Think, Pair, Share

Consider the following sets:

$$A = \{1, 2, 3, 4, 5, 6\}$$

$$B = \{2, 4, 6\}$$

$$C = \{1, 2, 7\}$$

Questions

1. Which set is a **subset** of A?
2. Is C a subset of A? Explain why.
3. How many elements are in A?
4. Is $\{2, 4\}$ a subset of B?
5. Is A a subset of A?
6. What do you think $\varnothing$ represents?
7. Can you find **three different subsets** of A?

1. B is a subset of A.
2. No, because $7 \notin A$.
3. $n(A) = 6$.
4. Yes.
5. Yes.
6. The empty set contains no elements.
7. Examples: $\{1\}$, $\{2, 3\}$, $\{4, 5, 6\}$.

Subsets and Cardinality

Set	集合
Element	元素
Member of	属于
Subset	子集
Proper Subset	真子集
Universal Set	全集
Empty Set (Null Set)	空集
Cardinality	基数（集合大小）
Finite Set	有限集
Infinite Set	无限集

2. What Is a Subset?

A set B is a **subset** of A if **every element of B is also an element of A**.

Example

$$A = \{1, 2, 3, 4, 5\}$$

Which of the following are subsets of A?

$$B = \{1, 3\}$$

$$C = \{2, 4, 5\}$$

$$D = \{1, 6\}$$

Answer

$$B \subseteq A$$

because 1 and 3 are in A.

$$C \subseteq A$$

because 2, 4 and 5 are in A.

But:

$$D \not\subseteq A$$

because 6 is **not** in A.

3. Proper Subsets

A **proper subset** is a subset that is **smaller than the original set**.

For example:

$$A = \{1, 2, 3, 4\}$$

and

$$B = \{1, 2\}$$

Then:

$$B \subset A$$

because every element of B is in A, but $B \neq A$.

Important distinction

$$B \subseteq A$$

allows $B = A$.

Whereas:

$$B \subset A$$

means B must be smaller than A.

4. The Empty Set

The **empty set** is a set containing **no elements**.

It is written:

$$\boxed{\varnothing}$$

or:

$$\boxed{\{\}}$$

For example, the set of positive integers less than 0 is:

$$\varnothing$$

The empty set is a subset of **every set**.

Therefore:

$$\boxed{\varnothing \subseteq A}$$

for any set A.

5. Cardinality

The **cardinality** of a set means the **number of elements in the set**.

We write:

$$n(A)$$

or sometimes:

$$|A|$$

Example

$$A = \{2, 4, 6, 8, 10\}$$

There are five elements, so:

$$\boxed{n(A) = 5}$$

Another Example

$$B = \{a, b, c, d\}$$

Therefore:

$$\boxed{n(B) = 4}$$

6. Cardinality and Venn Diagrams

Suppose:

$$n(U) = 30$$

$$n(A) = 18$$

$$n(B) = 15$$

and

$$n(A \cap B) = 7$$

We can calculate:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Therefore:

$$18 + 15 - 7 = 26$$

So:

$$\boxed{n(A \cup B) = 26}$$

The number outside both sets is:

$$30 - 26 = \boxed{4}$$

7. A Useful Formula

For two sets:

$$\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$$

Why do we subtract?

When we add $n(A)$ and $n(B)$, the intersection is counted **twice**.

So we subtract it once.

8. Worked Example

A survey of 50 students found:

- 32 play football.
- 25 play basketball.
- 15 play both.

Let:

$$F = \text{football}$$

$$B = \text{basketball}$$

Step 1: Find the number who play at least one sport.

[redacted]

Step 2: Find the number who play neither.

[redacted]

Step 3: Football but not basketball

[redacted]

Step 4: Basketball but not football

[redacted]

Question 1 – Subsets

Let:

$$A = \{1, 2, 3, 4, 5, 6\}$$

Determine whether each statement is true or false.

a) $\{1, 3\} \subseteq A$

b) $\{2, 5, 6\} \subseteq A$

c) $\{1, 7\} \subseteq A$

d) $\varnothing \subseteq A$

Question 2 – Cardinality

Find $n(A)$.

a)

$$A = \{3, 6, 9, 12, 15\}$$

b)

$$B = \{a, e, i, o, u\}$$

c)

$$C = \varnothing$$

Question 3 – Venn Diagram

In a class of 45 students:

- 28 study Mathematics.
- 20 study Science.
- 12 study both.

