

Wednesday, 22 April 2026

Inverse and composite functions

LO: To be able to find and use inverse and composite functions

Keywords

Inverse

Composite

Function

TURN and TALK

Solve:

$$x^3 - 6x^2 + 11x - 6 = 0$$

Let

$$f(x) = x^2 + (k - 2)x + 1$$

and

$$g(x) = x^2 + (2k + 1)x + k,$$

where k is a real constant.

The graphs of $f(x)$ and $g(x)$ intersect at exactly one point.

Find the x -coordinate of the point of intersection in terms of k .

Final Answer

$$x = 1, 2, 3$$

$$x^2 + (k - 2)x + 1 = x^2 + (2k + 1)x + k$$

Cancel x^2 :

$$(k - 2)x + 1 = (2k + 1)x + k$$

Rearrange:

$$(k - 2 - 2k - 1)x = k - 1$$

$$(-k - 3)x = k - 1$$

$$x = \frac{1 - k}{k + 3}$$

Final Answer

$$x = \frac{1 - k}{k + 3}$$

1. Composite function — 复合函数
2. $f(g(x))$ — 先内后外 (先做 g , 再做 f)
3. Inner function — 内函数
4. Outer function — 外函数
5. Inverse function — 反函数
6. $f^{-1}(x)$ — 反函数表示
7. One-to-one function — 一一对应函数
8. Horizontal line test — 水平线检验
9. Swap x and y — 交换 x 和 y
10. $f(f^{-1}(x)) = x$ — 复合恒等式 (函数与其反函数互相抵消)

Starter**Date:**

1. Solve
 $x^2 + 8x + 12 > 0$

2. Make a the subject of
the formula.

$$p = \frac{n^2 + a}{n + a}$$

3. $f(x)$ is $x^2 + 2x$
 $g(x) = 5x - 3$
Find $gf(x)$

4. $f(x)$ is $x^2 + 2x$
 $g(x) = 5x - 3$
Find $g^{-1}(x)$

5. Solve
 $x^2 + y^2 = 25$
 $y - 2x = 5$

In this case it means:
Multiply by 3
Then add 2

$$= 3x + 2$$

~~Misconception~~
an equation

Write these functions as function machines:

eg) $f(x) = \frac{2x - 4}{3}$

Multiply by 2 and subtract 4 then divide by 3

1) $f(x) = 7x + 8$

Multiply by 7 then add 8

2) $f(x) = \frac{5x}{3} + 6$

Multiply by 5, divide by 3 then add 6

3) $f(x) = -2x + 2$

Multiply by -2 then add 2

4) $f(x) = \frac{8x - 4}{4}$

Multiply by 8, minus 4 then divide by 4

5) $f(x) = \frac{x - 9}{3}$

Subtract 9 then divide by 3

Use these functions to find these values:

