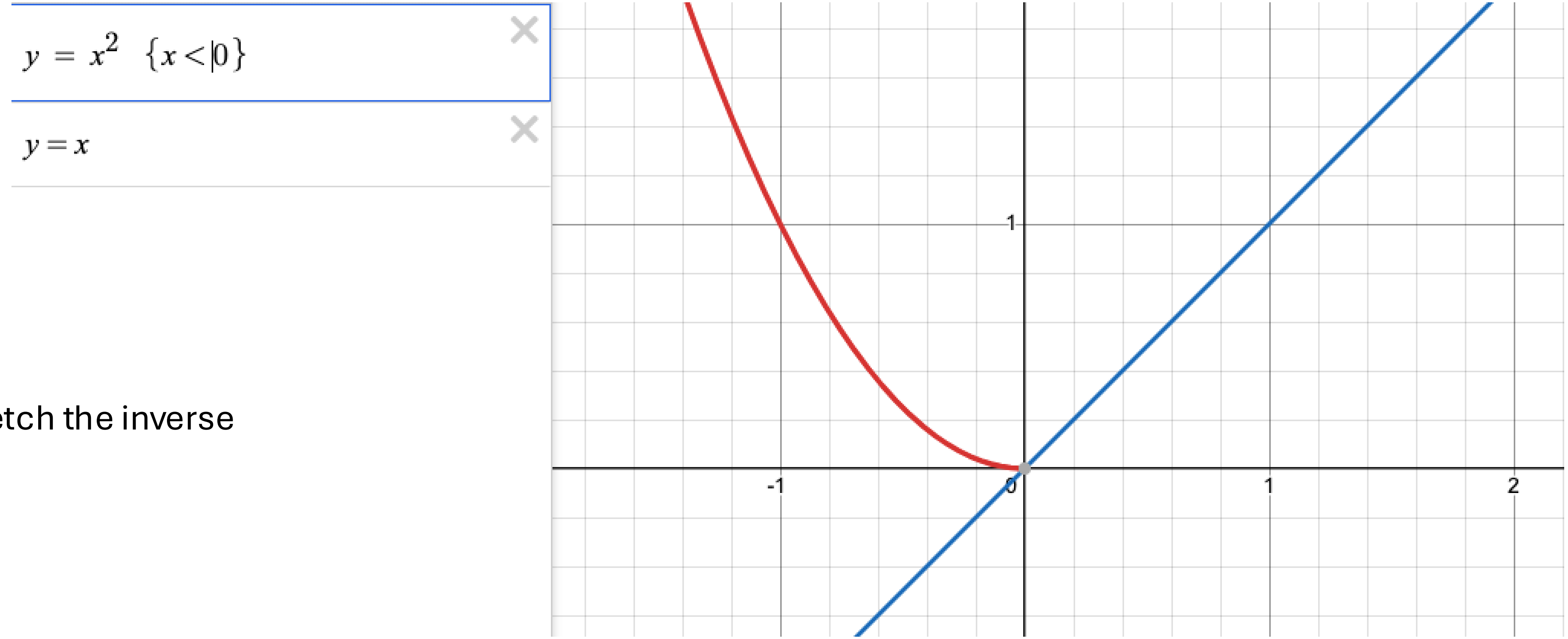
$y = x^2 \quad \{x > 0\}$

$y = x$



Sketch the inverse

Restricting Domains. - YOU DO



Sketch the inverse

Restricting Domains

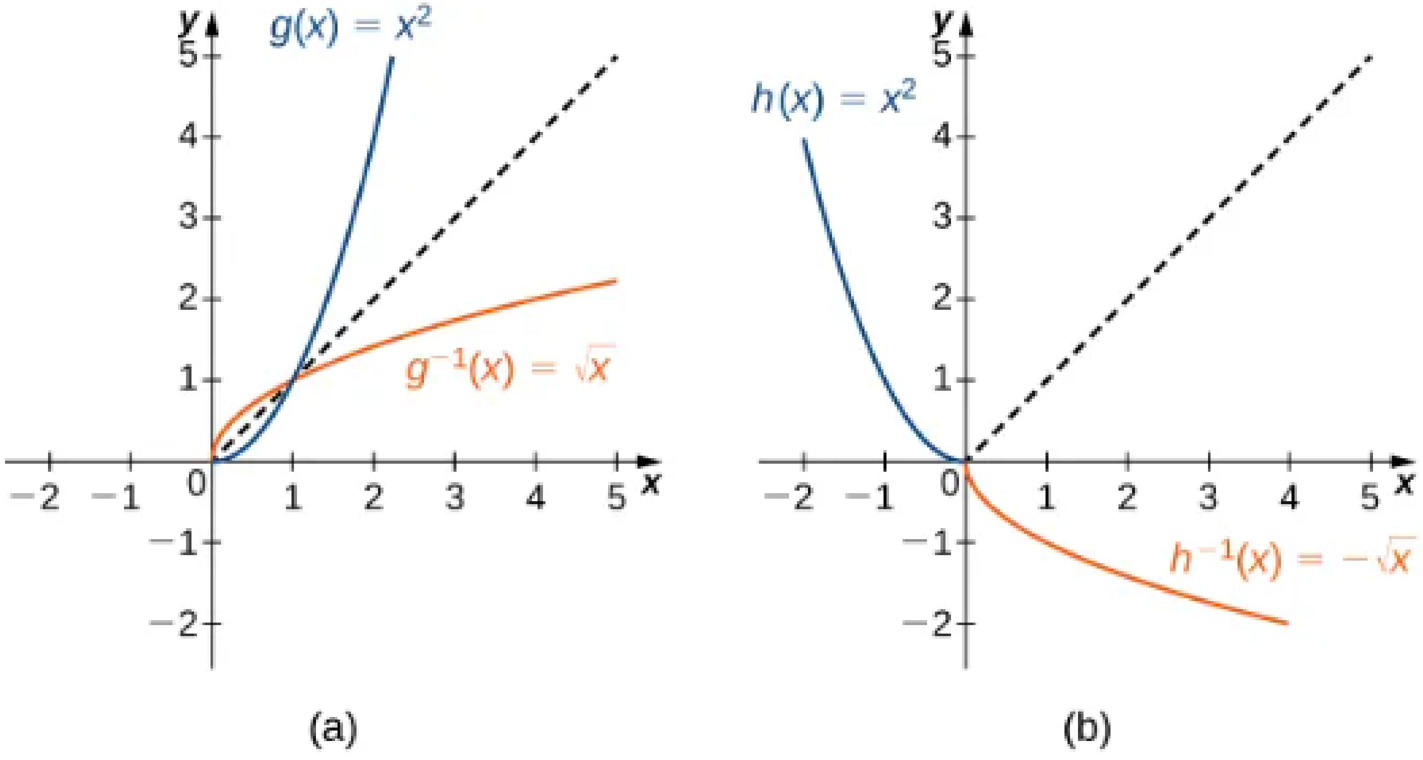
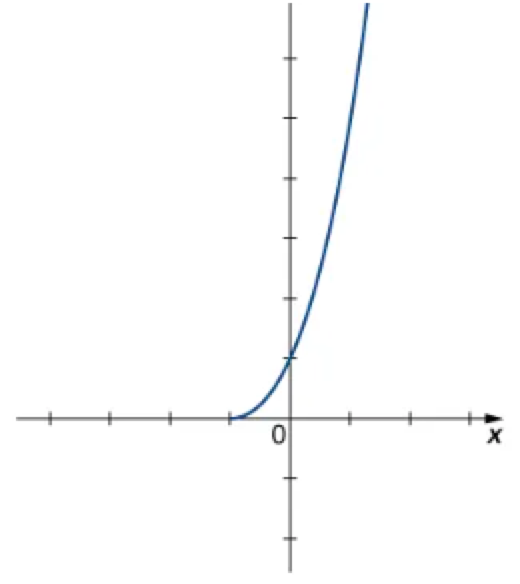


Figure 1.40 (a) For $g(x) = x^2$ restricted to $[0, \infty)$, $g^{-1}(x) = \sqrt{x}$. (b) For $h(x) = x^2$ restricted to $(-\infty, 0]$, $h^{-1}(x) = -\sqrt{x}$.

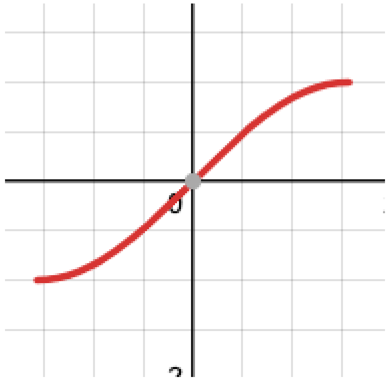
Consider the function $f(x) = (x+1)^2$.

a. Sketch the graph of f and use the horizontal line test to show that f is not one-to-one.
b. Show that f is one-to-one on the restricted domain $[-1, \infty)$. Determine the domain and range for the inverse of f on this restricted domain and find a formula for f^{-1}.