Find:

a) Mathematics only

b) Science only

c) Mathematics or Science

d) Neither Mathematics nor Science

Q1 – Subsets:

a) True

b) True

c) False

d) True

Q2 – Cardinality:

a) $n(A) = 5$

b) $n(B) = 5$

c) $n(C) = 0$

Q3 – Venn Diagram:

a) Mathematics only = **16**

b) Science only = **8**

c) Mathematics or Science = **36**

d) Neither = **9**

10. Stretch & Challenge

A school has 80 students.

- 48 study Mathematics.
- 35 study Physics.
- 20 study both Mathematics and Physics.
- 10 study neither.

Find:

a) The number who study Mathematics or Physics.

b) Check whether the information is consistent.

c) How many study Mathematics but not Physics?

d) How many study Physics but not Mathematics?

e) How many study exactly one of the two subjects?

Given 80 students:

- Mathematics = 48
- Physics = 35
- Both = 20
- Neither = 10

a) Mathematics or Physics:

$$48 + 35 - 20 = \boxed{63}$$

b) Consistent? **No.**
63 + 10 = 73, not 80. Therefore, the information is inconsistent.

c) Mathematics only:

$$48 - 20 = \boxed{28}$$

d) Physics only:

$$35 - 20 = \boxed{15}$$

e) Exactly one:

$$28 + 15 = \boxed{43}$$

Exit Ticket

Given:

$$U = \{1, 2, 3, \ldots, 15\}$$

$$A = \{2, 4, 6, 8, 10, 12, 14\}$$

$$B = \{2, 4, 6\}$$

Answer:

1. Is $B \subseteq A$?
2. Is B a proper subset of A?
3. Find $n(A)$.
4. Find $n(B)$.
5. Is $\varnothing \subseteq A$?
6. If $n(A) = 20$, $n(B) = 15$, and $n(A \cap B) = 8$, find $n(A \cup B)$.

1. $B \subseteq A$ → **Yes**
2. B is a proper subset of A → **Yes**
3. $n(A) = \boxed{7}$
4. $n(B) = \boxed{3}$
5. $\varnothing \subseteq A$ → **Yes**
6.

$$20 + 15 - 8 = \boxed{27}$$

$$\boxed{\text{Subset: every element of B is in A}}$$

$$\boxed{\varnothing = \text{no elements}}$$

$$\boxed{n(A) = \text{number of elements in A}}$$

$$\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$$

Sample Spaces

1. Sample Space	样本空间
2. Outcome	结果
3. Event	事件
4. Simple Event	简单事件
5. Compound Event	复合事件
6. Probability	概率
7. Union	并集
8. Intersection	交集
9. Complement	补集
10. Mutually Exclusive Events	互斥事件

1. **A sample space contains all possible outcomes** of a random experiment.
2. **An outcome is a single result** from the sample space.
3. **An event is a subset of the sample space** containing one or more outcomes.
4. **Probabilities are assigned to events,** not just individual outcomes.
5. **The probability of the entire sample space is always 1.**
6. **The probability of any event must be between 0 and 1 inclusive.**
7. **The complement rule states**

$$P(A^c) = 1 - P(A)$$

8. **Mutually exclusive events cannot occur simultaneously**, so:

$$P(A \cap B) = 0$$

9. **The addition rule for two events is**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

10. **Sample spaces can be represented using lists, tables, tree diagrams, or Venn diagrams** to organize outcomes and calculate probabilities.

Question 1: Rolling Two Dice

Two fair six-sided dice are rolled.

(a) Describe the sample space for this experiment.

(b) How many outcomes are in the sample space?

(c) Find the probability that the sum of the two dice is 7.

(d) Find the probability that at least one die shows a 6.

(a)

$$S = \{(1,1),(1,2),\ldots,(6,6)\}$$

All ordered pairs of outcomes.

(b)

$$6 \times 6 = 36$$

outcomes.

(c)

Sum of 7:

$$(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)$$

6 outcomes.

$$P(\text{sum} = 7) = \frac{6}{36} = \frac{1}{6}$$

(d)

Outcomes with at least one 6:

$$6 + 6 - 1 = 11$$

$$P(\text{at least one } 6) = \frac{11}{36}$$

True or False

- **A sample space contains all possible outcomes of a random experiment.**

True or False

- **An event can contain only one outcome.**

True or False

The probability of the entire sample space is 1.