$$\text{eg) } f(x) = \frac{2x - 4}{3} \quad f(8)$$

$$(8 \times 2 - 4) \div 3 = 4$$

$$1) \quad f(x) = 7x + 8 \quad f(2) \quad \mathbf{22}$$

$$2) \quad f(x) = \frac{5x}{3} + 6 \quad f(3) \quad \mathbf{11}$$

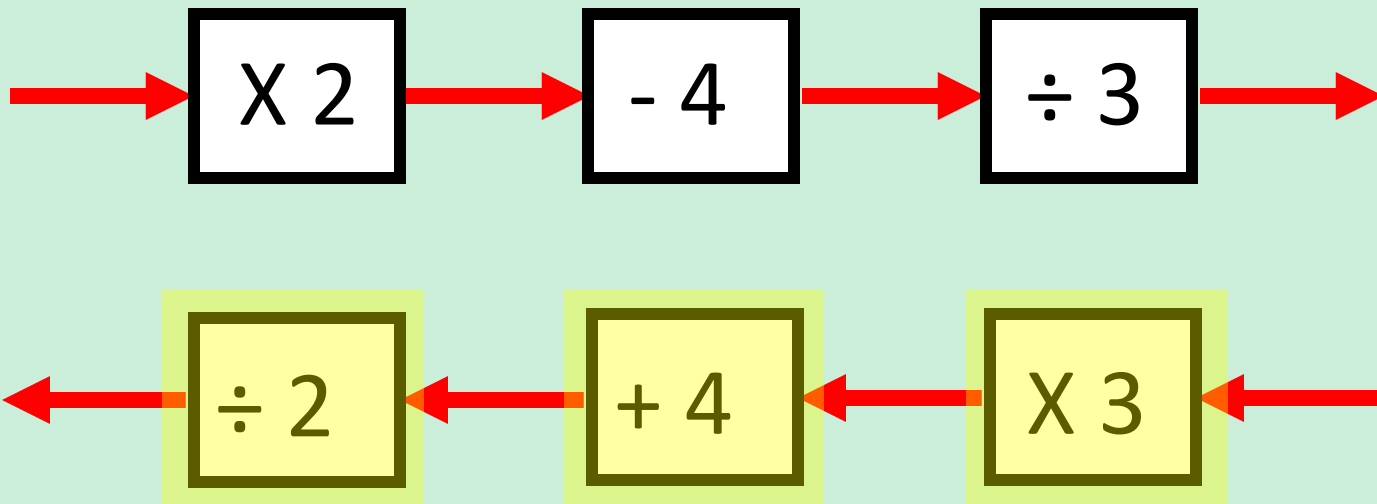
$$3) \quad f(x) = -2x + 2 \quad f(-2) \quad \mathbf{6}$$

$$4) \quad f(x) = \frac{8x - 4}{4} \quad f(4) \quad \mathbf{7}$$

$$5) \quad f(x) = \frac{x - 9}{3} \quad f(8) \quad \mathbf{-\frac{1}{3}}$$

For inverse functions we do the opposite, in reverse

$$\text{eg) } f(x) = \frac{2x - 4}{3}$$



$$f(x)^{-1} = \frac{3x + 4}{2}$$

Find $f(x)^{-1}$:

$$\text{eg) } f(x) = \frac{4x - 2}{5}$$

$$x4 > -2 > \div 5$$

$$\div 4 < +2 < x5$$

$$f(x)^{-1} = \frac{5x + 2}{4}$$

$$1) \quad f(x) = 7x + 8$$

$$f(x)^{-1} = \frac{x - 8}{7}$$

$$2) \quad f(x) = \frac{5x}{3} + 6$$

$$f(x)^{-1} = \frac{3x}{5} - 6$$

$$3) \quad f(x) = -2x + 2$$

$$f(x)^{-1} = \frac{x - 2}{-2}$$

$$4) \quad f(x) = \frac{8x - 4}{4}$$

$$f(x)^{-1} = \frac{4x + 4}{8}$$

For composite functions we substitute the right most function into the left

$$\text{eg) } f(x) = 2x + 5$$

$$g(x) = x^2 + 4$$

Find $fg(x)$

$g > f$ as right goes into left

$$f(x) = 2x + 5$$

$$g(x) = x^2 + 4$$

$$fg(x) = 2(x^2 + 4) + 5$$

$$fg(x) = 2x^2 + 8 + 5$$

Find the composite functions:

1) $f(x) = 7x + 8$ Find $fg(x)$ $fg(x) = 7x^2 + 22$
• $g(x) = x^2 + 2$

2) $h(x) = 5x + 6$ Find $fh(x)$ $fg(x) = 15x + 24$
• $f(x) = 3x + 6$

3) $f(x) = 2x - 1$ Find $fg(x)$ $fg(x) = 4x^2 - 1$
• $g(x) = 2x^2$

4) $g(x) = 8x + 1$ Find $fg(x)$ $fg(x) = 4x + 1$
• $f(x) = \frac{x+1}{2}$

Challenge

$$f(x) = 2x + 14$$

$$h(x) = x^2 + 6$$

Solve:

$$f(x) = h(x)$$

$$2x + 14 = x^2 + 6$$

$$x = 4, -2$$

Challenge

Let

$$f(x) = 2x - 3, \quad g(x) = \frac{1}{x+1}, \quad x \neq -1$$

(a) Find an expression for $(f \circ g)(x)$.

(b) Find $f^{-1}(x)$.

(c) Solve the equation:

$$(f \circ g)(x) = f^{-1}(x)$$

Answers

$$(a) \frac{2}{x+1} - 3$$

$$(b) \frac{x+3}{2}$$

$$(c) x = -5 \pm 2\sqrt{5}$$

True or False

$f(f^{-1}(x)) = x$ for all x in the domain.

False

TRUE or FALSE

- A function must be one-to-one to have an inverse.

TRUE