

Turn and Talk

Solve

$$\frac{x+1}{x-2} > 0$$

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$$(-\infty, -1) \cup (2, \infty)$$

REVIEW OF FUNCTIONS

function — 函数

equation — 方程

variable — 变量

independent variable — 自变量

dependent variable — 因变量

domain — 定义域

range — 值域

input — 输入

output — 输出

mapping — 映射

graph — 图像

coordinate plane — 坐标平面

ordered pair — 有序对

vertical line test — 垂直线检验

one-to-one function — 一一函数

many-to-one function — 多对一函数

inverse function — 反函数

composite function — 复合函数

piecewise function — 分段函数

linear function — 线性函数

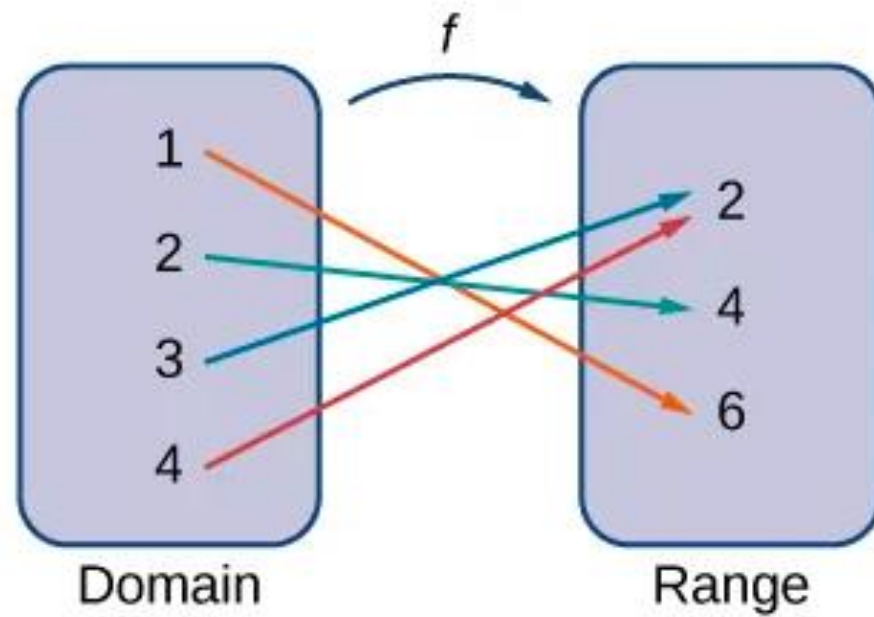
DEFINITION

A **function** f consists of a set of inputs, a set of outputs, and a rule for assigning each input to exactly one output. The set of inputs is called the **domain** of the function. The set of outputs is called the **range** of the function.

<https://openstax.org/books/calculus-volume-1/pages/1-1-review-of-functions>



Figure 1.2 A function can be visualized as an input/output device.



