Two independent random samples are taken from two populations. Which condition is **necessary** for constructing a confidence interval for $\mu_1 - \mu_2$?

A. The two samples must be the same size

B. The populations must have the same mean

C. The samples must be independent

D. The population standard deviations must be known

🔑 Difference in Sample Means — z vs t (Concise)

Use z (Normal):

- Population standard deviations **known** (σ_1, σ_2)
- Normal populations or large samples

Use t (MOST COMMON):

- Population standard deviations **unknown** → use s_1, s_2
- Normal populations or large samples

⚡ Exam Shortcut

👉 **Use t unless σ is given**

⚠️ Small Samples

If $n < 30$:

- No strong skew / no outliers required

🧠 One-line takeaway

👉 **Unknown σ → t (almost always)**

AP Statistics Decision-Making Exercise

Instructions: For each situation, decide whether you should use a **Normal (z) distribution** or a **t distribution**. Give a brief reason.

Questions

1. A random sample of 50 students is taken. The population standard deviation is known to be 12. You want to estimate the population mean.

2. A random sample of 18 students is taken. The population standard deviation is **unknown**, and the data are approximately normal. You want to test a claim about the population mean.

3. A random sample of 100 adults is taken. The population standard deviation is unknown. The sample standard deviation is 15. You want to construct a confidence interval for the population mean.

4. A population has a known standard deviation of 8. A random sample of 25 observations is taken. You want to test a claim about the population mean.

5. A random sample of 12 students is taken from a strongly skewed population. The population standard deviation is unknown. You want to make an inference about the population mean.

6. Two independent random samples are taken:

- Group A: $n = 30,\ s = 10$
- Group B: $n = 35,\ s = 12$

The population standard deviations are unknown. You want to compare the two population means.

7. Challenge: A random sample of 15 observations comes from a population that is approximately normal. The population standard deviation is unknown. Which distribution should be used, and what additional information would you check before proceeding?

Stretch and Challenge

Stretch & Challenge

Question 1 – How Large Can the Difference Be?

Two independent populations have:

- Population A: $\mu_A = 80,\ \sigma_A = 12,\ n_A = 36$
- Population B: $\mu_B = 75,\ \sigma_B = 15,\ n_B = 49$

a) Find the mean and standard deviation of the sampling distribution of $\bar{X}_A - \bar{X}_B$.

b) Find the probability that the difference between the two sample means is **greater than 8**.

c) Explain what your answer in part (b) means in context.

Question 2 – Detecting a Difference

The average time students spend completing a mathematics test is 52 minutes at School A and 48 minutes at School B. The population standard deviations are 8 minutes and 10 minutes, respectively.

Independent random samples of 64 students from School A and 100 students from School B are selected.

a) Describe the shape, centre, and spread of the sampling distribution of $\bar{X}_A - \bar{X}_B$.

b) Calculate the probability that $\bar{X}_A - \bar{X}_B > 6$.

c) The researcher obtains a difference of 6.5 minutes. Explain whether this result would be unusual based on the sampling distribution.

A nutritionist wants to compare the average daily calorie intake of adults in two cities, City A and City B.

- A random sample of 40 adults from City A has a mean of 2,400 calories with a standard deviation of 300 calories.
- A random sample of 35 adults from City B has a mean of 2,300 calories with a standard deviation of 250 calories.

Assume the conditions for inference are satisfied.

(a) Define the parameter of interest.

(b) State the null and alternative hypotheses to test whether the mean daily calorie intake differs between the two cities.

(c) Identify the appropriate test and justify your choice.

(d) Calculate the standard error of the difference in sample means.

(e) Write the formula for the test statistic (do not calculate the final t-value).

(f) Explain the conditions required for using this test.

Stretch and Challenge

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Table B *t* **distribution critical values**

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

🔑 Experimental Design & Causation — Key Points (Concise)

- **Only experiments can show causation（只有实验能说明因果关系）**
- **Random assignment（随机分配）** → balances confounding variables
- **Control group（对照组）** → provides comparison
- **Placebo & blinding（安慰剂与盲法）** → reduce bias
- **Confounding variables（混杂变量）** → must be controlled
- **Well-designed experiment = control + randomization + replication（控制+随机+重复）**
- **Observational studies（观察性研究）** → show association, NOT causation
- **Large sample size（大样本）** → more reliable results

⚡ One-line takeaway

👉 **Causation requires random assignment and control**

A school wants to test whether a new study app improves students' test scores.
They select 80 students and allow them to **choose** whether to use the app or not. After one month, students who used the app scored higher on average.

Questions

(a) Is this an experiment or an observational study? Explain.

(b) Identify a potential **confounding variable**.

(c) Can the school conclude that the app **causes** higher scores? Explain.

(d) Describe how you would redesign the study to allow for a **causal conclusion**.