UNIT 1 KNOWLEDGE – CALCULUS 12 – LIMITS AND CONTINUITY

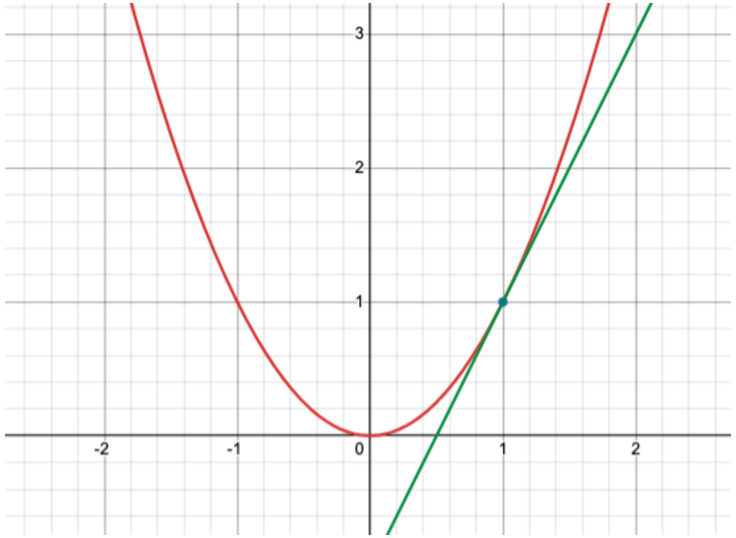
- 1.1 • Introducing Calculus: Can Change Occur at an Instant? ✓
- 1.2 • Defining Limits and Using Limit Notation ✓
- 1.3 • Estimating Limit Values from Graphs ✓
- 1.4 • Estimating Limit Values from Tables ✓
- 1.5 • Determining Limits Using Algebraic Properties of Limits ✓
- 1.6 • Determining Limits Using Algebraic Manipulation ✓
- 1.7 • Selecting Procedures for Determining Limits ✓
- 1.8 • Determining Limits Using the Squeeze Theorem ✓
- 1.9 • Connecting Multiple Representations of Limits ✓
- 1.10 • Exploring Types of Discontinuities ✓
- 1.11 • Defining Continuity at a Point ✓
- 1.12 • Confirming Continuity over an Interval ✓
- 1.13 • Removing Discontinuities ✓
- 1.14 • Connecting Infinite Limits and Vertical Asymptotes ✓
- 1.15 • Connecting Limits at Infinity and Horizontal Asymptotes ✓
- 1.16 • Working with the Intermediate Value Theorem (IVT)

Turn and Talk

Use first principles to find the gradient function of

$$f(x) = x^2$$

Then find the gradient at $x = 1$.



We use first principles:

$\lim_{h\to 0} \frac{(x+h)^2-x^2}{h}$

$$= \frac{x^2+2xh+h^2-x^2}{h} = \frac{2xh+h^2}{h} = 2x+h$$

As $h \to 0$:

Gradient function = $2x$

📍 **At $x = 1$:**

$$2(1) = 2$$

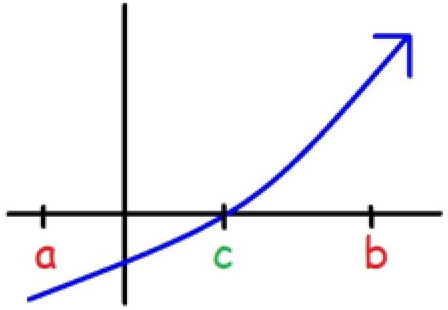
1. Continuous function – 连续函数
2. Closed interval [a, b] – 闭区间 [a, b]
3. Value between f(a) and f(b) – f(a) 和 f(b) 之间的值
4. Existence of a root – 根的存在
5. Zero / root – 零点 / 根
6. Function value – 函数值
7. Intermediate value – 中间值
8. Guaranteed – 保证
9. Assumption / condition – 假设 / 条件
10. Sign change – 符号变化

What Will We Learn?

