Turn and Talk

Question 1 – Power Rule

Find the derivative of

$$f(x) = 7x^5 - 3x^2 + 4x^{-1}$$

Question 2 – Chain Rule

Find $\frac{dy}{dx}$ if

$$y = (3x^2 + 1)^5$$

Question 3 – Product Rule

Differentiate

$$y = (2x^3)(x^2 + 4)$$

Turn and Talk

Question 1 – Power Rule (Answer)

$$f'(x) = 35x^4 - 6x - 4x^{-2}$$

Question 2 – Chain Rule (Answer)

$$\frac{dy}{dx} = 30x(3x^2 + 1)^4$$

Question 3 – Product Rule (Answer)

$$y' = 10x^4 + 24x^2$$

UNIT 2 KNOWLEDGE - CALCULUS 12 – DIFFERENTIATION: DEFINITION AND FUNDAMENTAL PROPERTIES

2.1
- Defining Average and Instantaneous Rates of Change at a Point ✓

2.2
- Defining the Derivative of a Function and Using Derivative Notation ✓

2.3
- Estimating Derivatives of a Function at a Point ✓

2.4
Connecting Differentiability and Continuity: Determining When Derivatives Do and Do Not Exist ✓

2.5
- Applying the Power Rule ✓

2.6
- Derivative Rules: Constant, Sum, Difference, and Constant Multiple ✓

2.7
- Derivatives of cos x, sin x, e^x , and ln x ✓

2.8
- The Product Rule ✓

2.9
- The Quotient Rule

2.10
- Finding the Derivatives of Tangent, Cotangent, Secant, and/or Cosecant Functions

What Will We Learn?

- What's the Quotient Rule?

- What's the method by which the derivative of a quotient of two functions can be calculated?

The Quotient Rule

The quotient, $\frac{f}{g}$ of two differentiable functions f and g is itself differentiable at all values for x for which $g(x) \neq 0$. Moreover, the derivative of $\frac{f}{g}$ is given by the denominator times the derivative of the numerator minus the numerator times the derivative of the denominator, all divided by the square of the denominator.

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Example: The Quotient Rule

Find the derivative of $y = \frac{5x+2}{x^2-1}$

$$Quotient\ rule: \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$y' = \frac{5(x^2-1) - (5x+2)\cdot 2x}{(x^2-1)^2}$$

$$= \frac{5x^2 - 5 - 10x^2 - 4x}{(x^2-1)^2}$$

$$= \frac{-5x^2 - 4x - 5}{(x^2-1)^2}$$

Another Example

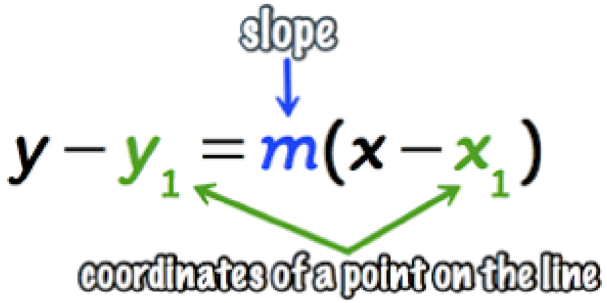
Let $y = \dfrac{x^2 + x - 2}{x^3 + 6}$. Then

Example: The Quotient Rule

Find the equation of the tangent line to $y = \frac{e^x}{1+x^2}$ at $\left(1, \frac{e}{2}\right)$.

$$Quotient\ rule: \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

1. Find the slope.
2. Plug in the $x -$ value of the pair.



Example: The Quotient Rule

Find the equation of the tangent line to $y = \frac{e^x}{1+x^2}$ at $\left(1, \frac{e}{2}\right)$.

$$y - \frac{e}{2} = 0(x-1)$$
$$y = \frac{e}{2}$$

At the point of the ordered pair, there's a 0 tangent line (horizontal tangent)

Key Takeaways

The Quotient Rule

The quotient, $\frac{f}{g}$ of two differentiable functions f and g is itself differentiable at all values for x for which $g(x) \neq 0$. Moreover, the derivative of $\frac{f}{g}$ is given by the denominator times the derivative of the numerator minus the numerator times the derivative of the denominator, all divided by the square of the denominator.

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

What Will We Learn?