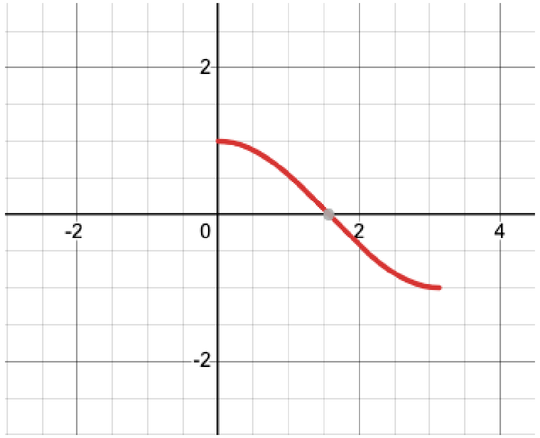


The domain and range of f^{-1} are given by the range and domain of f, respectively. Therefore, the domain of f^{-1} is $[0, \infty)$ and the range of f^{-1} is $[-1, \infty)$. To find a formula for f^{-1}, solve the equation $y = (x+1)^2$ for x. If $y = (x+1)^2$, then $x = -1 \pm \sqrt{y}$. Since we are restricting the domain to the interval where $x \geq -1$, we need $\pm\sqrt{y} \geq 0$. Therefore, $x = -1 + \sqrt{y}$. Interchanging x and y, we write $y = -1 + \sqrt{x}$ and conclude that $f^{-1}(x) = -1 + \sqrt{x}$.

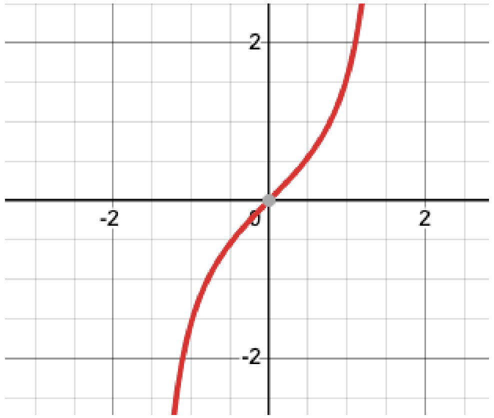
Inverse Trigonometric Functions



sin(x)



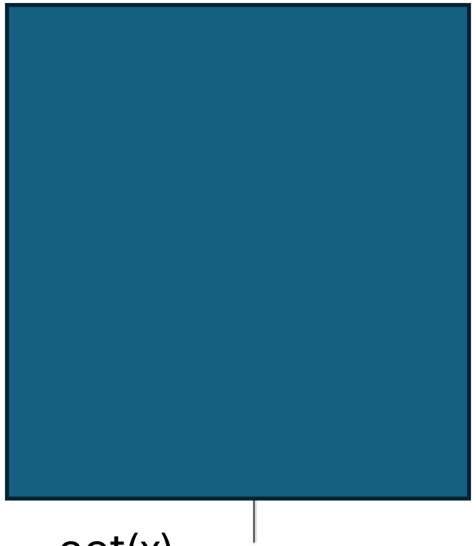
cos(x)



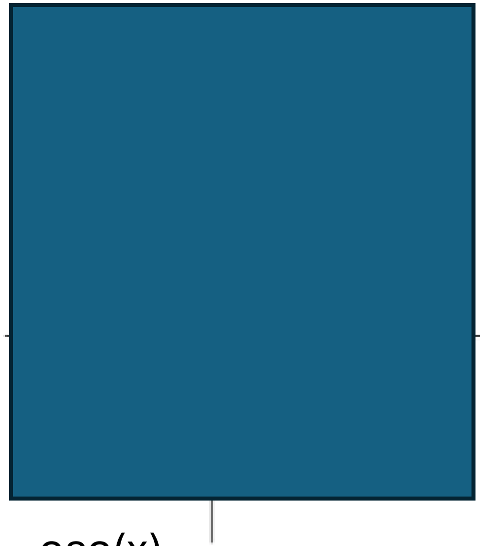
tan(x)

Inverse Trigonometric Functions

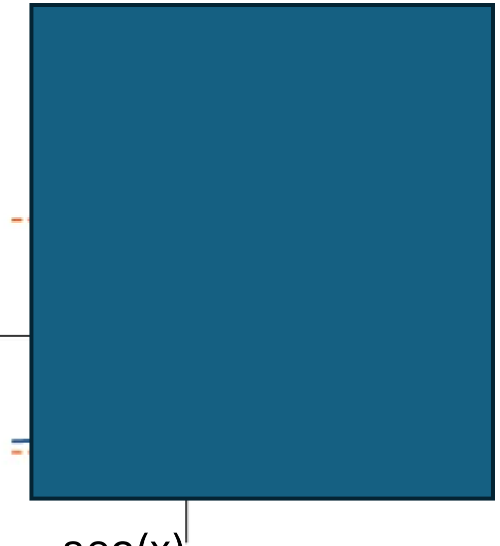
sin(x)