- What does the Intermediate Value Theorem really mean?

- When can we apply the Intermediate Value Theorem? What are the conditions?

- How can we explain the behavior of a function on an interval using the Intermediate Value Theorem



$$f(a) < K < f(b)$$

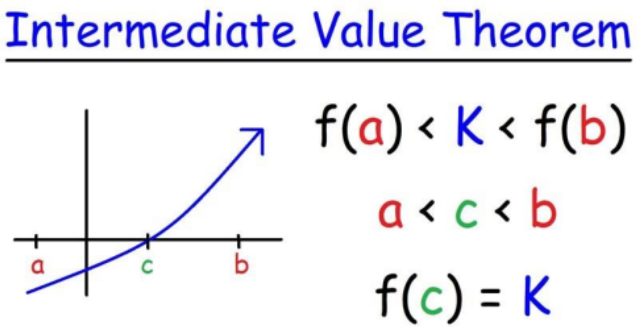
$$a < c < b$$

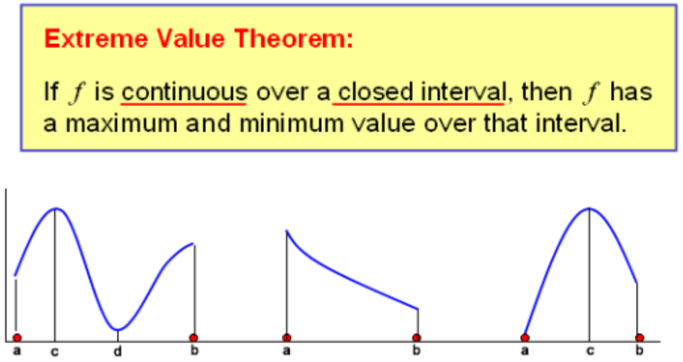
$$f(c) = K$$

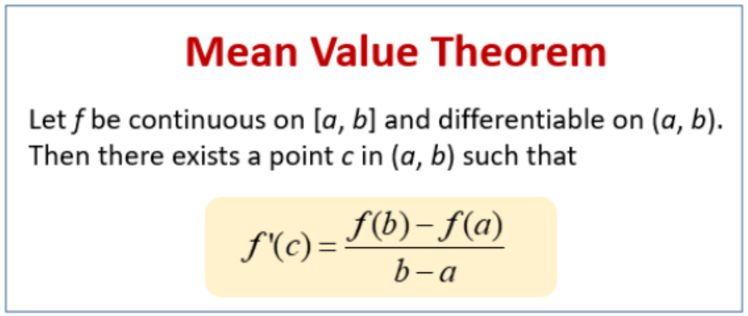
Main Existence Theorems in Calculus

- There are three main existence theorems in calculus: **the intermediate value theorem (unit 1), the extreme value theorem (unit 5), and the mean value theorem (unit 5).**

- They all guarantee the existence of a point on the graph of a function that has certain features, which is why they are called this way.