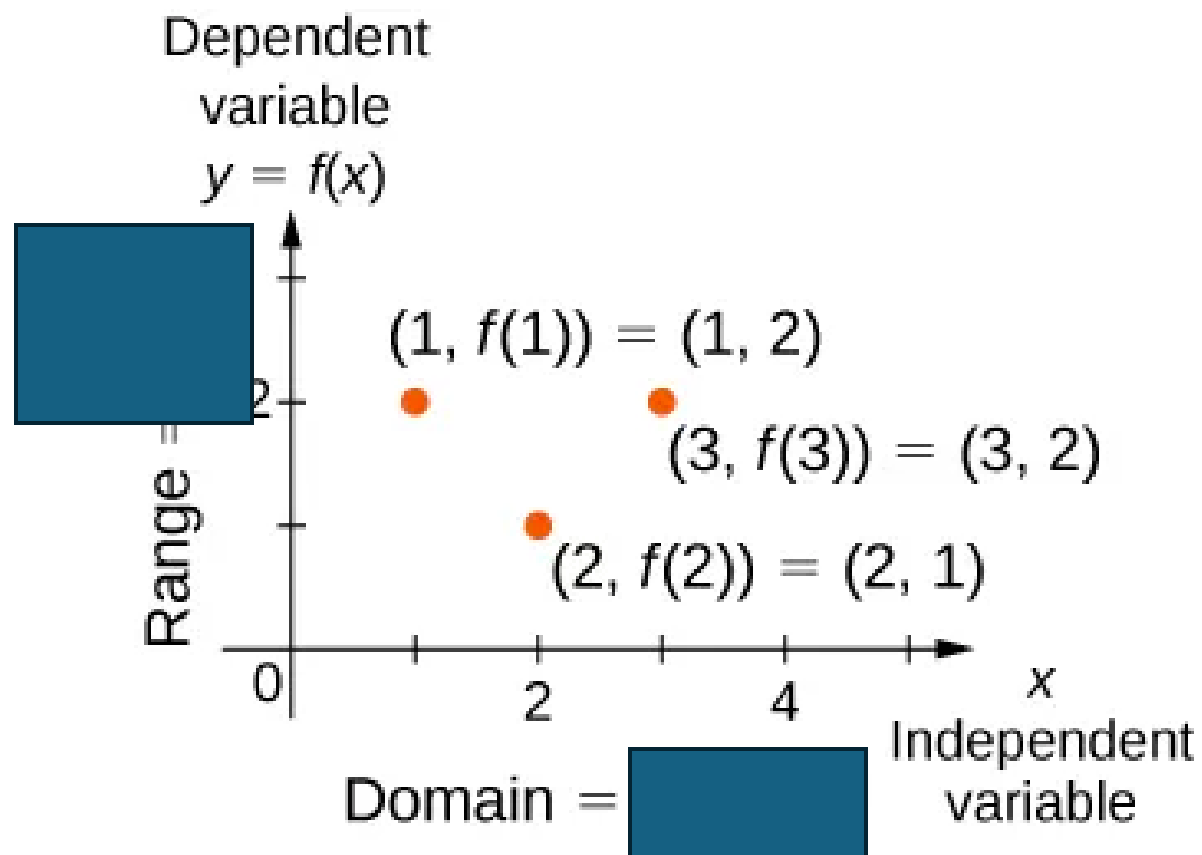


Figure 1.4 In this case, a graph of a function f has a domain of $\{1, 2, 3\}$ and a range of $\{1, 2\}$. The independent variable is x and the dependent variable is y .

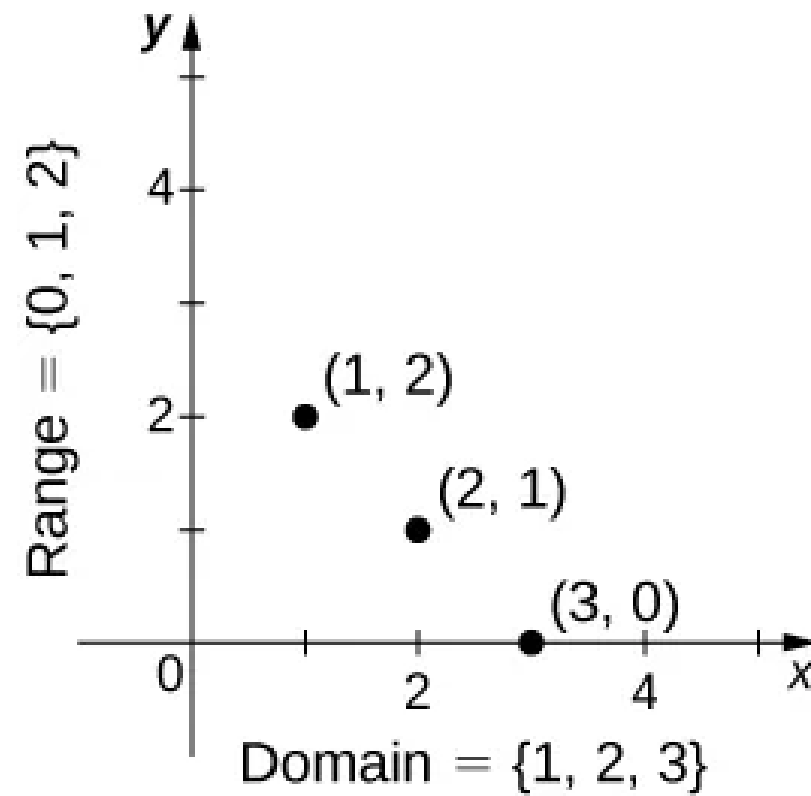
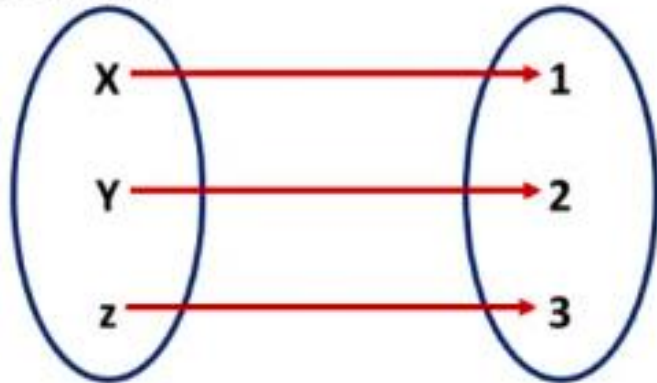
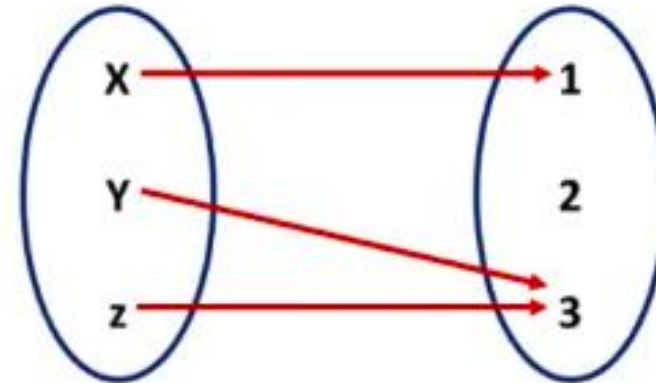


Figure 1.5 Here we see a graph of the function f with domain $\{1, 2, 3\}$ and rule $f(x) =$. The graph consists of the points $(x, f(x))$ for all x in the domain.

