PRECALCULUS · CHALLENGING REVIEW

Sets, Probability, Transformations and Modelling

Challenging review — a six-part mega-question plus a transformations refresher

6
parts, question first

2
supporting questions

8
true / false checks

Review before the mid-term. Covers sets, Venn diagrams, power sets, probability, transformations and quadratic modelling.

How this lesson runs

Key concept, worked example, questions to try — then true or false

Starter	One quick recall question on the shape of quadratic functions — attempt before the reveal.
Key points	Each section opens with the ideas you need, main point highlighted first, with formulae on their own slide.
Question	Six challenging parts appear on their own slides. Attempt each before turning the page.
Solution	The next slide gives the answer at 18pt, with a trap box naming the classic error.
True or false	The lesson ends with eight statements spanning sets, probability, transformations and modelling — one per slide, answer on the slide that follows.

Key topic

What this lesson is about, in one page

The skill	Organise a two-category survey in a Venn diagram, then read off unions, complements and 'neither'.
The probability	Read probabilities straight from the Venn region counts, including conditional probabilities.
The counting	Count subsets and power sets: 2^n possibilities, $n(n-1)$ proper subsets.
The model	Cap the survey with a quadratic enrolment model — find the year and size of the peak.

Vocabulary

Key terms with Chinese translations

union	**并集**	$A \cup B$: elements in A or B (or both)
intersection	**交集**	$A \cap B$: elements in both A and B
complement	**补集**	A': everything in the universal set but not in A
Venn diagram	**维恩图**	overlapping circles showing how sets overlap
subset	**子集**	every element of one set belongs to another
power set	**幂集**	the set of all subsets of a set, with 2^n elements
conditional probability	**条件概率**	$P(A \mid B)$: the probability of A given that B has happened
transformation	**变换**	an operation that moves or reshapes a graph
vertex form	**顶点式**	$y = a(x - h)^2 + k$, with the vertex at (h, k)
parabola	**抛物线**	the U-shaped graph of a quadratic function

S

Starter · The shape of quadratic functions

What you must remember before the modelling part

- One quick recall question
- Vertex, axis of symmetry and the role of a

Key points · the shape of a quadratic

Everything about $y = ax^2 + bx + c$ in five lines

1	**The parabola**	Every quadratic graph is a parabola. $a > 0$ opens upward; $a < 0$ opens downward.
2	**Axis of symmetry**	The mirror line $x = -b/(2a)$. The vertex sits on it.
3	**Vertex**	At $x = -b/(2a)$. It is the minimum when $a > 0$ and the maximum when $a < 0$.
4	**Roots (zeros)**	Solve $ax^2 + bx + c = 0$. The discriminant $b^2 - 4ac$ decides: two, one or no real roots.
5	**y-intercept**	The curve crosses the y-axis at $(0, c)$.

Question S1

5 marks

Starter · Shape of quadratics

THE QUESTION

For the quadratic $f(x) = -2x^2 - 8x + 10$:

(a) Does the parabola open upward or downward? How do you know?
(b) Find the equation of the axis of symmetry.
(c) Find the coordinates of the vertex.
(d) Find the y-intercept.
(e) Solve $f(x) = 0$ and state the roots.

Question S1 — solution

5 marks

Starter · Shape of quadratics

ANSWERS ONLY

- (a) $a = -2 < 0$, so it opens downward and the vertex is a maximum.
- (b) $x = -(-8)/(2\cdot(-2)) = -2$.
- (c) $f(-2) = -8 + 16 + 10 = 18$, so the vertex is $(-2, 18)$.
- (d) $(0, 10)$.
- (e) $-2x^2 - 8x + 10 = -2(x - 1)(x + 5)$, so the roots are $x = 1$ and $x = -5$.

TRAP Signs in $-b/(2a)$: with $a = -2$ and $b = -8$ the two negatives cancel then flip, $x = -2$. A sign slip here costs the vertex mark too.

01

The Challenging Mega-Question

A languages survey, a power-set twist and an enrolment model

- 200 students: 120 study French (F), 95 study Spanish (S), 55 study both
- Key points and formulae first — then parts (a) to (f), each question first

Key points · sets, probability and counting

Four ideas that unlock all six parts

1	**Inclusion–exclusion**	$n(F \cup S) = n(F) + n(S) - n(F \cap S)$. Subtract the overlap once — it was counted twice.
2	**'Neither' by complement**	$n((F \cup S)') = n(U) - n(F \cup S)$: everything outside both circles.
3	**Power sets double**	A set with n elements has 2^n subsets and $2^n - 1$ proper subsets. One more element doubles the count.
4	**Vertex form peaks**	$n(t) = a(t - p)^2 + q$ has its maximum at (p, q) when $a < 0$ — read the answer straight off.