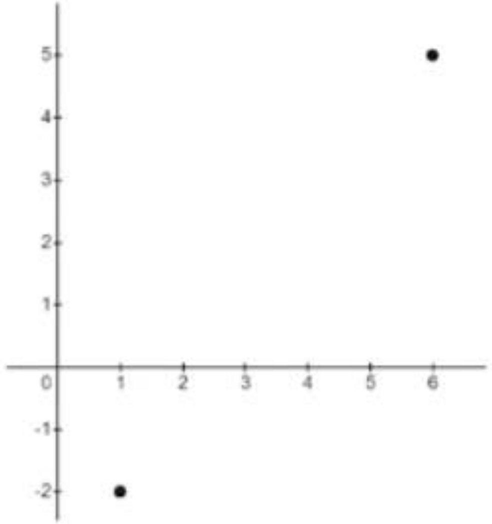


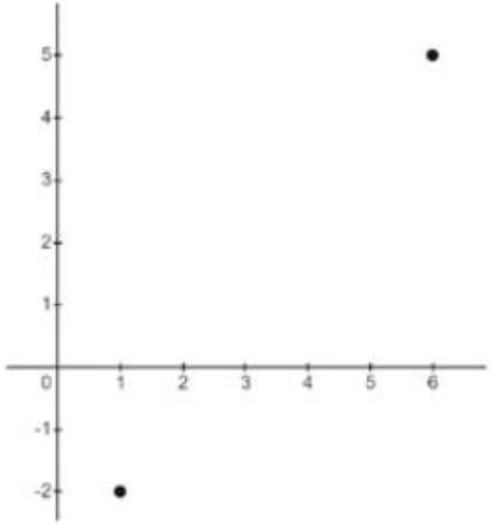




Working with the Intermediate Value Theorem

Sketch a possible graph of $f(x)$ where $f(1) = -2\ and\ f(6) = 5.$





Working with the Intermediate Value Theorem

Sketch a possible graph of $f(x)$

where $f(1) = -2$, $f(6) = 5$, $f(x)$ *is* ***continuous*** *on* $[1,6]$.

Working with the Intermediate Value Theorem

Could you sketch a possible graph of $f(x)$

where $f(1) = -2, \quad f(6) = 5,$

$f(x)$ *is continuous on* $[1,6]$ *and* **NEVER LET let the function = 0**?

Working with the Intermediate Value Theorem

Could you sketch a possible graph of $f(x)$

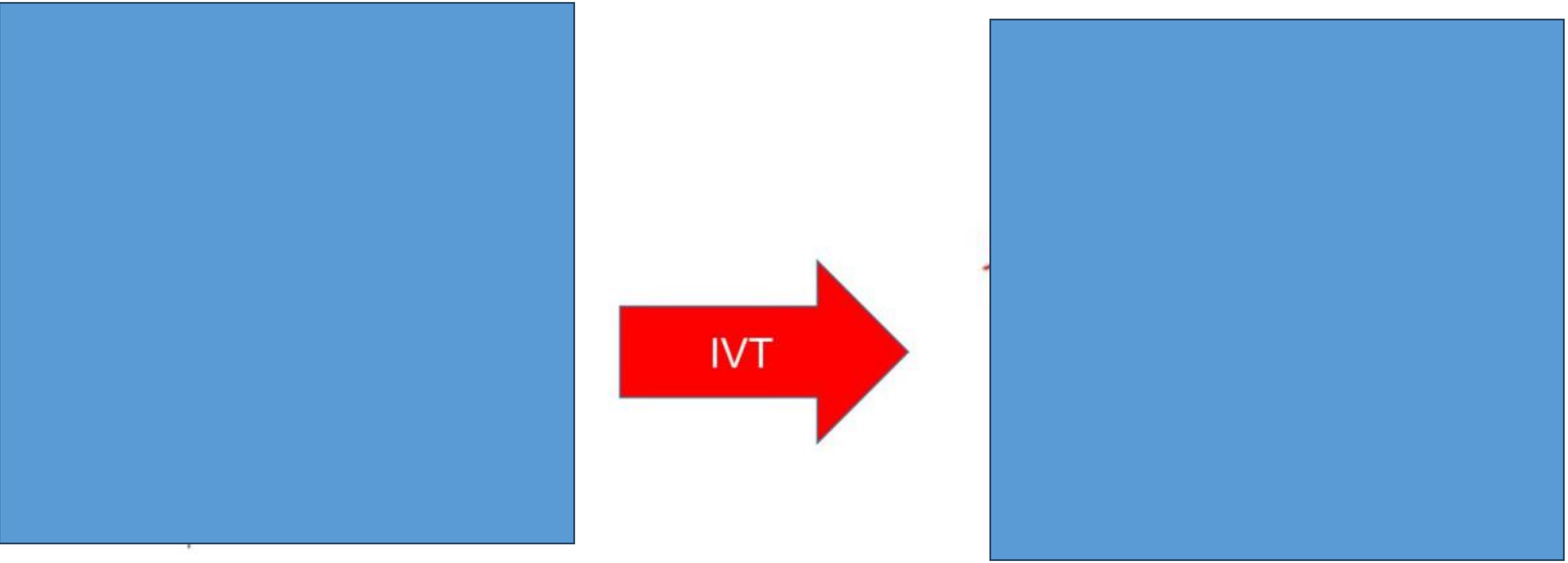
where $f(1) = -2, \quad f(6) = 5, \ f(x)$ *is continuous on* $[1,6]$ *and* ***never let the function = 0***?