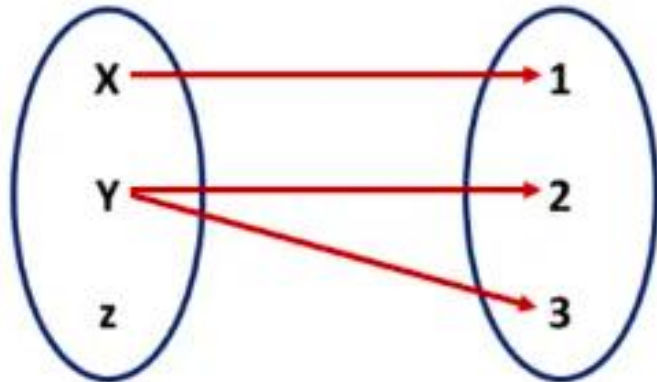
MAPPING OF ELEMENTS



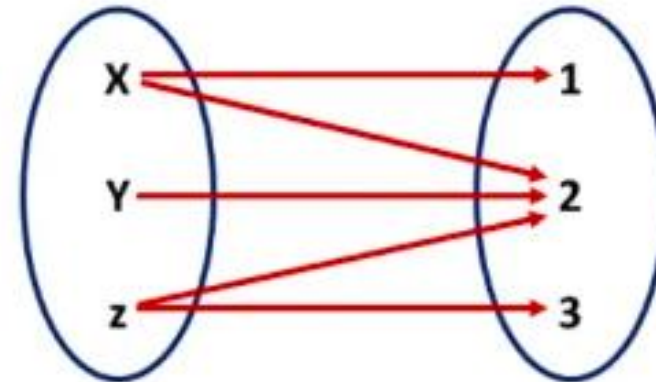
ONE TO ONE



MANY TO ONE



ONE TO MANY



MANY TO MANY

Here are some **equations**. Decide which **define functions** (of x):

1. $y = 2x + 3$

2. $x^2 + y^2 = 1$

3. $y^2 = x$

4. $y = \sqrt{x}$

5. $y = \pm\sqrt{x}$

6. $x = 5$

7. $y = |x|$

1. $y = 2x + 3$ ✓

2. $x^2 + y^2 = 1$ ✗

3. $y^2 = x$ ✗

4. $y = \sqrt{x}$ ✓

5. $y = \pm\sqrt{x}$ ✗

6. $x = 5$ ✗

7. $y = |x|$ ✓

$$\{x|1 < x < 5\}.$$

A set such as this, which contains all numbers greater than a and less than b , can also be denoted using the interval notation (a, b) . Therefore,

$$(1, 5) = \{x|1 < x < 5\}.$$

The numbers 1 and 5 are called the *endpoints* of this set. If we want to consider the set that includes the endpoints, we would denote this set by writing

$$[1, 5] = \boxed{}.$$

We can use similar notation if we want to include one of the endpoints, but not the other. To denote the set of nonnegative real numbers, we would use the set-builder notation

$$\boxed{}$$

The smallest number in this set is zero, but this set does not have a largest number. Using interval notation, we would use the symbol ∞ , which refers to positive infinity, and we would write the set as

$$[0, \infty) = \boxed{}$$

Piecewise Functions

Some functions are defined using different equations for different parts of their domain. These types of functions are known as *piecewise-defined functions*. For example, suppose we want to define a function f with a domain that is the set of all real numbers such that $f(x) = 3x + 1$ for $x \geq 2$ and $f(x) = x^2$ for $x < 2$. We denote this function by writing

$$f(x) = \begin{cases} 3x + 1 & x \geq 2 \\ x^2 & x < 2 \end{cases}.$$

EXAMPLE 1.1

Evaluating Functions

For the function $f(x) = 3x^2 + 2x - 1$, evaluate

- a. $f(-2)$
- b. $f(\sqrt{2})$
- c. $f(a + h)$

EXAMPLE 1.1

Evaluating Functions

For the function $f(x) = 3x^2 + 2x - 1$, evaluate

- a. $f(-2)$
- b. $f(\sqrt{2})$
- c. $f(a + h)$

a. $f(-2) = 3(-2)^2 + 2(-2) - 1 = 12 - 4 - 1 = 7$

b. $f(\sqrt{2}) = 3(\sqrt{2})^2 + 2\sqrt{2} - 1 = 6 + 2\sqrt{2} - 1 = 5 + 2\sqrt{2}$

c. $f(a + h) = 3(a + h)^2 + 2(a + h) - 1 = 3(a^2 + 2ah + h^2) + 2a + 2h - 1$
 $= 3a^2 + 6ah + 3h^2 + 2a + 2h - 1$

Finding Domain and Range

For each of the following functions, determine the i. domain and ii. range.

a. $f(x) = (x - 4)^2 + 5$

b. $f(x) = \sqrt{3x + 2} - 1$

c. $f(x) = \frac{3}{x - 2}$

Finding Domain and Range

For each of the following functions, determine the i. domain and ii. range.

a. $f(x) = (x - 4)^2 + 5$

b. $f(x) = \sqrt{3x + 2} - 1$

c. $f(x) = \frac{3}{x - 2}$

Domain: $-\infty < x < \infty$

Range $y \geq 5$

Domain: $x > -\frac{2}{3}$

Range $y \geq -1$

c. Consider $f(x) = 3/(x - 2)$.

i. Since $3/(x - 2)$ is defined when the denominator is nonzero, the domain is $\{x \mid x \neq 2\}$.

ii. To find the range of f , we need to find the values of y such that there exists a real number x in the domain with the property that

$$\frac{3}{x - 2} = y.$$

Solving this equation for x , we find that

$$x = \frac{3}{y} + 2.$$

Therefore, as long as $y \neq 0$, there exists a real number x in the domain such that $f(x) = y$.

Thus, the range is $\{y \mid y \neq 0\}$.