Formulae to use

Everything you need on one page — no calculator formulae here

Inclusion–exclusion	$n(F \cup S) = n(F) + n(S) - n(F \cap S)$
Complement rule	$n(A') = n(U) - n(A)$
Conditional probability	$P(F\|S) = n(F \cap S) \,/\, n(S)$
Subsets and power set	a set with n elements has 2^n subsets, $2^n - 1$ proper subsets
Set-element count	$\{a, b, c\}$ has $n = 3 \rightarrow 2^3 = 8$ subsets, 7 proper
Vertex form	$n(t) = a(t - p)^2 + q$ — vertex (p, q), maximum when $a < 0$

Question (a)

3 marks

Languages survey · part (a)

THE QUESTION

In a school, 200 students were surveyed on language study:

- 120 study French (F)
- 95 study Spanish (S)
- 55 study both

(a) Represent the survey on a Venn diagram, and state how many students study neither language.

Question (a) — solution

3 marks

Languages survey · part (a)

ANSWERS ONLY

- F only: 120 − 55 = 65; S only: 95 − 55 = 40; both: 55
- n(F ∪ S) = 65 + 55 + 40 = 160
- Neither: 200 − 160 = 40 students

TRAP TRAP: fill the OVERLAP first (55), then subtract outwards. Writing 120 and 95 as region values double-counts the 55.

Question (b)

2 marks

Languages survey · part (b)

THE QUESTION

Still the same survey: 200 students, $n(F) = 120$, $n(S) = 95$, $n(F \cap S) = 55$.

(b) Find $n(F')$ and $n((F \cup S)')$.

Question (b) — solution

2 marks

Languages survey · part (b)

ANSWERS ONLY

- $n(F') = 200 - 120 = 80$
- $n((F \cup S)') = 200 - 160 = 40$

TRAP TRAP: $(F \cup S)'$ is 'neither', not 'not French'. The complement applies to the WHOLE union, so subtract the union 160, not 120.

Question (c)

3 marks

Languages survey · part (c)

THE QUESTION

Still the same survey: 200 students, n(F) = 120, n(S) = 95, n(F ∩ S) = 55.

(c) A student is chosen at random. Find:

(i) P(studies exactly one of the two languages)

(ii) P(F | S), correct to 3 decimal places

Question (c) — solution

3 marks

Languages survey · part (c)

ANSWERS ONLY

- (i) exactly one = 65 + 40 = 105, so P = 105/200 = 0.525
- (ii) P(F | S) = 55/95 = 11/19 ≈ 0.579
- (The condition shrinks the sample space to the 95 Spanish students.)

TRAP TRAP: P(F | S) divides by n(S) = 95, NOT by 200. The 'given' re-definitions the denominator — this is the most-missed mark in the topic.

Question (d)

3 marks

Power sets · part (d)

THE QUESTION

The survey team now studies the set $A = \{a, b, c\}$.

(d) Find the number of subsets of A, and the number of proper subsets of A.

Question (d) — solution

3 marks

Power sets · part (d)

ANSWERS ONLY

- Number of subsets: $2^3 = 8$
- Number of proper subsets: $2^3 - 1 = 7$ (every subset except A itself)
- (The 8 subsets: ∅, {a}, {b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c}.)

TRAP TRAP: the improper subset A itself is not 'proper'. Forgetting to subtract it turns 7 into 8 — and n elements into $2^n - 1$, not 2^n.

Question (e)

3 marks

Power sets · part (e)

THE QUESTION

Still the set A = {a, b, c}, with $2^3 = 8$ subsets.

(e) A fourth element d is added to form B = {a, b, c, d}. By how many times does the number of subsets increase? State n(B) and the new number of subsets.

Question (e) — solution

3 marks

Power sets · part (e)

ANSWERS ONLY

- Adding one element doubles the subset count: $2^4 = 16$
- n(B) = 4, so n(P(B)) = 2^4 = 16 subsets
- Increase: 16 ÷ 8 = 2 — each new element can be 'in' or 'out' of a subset, so the possibilities double.

TRAP TRAP: the doubling is the POINT. The new subsets are the old 8 plus each old subset with d added — a fresh 'in/out' choice for d.

Question (f)

4 marks

Enrolment model · part (f)

THE QUESTION

To finish, enrolment in the languages programme is modelled by

$$n(t) = -2t^2 + 40t + 60,$$

where n is the number of students and t the number of years since the programme began.