It's IMPOSSIBLE!

- To have a continuous function in the interval, ***we have to pass through 0***.
- Every single no. between -2 and 5 MUST BE taken on by the function if continuous.
- That's exactly what the intermediate value theorem (IVT) is all about.

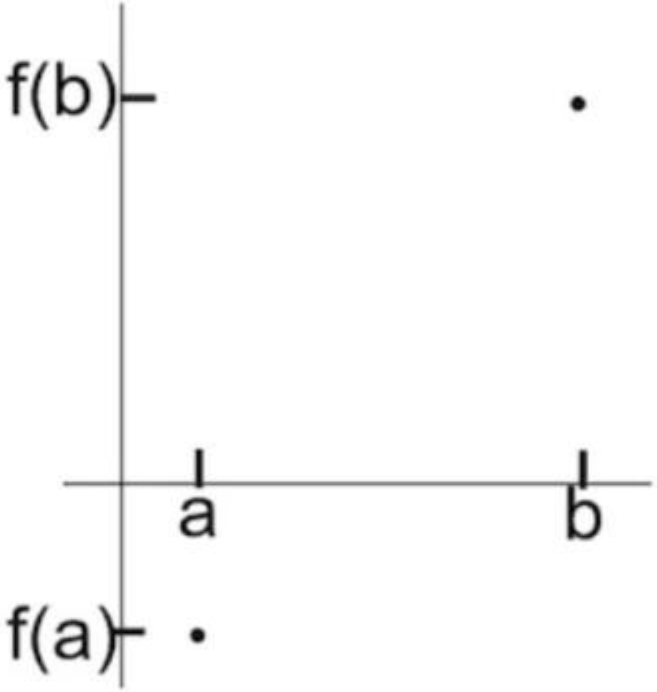
Working with the Intermediate Value Theorem

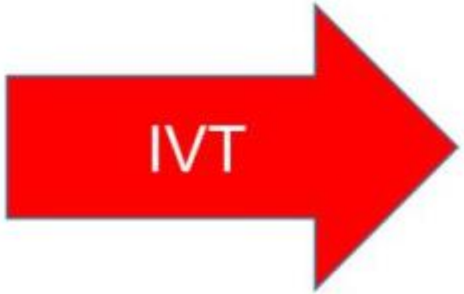
If f is a continuous function on the closed interval $[a, b]$ and d is a number between $f(a)$ and $f(b)$, then the Intermediate Value Theorem guarantees that there is at least one number c between a and b, such that $f(c) = d$.