Representing Functions

Typically, a function is represented using one or more of the following tools:

- A table
- A graph
- A formula

Hours after Midnight	Temperature ($^{\circ}F$)	Hours after Midnight	Temperature ($^{\circ}F$)
0	58	12	84
1	54	13	85
2	53	14	85
3	52	15	83
4	52	16	82
5	55	17	80
6	60	18	77
7	64	19	74
8	72	20	69
9	75	21	65
10	78	22	60
11	80	23	58

Table 1.1 Temperature as a Function of Time of Day

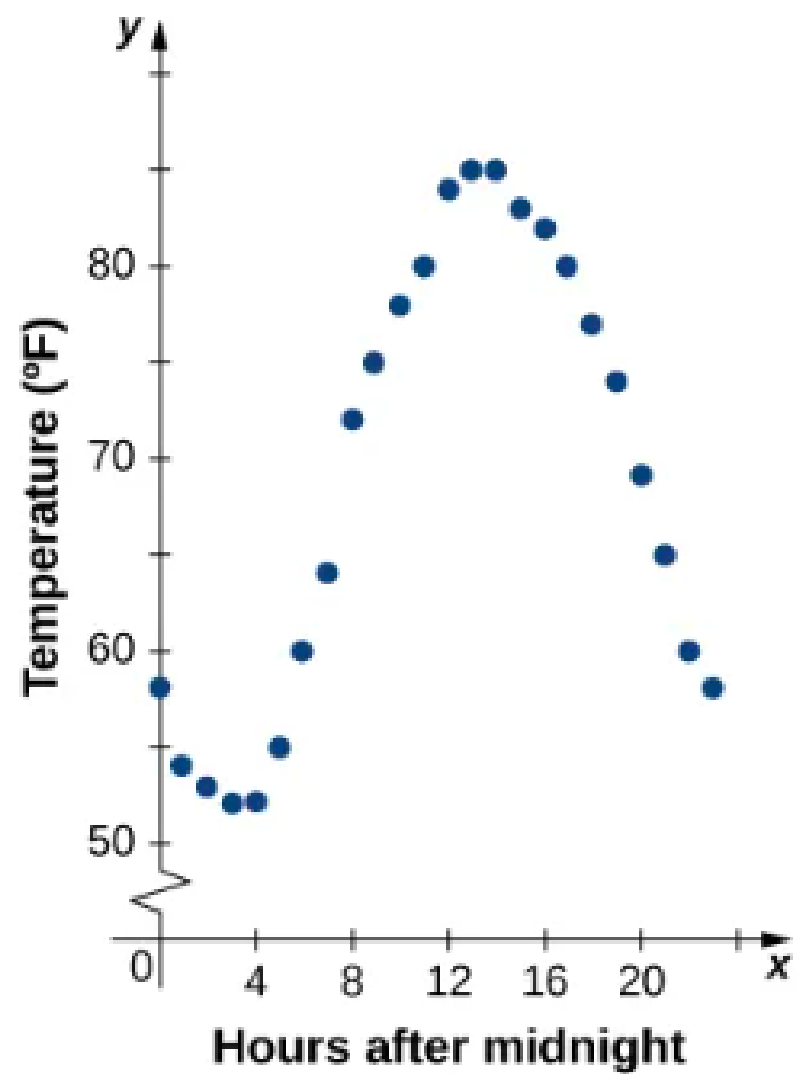
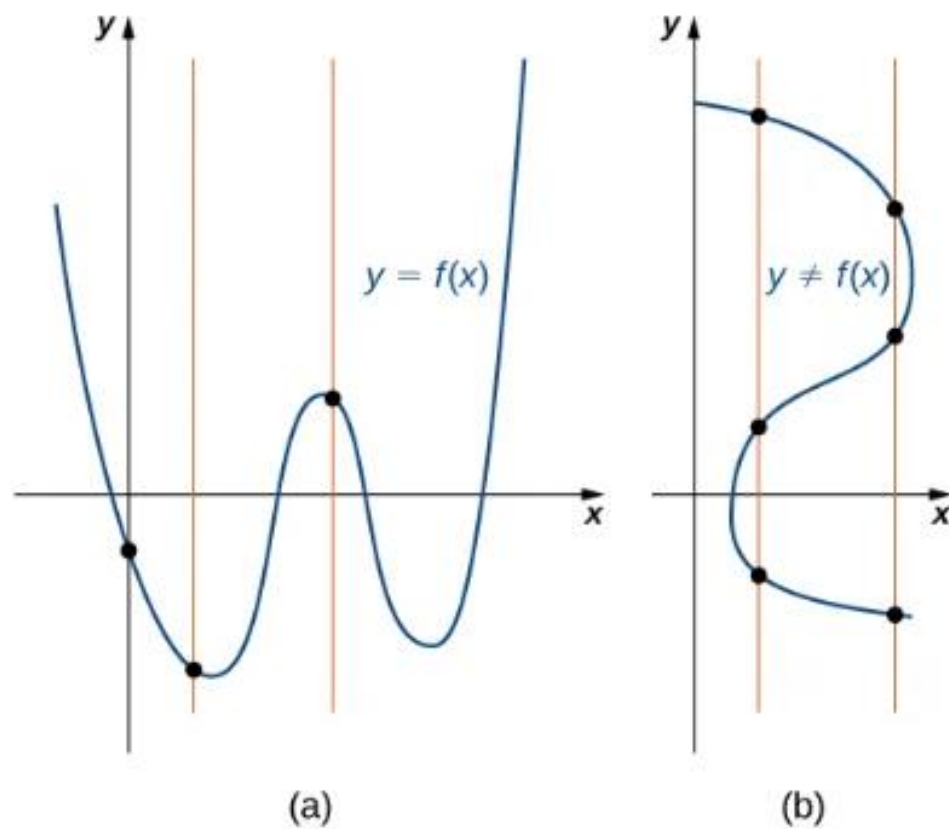


Figure 1.6 The graph of the data from [Table 1.1](#) shows temperature as a function of time.