(f) Find the year in which enrolment is at its maximum, and state that maximum.

Question (f) — solution

4 marks

Enrolment model · part (f)

ANSWERS ONLY

- a = −2 < 0, so the vertex is a maximum
- t = −40/(2 × (−2)) = 10
- n(10) = −2(100) + 400 + 60 = 260
- Maximum enrolment: 260 students, in year 10

TRAP TRAP: the vertex t-value is the YEAR of the peak, not the peak itself — quote both n(10) = 260 and t = 10. A sign slip in −b/(2a) sends you to year −10.

02

Supporting Practice

Two shorter questions to secure the method

- T1: the thrown ball — a full transformations revision question
- P2: a second survey — conditional probability on new numbers

Question T1

5 marks

Supporting practice · transformations

THE QUESTION

A ball is thrown. Its height, in metres, after t seconds is

$h(t) = -5(t - 1.5)^2 + 16.25$.

(a) Starting from $f(t) = t^2$, state the sequence of transformations that produces the graph of h.
(b) State the maximum height of the ball and the time at which it is reached.
(c) Find, exactly and to 2 decimal places, the time at which the ball hits the ground.
(d) State the domain and range of h in the context of the model.

Question T1 — solution

5 marks

Supporting practice · transformations

ANSWERS ONLY

- (a) Translate 1.5 RIGHT · reflect in the t-axis and stretch by 5 (the −5) · translate 16.25 UP
- (b) Maximum 16.25 m at t = 1.5 s (the vertex, a < 0)
- (c) $-5(t-1.5)^2 + 16.25 = 0 \rightarrow t = 1.5 + \sqrt{3.25} = (3+\sqrt{13})/2 \approx 3.30$ s
- (d) Domain $0 \le t \le (3+\sqrt{13})/2$; range $0 \le h \le 16.25$

TRAP TRAP: $1.5 - \sqrt{3.25} \approx -0.30$ s is BEFORE launch — reject it. And quote the flight domain, never 'all reals', for a model.

Question P2

4 marks

Supporting practice · survey probability

THE QUESTION

Of 60 members at a sports club, 33 play cricket (C), 27 play rugby (R) and 12 play both.

(a) Draw the Venn diagram and state how many play neither sport.
(b) A member is chosen at random. Find P(C | R) as a fraction in lowest terms.
(c) Find the probability the member plays neither sport.

Question P2 — solution

4 marks

Supporting practice · survey probability

ANSWERS ONLY

- (a) C only: 21, both: 12, R only: 15, neither: 60 − 48 = 12
- (b) P(C|R) = 12/27 = 4/9
- (c) P(neither) = 12/60 = 1/5

TRAP TRAP: P(C|R) divides by 27, not 60. And the Venn regions 21/12/15 must each be used for their own probability questions — never the raw totals.

Mark scheme

Marks by question, with what earns each mark

S1	**5**	1 per part: direction, axis, vertex, intercept, roots
(a)	**3**	1 Venn regions 65/55/40 · 1 union 160 · 1 neither = 40
(b)	**2**	1 $n(F')$ = 80 · 1 $n((F \cup S)')$ = 40 via the union
(c)	**3**	1 exactly-one count 105 · 1 P = 0.525 · 1 $P(F \mid S)$ = 55/95 ≈ 0.579
(d)	**3**	1 2^3 = 8 subsets · 1 proper = 7 · 1 list or justification
(e)	**3**	1 doubling stated · 1 2^4 = 16 · 1 n(B) = 4 with reason
(f)	**4**	1 $a < 0$ maximum · 1 t = 10 · 1 n(10) = 260 · 1 both year and size quoted
T1	**5**	1 transformations · 1 max 16.25 m at 1.5 s · 1 exact landing $(3+\sqrt{13})/2$ · 1 ≈ 3.30 s · 1 domain and range in context
P2	**4**	1 Venn regions · 1 neither 12 · 1 $P(C \mid R)$ = 4/9 · 1 P(neither) = 1/5

Main question total: 18 marks. Supporting practice: 9 marks. Starter: 5 practice marks (not assessed).