- How can we compute derivatives using the quotient rule given graphical and numerical representations of functions?

Example: Using the Quotient Rule with a Table of Values

The values of two differentiable functions, $f(x)$ and $g(x)$ along with their derivatives are given in the table below for several values of x.

Given $h(x) = \frac{f(x)}{g(x)}, find\ \ h'(2)$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	5	−1	1	2
2	4	−1	3	$\frac{3}{2}$
3	3	−1	4	1
4	2	−1	5	1
5	1	0	6	$-\frac{1}{2}$
6	2	1	4	−2

Example: Using the Quotient Rule with a Table of Values

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
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4	2	−1	5	1
5	1	0	6	$-\frac{1}{2}$
6	2	1	4	−2

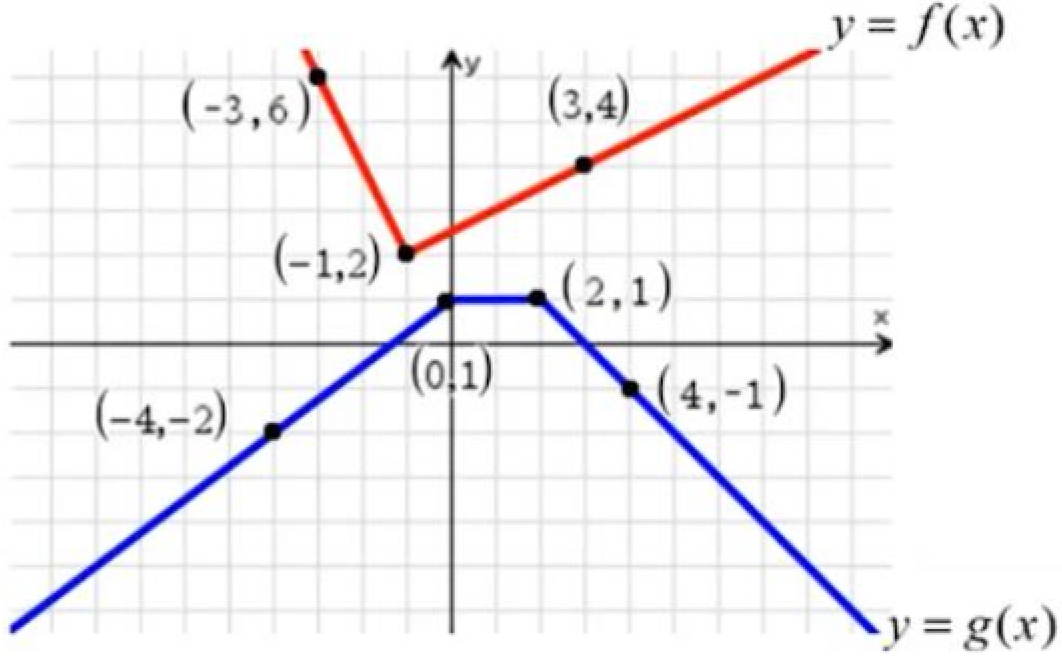
The values of two differentiable functions, $f(x)$ *and* $g(x)$ along with their derivatives are given in the table below for several values of x.

Given $j(x) = \frac{g(x)}{\sqrt{x}}$, *find* $j'(4)$.

Example: Using the Quotient Rule with Graph

Given the graphs of the differentiable functions $f(x)$ and $g(x)$ below and let $A(x) = \frac{g(x)}{f(x)}$. Determine the derivative value for $A'(1)$.

From graph, use slope (m) to find $f'(1)$ and $g'(1)$



Key Takeaways

The Quotient Rule

The quotient, $\frac{f}{g}$ of two differentiable functions f and g is itself differentiable at all values for x for which $g(x) \neq 0$. Moreover, the derivative of $\frac{f}{g}$ is given by the denominator times the derivative of the numerator minus the numerator times the derivative of the denominator, all divided by the square of the denominator.

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

PROOF

Step 1: Rewrite the function

Let

Step 2: Differentiate using product + chain rules

Turn and Talk

Step 3: Substitute

Step 4: Combine into a single fraction

TRUE or FALSE

- The quotient rule can be derived using the product rule and chain rule

TRUE or FALSE

- The product rule only works when both functions are polynomials