RULE: VERTICAL LINE TEST

Given a function f , every vertical line that may be drawn intersects the graph of f no more than once. If any vertical line intersects a set of points more than once, the set of points does not represent a function.

We can use this test to determine whether a set of plotted points represents the graph of a function ([Figure 1.8](#)).



EXAMPLE 1.3

Finding Zeros and y -Intercepts of a Function

Consider the function $f(x) = -4x + 2$.

- Find all zeros of f .
- Find the y -intercept (if any).
- Sketch a graph of f .

EXAMPLE 1.3

Finding Zeros and y -Intercepts of a Function

Consider the function $f(x) = -4x + 2$.

- Find all zeros of f .
- Find the y -intercept (if any).
- Sketch a graph of f .

- To find the zeros, solve $f(x) = -4x + 2 = 0$. We discover that f has one zero at $x = 1/2$.
- The y -intercept is given by $(0, f(0)) = (0, 2)$.
- Given that f is a linear function of the form $f(x) = mx + b$ that passes through the points $(1/2, 0)$ and $(0, 2)$, we can sketch the graph of f ([Figure 1.9](#)).

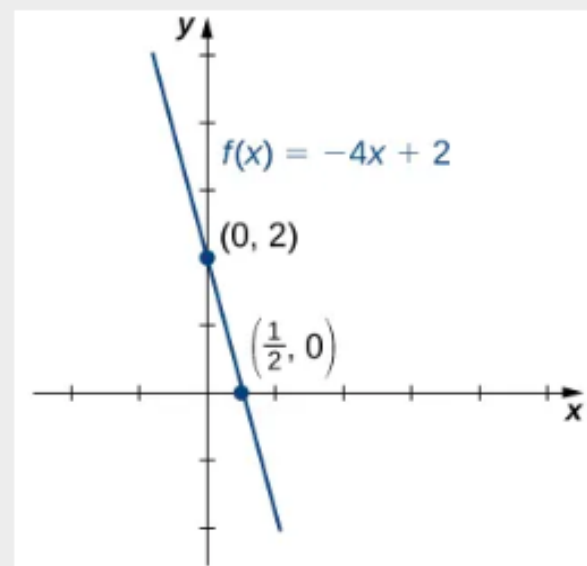


Figure 1.9 The function $f(x) = -4x + 2$ is a line with x -intercept $(1/2, 0)$ and y -intercept $(0, 2)$.

Increasing/ Decreasing Function

DEFINITION

We say that a function f is **increasing on the interval** I if for all $x_1, x_2 \in I$,

$$f(x_1) \leq f(x_2) \text{ when } x_1 < x_2.$$

We say f is **strictly increasing on the interval** I if for all $x_1, x_2 \in I$,

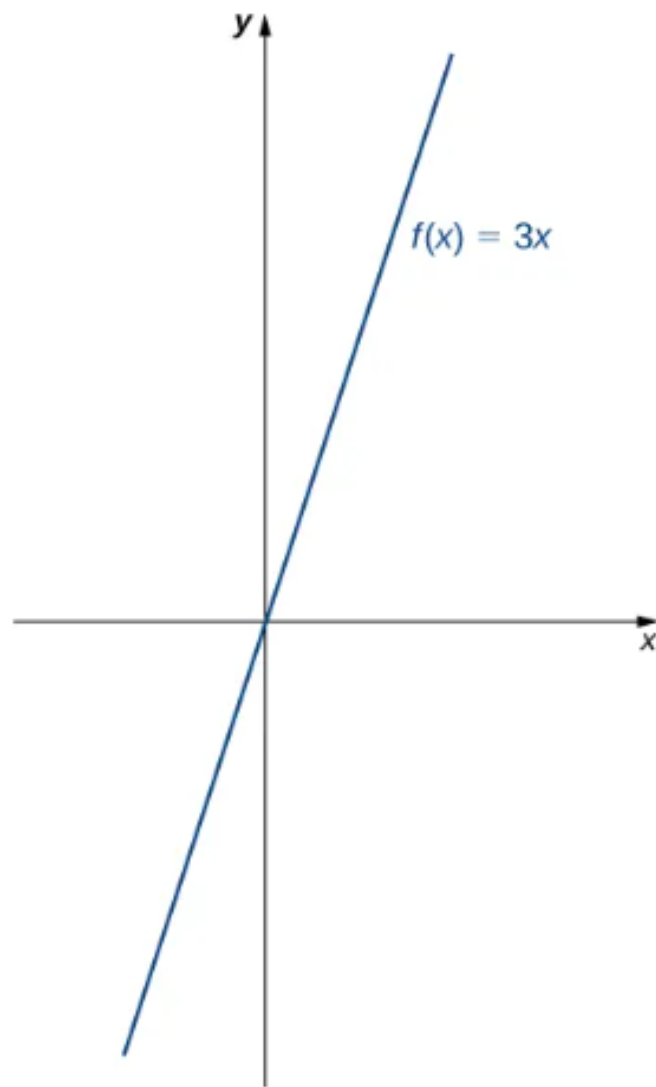
$$f(x_1) < f(x_2) \text{ when } x_1 < x_2.$$