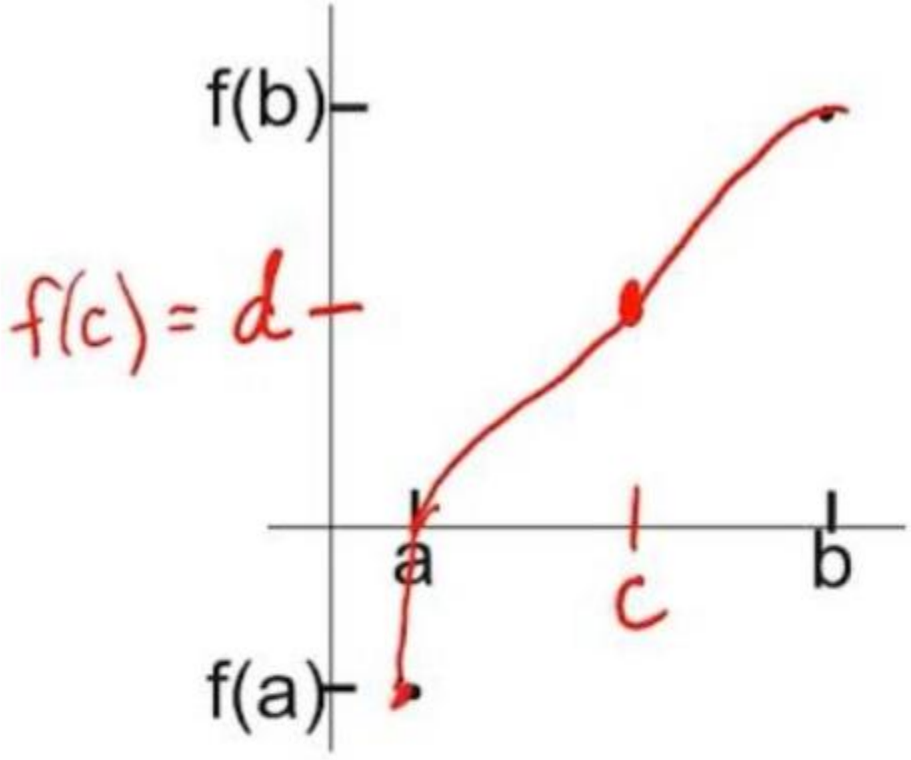


Working with the Intermediate Value Theorem

In another words, if the function is ***continuous*** from a *to* b, there is ***NO WAY*** the function can get from $f(a)$ to $f(b)$ **without** taking on ***every y-value*** between them ***at least once***.







Let’s see an example!

Working with the Intermediate Value Theorem

Let f be a function that is ***continuous*** on the closed interval [-4,3] with $f(-4) = 7$ and $f(3) = -1$. Is there a number k between -4 and 3 where f is guaranteed to have a zero?

Summary

- The Intermediate Value Theorem can be applied if the condition of continuity on a closed interval is known.
- This theorem is often required when asked if a specific function value is guaranteed to exist on an open interval.
- There are three main existence theorems in calculus: **the intermediate value theorem, the extreme value theorem (unit 5), and the mean value theorem (unit 5).**
- They all guarantee the existence of a point on the graph of a function that has certain features, which is why they are called this way.