Answer key

One line per question

ANSWERS

- S1 down; axis $x = -2$; vertex $(-2, 18)$; $(0, 10)$; roots 1 and -5
- (a) regions 65 / 55 / 40; $n(F \cup S) = 160$; neither = 40
- (b) $n(F') = 80$; $n((F \cup S)') = 40$
- (c) P(exactly one) = 105/200 = 0.525; $P(F|S) = 55/95 = 11/19 \approx 0.579$
- (d) 8 subsets; 7 proper subsets
- (e) doubles to $2^4 = 16$ subsets; $n(B) = 4$

QUICK VERIFICATION

- (f) maximum 260 students in year 10 (vertex (10, 260))
- T1 right 1.5 · reflect + stretch 5 · up 16.25; max 16.25 m at 1.5 s;
- $t = (3+\sqrt{13})/2 \approx 3.30$ s; domain $0 \le t \le (3+\sqrt{13})/2$, range $0 \le h \le 16.25$
- P2 regions 21/12/15, neither 12; $P(C|R) = 4/9$; P(neither) = 1/5
- Check (a): 65 + 55 + 40 + 40 = 200 ✓ every student accounted for
- Check (c): 105/200 = 21/40 ✓ and 55/95 = 11/19 ✓
- Check (f): $-b/(2a) = -40/(-4) = 10$ ✓ and $n(10) = 260$ ✓

True or false · 1

T / F

Decide, then justify your decision — answer on the next slide

$n(A \cup B) = n(A) + n(B)$ for any two finite sets A and B.

True or false · 1 — answer

FALSE

FALSE

Only when A and B do not overlap. The overlap is counted twice, so inclusion–exclusion subtracts n(A ∩ B) — in part (a), 120 + 95 would have over-counted.

True or false · 2

T / F

Decide, then justify your decision — answer on the next slide

If n(U) = 30, n(A) = 18 and n(B) = 15 with n(A ∩ B) = 7, then 4 elements are in neither set.

True or false · 2 — answer

TRUE

TRUE

n(A ∪ B) = 18 + 15 − 7 = 26, so neither = 30 − 26 = 4. Exactly the 'neither' method from part (a).

True or false · 3

T / F

Decide, then justify your decision — answer on the next slide

A set with n elements has $2^n - 1$ subsets.

True or false · 3 — answer

FALSE

FALSE

It has 2^n subsets and $2^n - 1$ PROPER subsets — the whole set itself is a subset but not a proper one. Part (d): 8 subsets, 7 proper.

True or false · 4

T / F

Decide, then justify your decision — answer on the next slide

$P(A \mid B) = n(A \cap B)/n(A)$, dividing by the size of A.

True or false · 4 — answer

FALSE

FALSE

The 'given' event defines the denominator: $P(A|B) = n(A \cap B)/n(B)$. Part (c) divided by 95, the Spanish count — not by 200.

True or false · 5

T / F

Decide, then justify your decision — answer on the next slide

The graph of $-f(x)$ is the reflection of the graph of f in the x-axis.

True or false · 5 — answer

TRUE

TRUE

Yes — every output flips sign. In T1, the −5 both reflected the parabola and stretched it by a factor of 5.

True or false · 6

T / F

Decide, then justify your decision — answer on the next slide

Replacing x by x − c in f(x) shifts the graph LEFT by c.

True or false · 6 — answer

FALSE

FALSE

Inside changes oppose the sign: f(x − c) shifts RIGHT by c. In T1, (t − 1.5) put the vertex at t = +1.5.

True or false · 7

T / F

Decide, then justify your decision — answer on the next slide

The vertex of $y = a(x - h)^2 + k$ is the point (h, k), and it is a maximum when $a < 0$.

True or false · 7 — answer

TRUE

TRUE

Exactly. The bracket is zero at $x = h$, giving $y = k$ — and in part (f) the vertex $(10, 260)$ was the enrolment peak because $a = -2 < 0$.

True or false · 8

T / F

Decide, then justify your decision — answer on the next slide

The domain of $h(t) = -5(t - 1.5)^2 + 16.25$, as a model of a thrown ball, is all real numbers.

True or false · 8 — answer

FALSE

FALSE

In context the flight runs from launch to landing: $0 \le t \le (3 + \sqrt{13})/2 \approx 3.30$. All reals is the mathematical domain, not the model's.

FIVE THINGS TO REMEMBER

Do these and sets, probability, transformations and modelling will not cost you marks

- Fill the Venn OVERLAP first, then subtract outwards — and check the regions sum to n(U).
- 'Neither' is the complement of the union: subtract n(A ∪ B) from n(U), never just n(A).
- A set with n elements has 2^n subsets and $2^n - 1$ proper subsets; one extra element doubles the count.
- P(A|B) divides by n(B) — the 'given' event re-definitions the denominator.
- Inside shifts oppose their sign and −f(x) reflects in the x-axis; the vertex of $a(t - p)^2 + q$ is the peak when $a < 0$.

Matching worksheet: Sets, Probability and Modelling Review Worksheet — 18 marks, 55 minutes.