We say that a function f is **decreasing on the interval** I if for all $x_1, x_2 \in I$,

$$f(x_1) \geq f(x_2) \text{ if } x_1 < x_2.$$

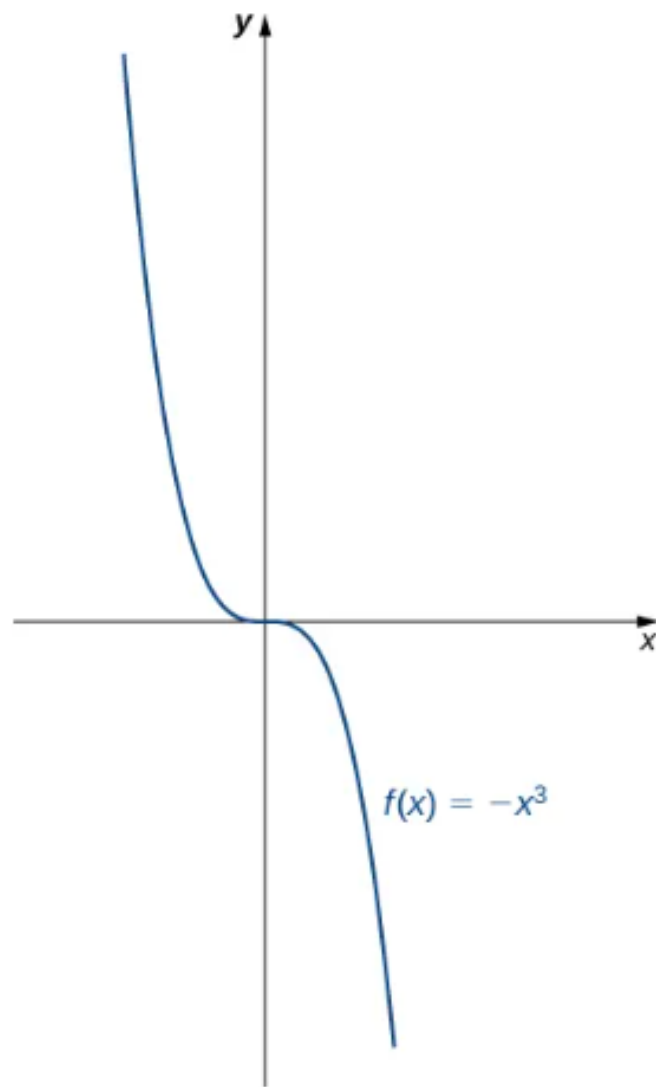
We say that a function f is **strictly decreasing on the interval** I if for all $x_1, x_2 \in I$,

$$f(x_1) > f(x_2) \text{ if } x_1 < x_2.$$



(a)

INCREASING



(b)

DECREASING

Combining Functions

$$(f + g)(x) = f(x) + g(x)$$

Sum

$$(f - g)(x) = f(x) - g(x)$$

Difference

$$(f \cdot g)(x) = f(x) g(x)$$

Product

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \text{ for } g(x) \neq 0$$

Quotient

EXAMPLE 1.6

Combining Functions Using Mathematical Operations

Given the functions $f(x) = 2x - 3$ and $g(x) = x^2 - 1$, find each of the following functions and state its domain.

- a. $(f + g)(x)$
- b. $(f - g)(x)$
- c. $(f \cdot g)(x)$
- d. $\left(\frac{f}{g}\right)(x)$