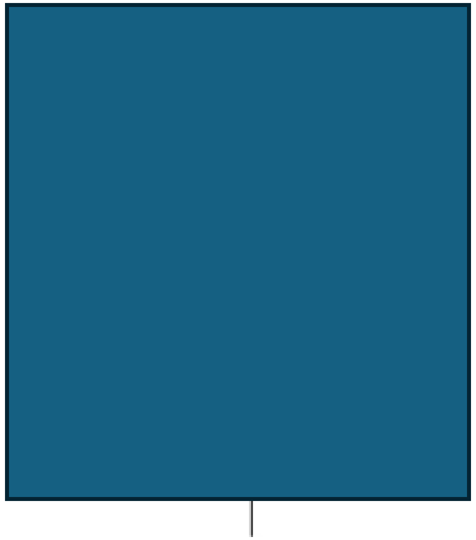
cos(x)



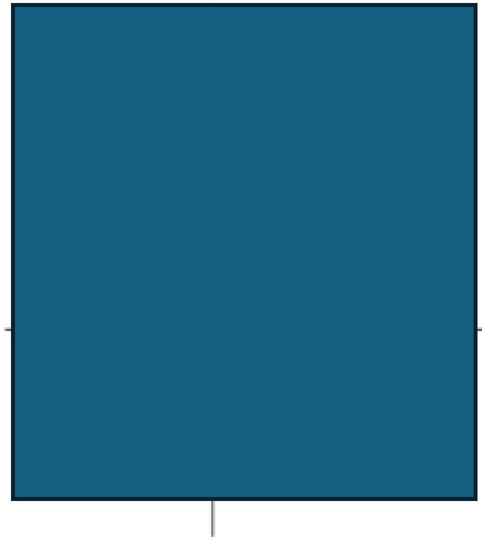
tan(x)



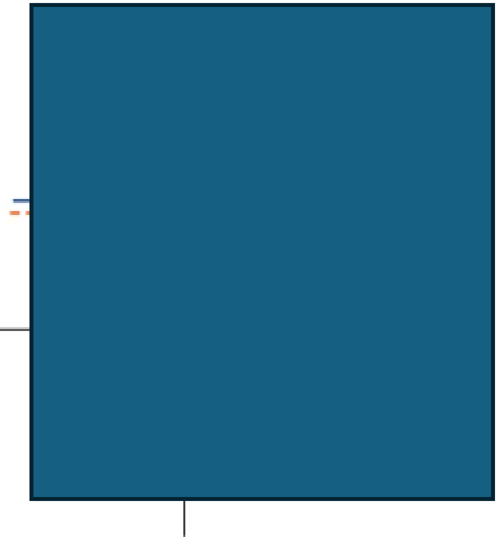
cot(x)



csc(x)



sec(x)



Evaluating Expressions Involving Inverse Trigonometric Functions

Evaluate each of the following expressions.

a. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
b. $\tan\left(\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)\right)$
c. $\cos^{-1}\left(\cos\left(\frac{5\pi}{4}\right)\right)$
d. $\sin^{-1}\left(\cos\left(\frac{2\pi}{3}\right)\right)$

Evaluating $\sin^{-1}\left(-\sqrt{3}/2\right)$ is equivalent to finding the angle θ such that $\sin\theta = -\sqrt{3}/2$ and $-\pi/2 \leq \theta \leq \pi/2$. The angle $\theta = -\pi/3$ satisfies these two conditions. Therefore, $\sin^{-1}\left(-\sqrt{3}/2\right) = -\pi/3$.

First we use the fact that $\tan^{-1}\left(-1/\sqrt{3}\right) = -\pi/6$. Then $\tan(-\pi/6) = -1/\sqrt{3}$. Therefore, $\tan\left(\tan^{-1}\left(-1/\sqrt{3}\right)\right) = -1/\sqrt{3}$.

To evaluate $\cos^{-1}(\cos(5\pi/4))$, first use the fact that $\cos(5\pi/4) = -\sqrt{2}/2$. Then we need to find the angle θ such that $\cos(\theta) = -\sqrt{2}/2$ and $0 \leq \theta \leq \pi$. Since $3\pi/4$ satisfies both these conditions, we have $\cos^{-1}(\cos(5\pi/4)) = \cos^{-1}\left(-\sqrt{2}/2\right) = 3\pi/4$.

Since $\cos(2\pi/3) = -1/2$, we need to evaluate $\sin^{-1}(-1/2)$. That is, we need to find the angle θ such that $\sin(\theta) = -1/2$ and $-\pi/2 \leq \theta \leq \pi/2$. Since $-\pi/6$ satisfies both these conditions, we can conclude that $\sin^{-1}(\cos(2\pi/3)) = \sin^{-1}(-1/2) = -\pi/6$.