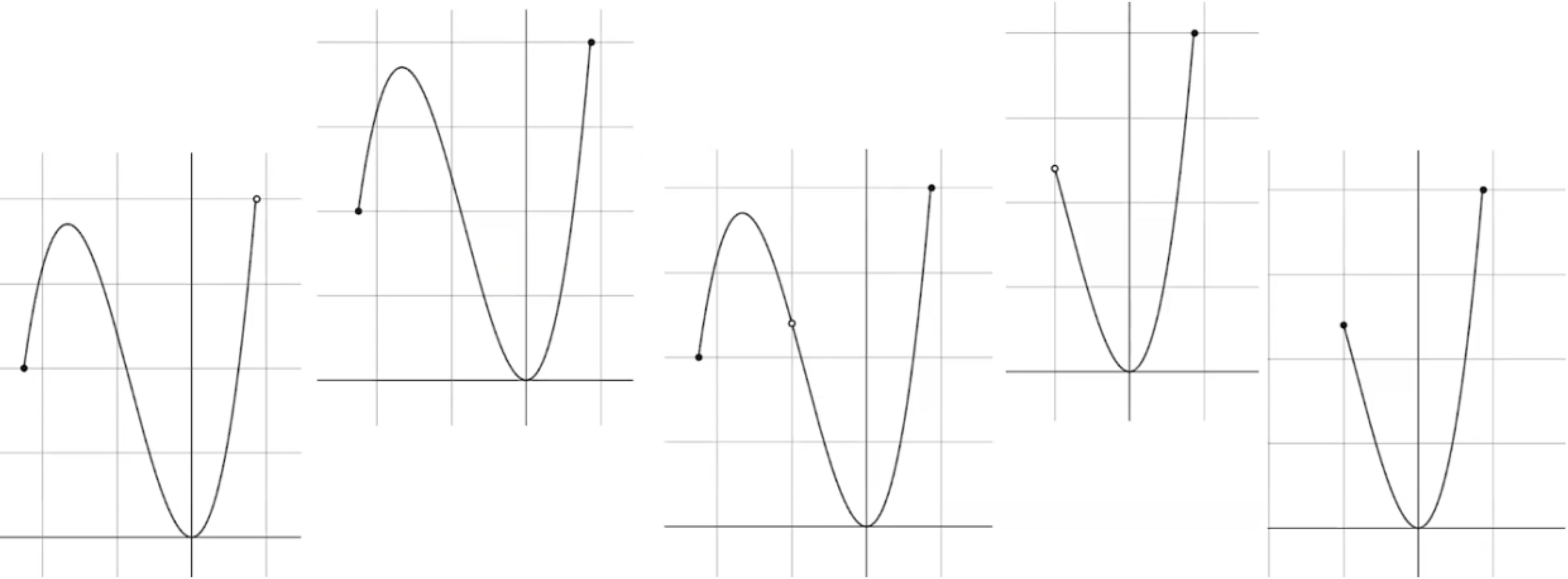
Next-> What Will We Learn?

- We'll look at ways questions about IVT might arise.

- We'll also see how to show proper justification and confirm the theorem conditions have been met.

Working with the Intermediate Value Theorem

How many of the graphs below represent functions that meet the criteria necessary to apply the Intermediate Value Theorem?



Working with the Intermediate Value Theorem

The function f and g are **continuous** for all real numbers and the table gives values of f and g at selected values of x. The function h is given by $h(x) = 2x - g(f(x))$. Explain why there must be a value k for $2 < k < 4$ such that $h(k) = 6$.

x	f(x)	g(x)
1	0	2
2	1	5
3	-1	1
4	3	-2

Steps

1. Check the conditions
2. Examine the endpoints at h(2) and h(4)

Working with the Intermediate Value Theorem

Let k be a function that is continuous on the closed interval $[-1,10]$ with $k(-1)=2$ and $k(10)=12$. Which of the following is guaranteed by the Intermediate Value Theorem?

A. $k(2)=7$

B. $k(x)$ attains a minimum on the open interval $(-1,10)$.

C. $k(x)=10$ has at least one solution in the open interval $(-1,10)$.

D. $k(x)=-1$ has at least one solution in the open interval $(2,12)$.

Working with the Intermediate Value Theorem

The function $w(t)$ is continuous for all times, t , in months, for $t \geq 0$. The table below gives values of w at selected values of t, for the weight of a baby in pounds.

Is there a time, t, during age 0 to 9 months of age that the baby weighed 10 pounds? Explain your reasoning.

t (months)	0	3	6	9
w(t) (pounds)	7.5	9.2	10.5	12

Summary

- The Intermediate Value Theorem can be applied if the condition of continuity on a closed interval is known.
- The functions using IVT may be given in multiple representations.
- The IVT is the first of three “existence theorems” in calculus. The other two existence theorems, Mean value Theorem and the Extreme Value Theorem, will be covered in Unit 5.

TRUE or FALSE

- If $f(x)$ is continuous on $[0, 2]$ and $f(0) = -1$, $f(2) = 1$, does there exist exactly one $c \in (0, 2)$ such that $f(c) = 0$?

FALSE

TRUE or FALSE

If $f(x)$ is continuous on $[a, b]$ and $f(a) > f(b)$, does $f(x)$ take every value between $f(a)$ and $f(b)$
?

TRUE