EXAMPLE 1.6

Combining Functions Using Mathematical Operations

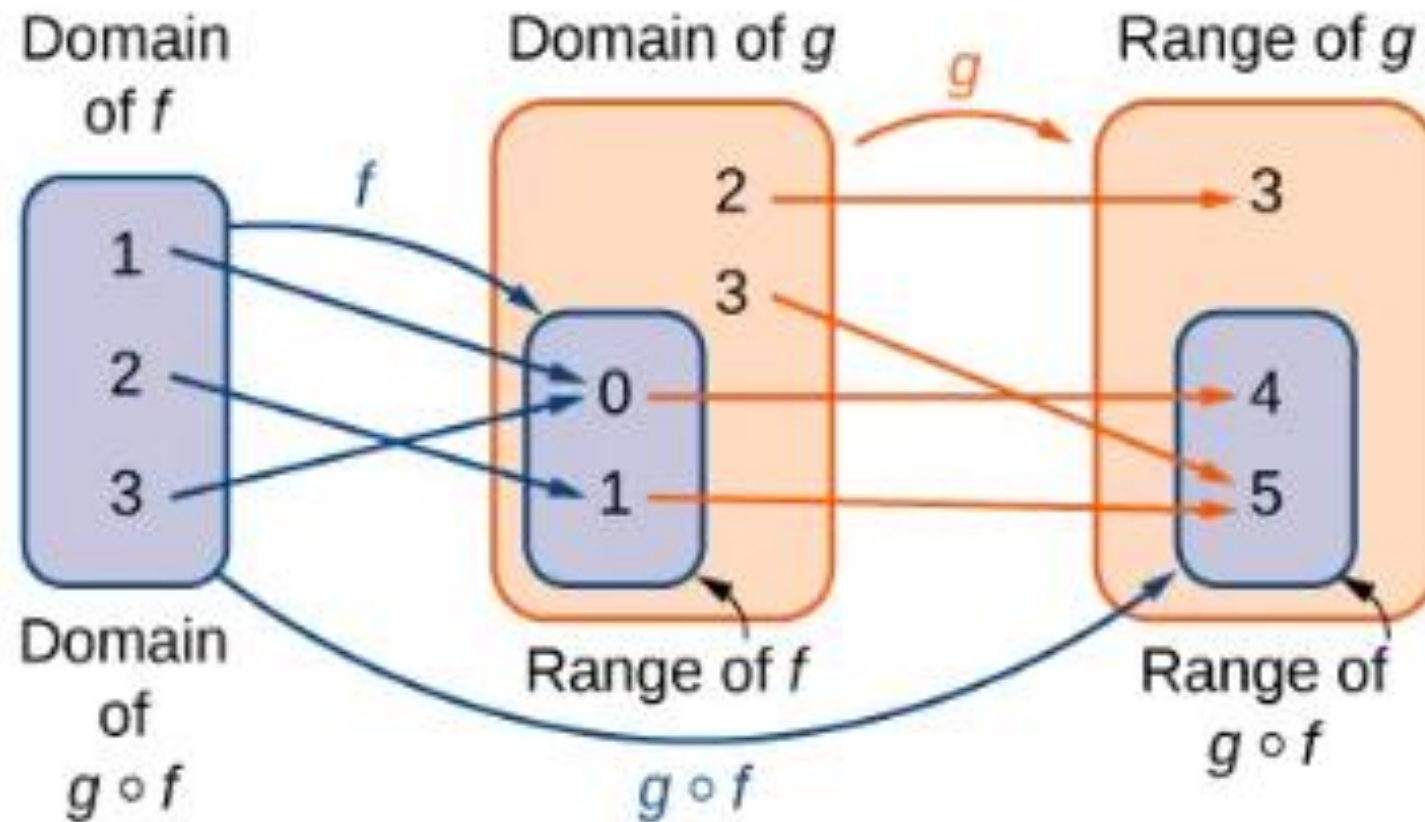
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- a. $(f + g)(x)$
- b. $(f - g)(x)$
- c. $(f \cdot g)(x)$
- d. $\left(\frac{f}{g}\right)(x)$

Solution

- a. $(f + g)(x) = (2x - 3) + (x^2 - 1) = x^2 + 2x - 4$. The domain of this function is the interval $(-\infty, \infty)$.
- b. $(f - g)(x) = (2x - 3) - (x^2 - 1) = -x^2 + 2x - 2$. The domain of this function is the interval $(-\infty, \infty)$.
- c. $(f \cdot g)(x) = (2x - 3)(x^2 - 1) = 2x^3 - 3x^2 - 2x + 3$. The domain of this function is the interval $(-\infty, \infty)$.
- d. $\left(\frac{f}{g}\right)(x) = \frac{2x-3}{x^2-1}$. The domain of this function is $\{x|x \neq \pm 1\}$.

Composite Functions



Compositions of Functions Defined by Formulas

Consider the functions $f(x) = x^2 + 1$ and $g(x) = 1/x$.

- Find $(g \circ f)(x)$ and state its domain and range.
- Evaluate $(g \circ f)(4)$, $(g \circ f)(-1/2)$.
- Find $(f \circ g)(x)$ and state its domain and range.
- Evaluate $(f \circ g)(4)$, $(f \circ g)(-1/2)$.

Compositions of Functions Defined by Formulas

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- Find $(f \circ g)(x)$ and state its domain and range.
- Evaluate $(f \circ g)(4)$, $(f \circ g)(-1/2)$.

$$g(f(x)) = \frac{1}{x^2 + 1}$$

Domain: $-\infty < x < \infty$

Range $0 < g(f(x)) < 1$

$$\begin{aligned} \text{b. } (g \circ f)(4) &= g(f(4)) = g(4^2 + 1) = g(17) = \frac{1}{17} \\ (g \circ f)\left(-\frac{1}{2}\right) &= g\left(f\left(-\frac{1}{2}\right)\right) = g\left(\left(-\frac{1}{2}\right)^2 + 1\right) = g\left(\frac{5}{4}\right) = \frac{4}{5} \end{aligned}$$

Since $1/\sqrt{y-1}$ is a real number if and only if $y > 1$, the range of f is the set $\{y|y > 1\}$.

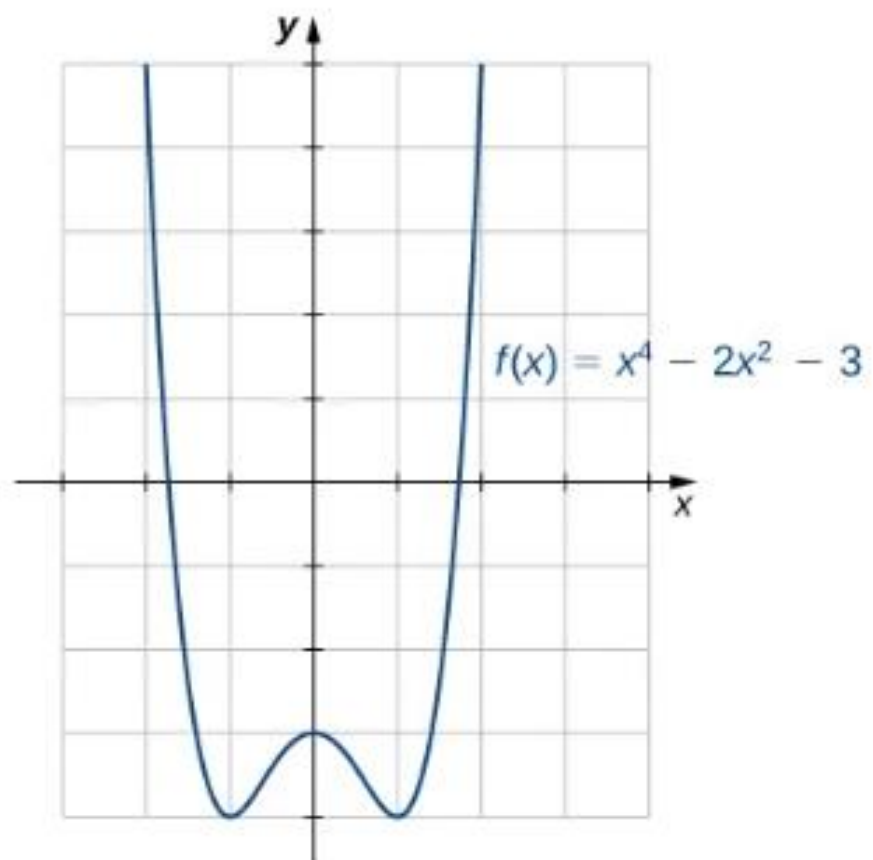
$$\begin{aligned} \text{d. } (f \circ g)(4) &= f(g(4)) = f\left(\frac{1}{4}\right) = \left(\frac{1}{4}\right)^2 + 1 = \frac{17}{16} \\ (f \circ g)\left(-\frac{1}{2}\right) &= f\left(g\left(-\frac{1}{2}\right)\right) = f(-2) = (-2)^2 + 1 = 5 \end{aligned}$$

Symmetry of Functions

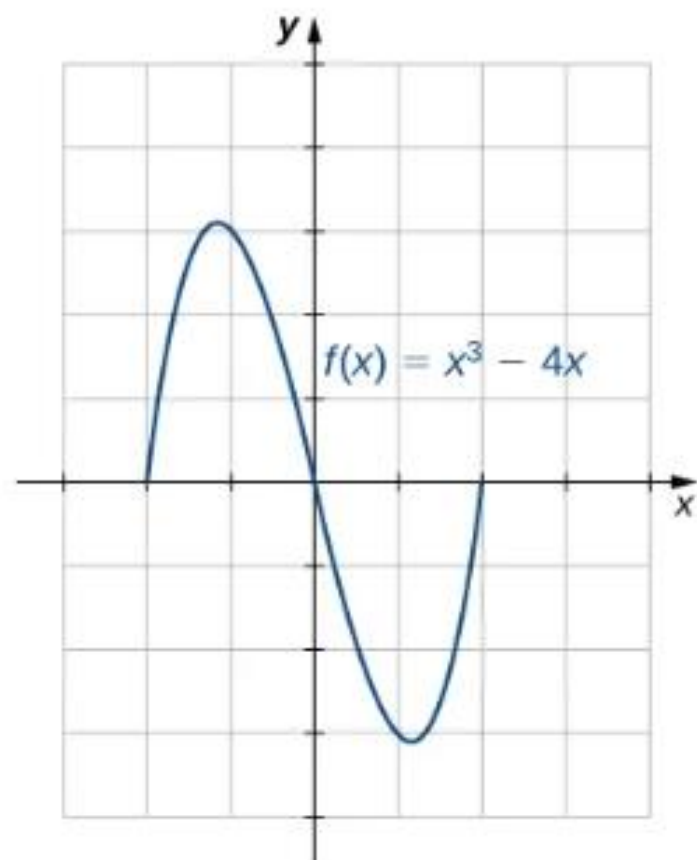
DEFINITION

If $f(x) = f(-x)$ for all x in the domain of f , then f is an **even function**. An even function is symmetric about the y -axis.

If $f(-x) = -f(x)$ for all x in the domain of f , then f is an **odd function**. An odd function is symmetric about the origin.



(a) Symmetry about the y-axis



(b) Symmetry about the origin

Even and Odd Functions

Determine whether each of the following functions is even, odd, or neither.

a. $f(x) = -5x^4 + 7x^2 - 2$

b. $f(x) = 2x^5 - 4x + 5$

c. $f(x) = \frac{3x}{x^2+1}$

Even and Odd Functions

Determine whether each of the following functions is even, odd, or neither.

a. $f(x) = -5x^4 + 7x^2 - 2$

b. $f(x) = 2x^5 - 4x + 5$

c. $f(x) = \frac{3x}{x^2+1}$

Solution

To determine whether a function is even or odd, we evaluate $f(-x)$ and compare it to $f(x)$ and $-f(x)$.

a. $f(-x) = -5(-x)^4 + 7(-x)^2 - 2 = -5x^4 + 7x^2 - 2 = f(x)$. Therefore, f is even.

b. $f(-x) = 2(-x)^5 - 4(-x) + 5 = -2x^5 + 4x + 5$. Now, $f(-x) \neq f(x)$. Furthermore, noting that $-f(x) = -2x^5 + 4x - 5$, we see that $f(-x) \neq -f(x)$. Therefore, f is neither even nor odd.

c. $f(-x) = 3(-x)/((-x)^2 + 1) = -3x/(x^2 + 1) = -[3x/(x^2 + 1)] = -f(x)$. Therefore, f is odd.