F1

Annie Bai	Franklin Chen	Charlie Cheng	Zoey Cheng	Harry Cui	King Deng	Aisling Fu	Ken He	Ivy Jing
Lee Li	Ziheng Li	Lewis Liu	Sam Liu	Ava Peng	Peter Shen	Sylvia Song	Coco Wang	Eileen Wang
Jason Wang	Luna Wang	Cathy Yan	Melody You	Allen Zhang	Lucas Zhang	Lynette Zhang	Ryder Zhang	Flora Zhou
Cyntina Zuo								

F2

Angelina Wang	Azura Cheng	Bonnie Zhao	Cecilia Jia	Diana Li	Oscar Liu	Zavier Liu	Tom Ou	Mia Peng
Joshua Hui	Julius Lv	Justin Jia	Lucas Li	Lyra Zhang	Iris Shao	Dylan Suo	PeytonAson Wang	Struck Wang
Selene Hou	Serena Feng	Silas Lv	Simon Wang	Stella Sun	Yola Wang	Mark Xiao	Elsa Ye	Emily Zhang
Zinnia Dong	Flora Zhang							

F3

Allen Peng	Aurora Yuan	Brittney Wei	Cynthia Liu	Erya Hu	Eva Gai	Felicity Pan	Fiona Ding	Freya Fan
Gordon Yao	Helios Luo	Honey Ruan	Iris Xie	Jasper Wu	Kaven Zhang	Kevin Gao	Leo Yang	Linger Li
Lydia Wei	Maggie Gao	Micheal Zhao	Ray Meng	Rose Jiang	Ross Ma	Roy Liu	Ryan Wang	Sky Bai
Star Su	Stella Xi							

F4

Aris Cheng	Tracy Dang	Jason Du	Alanna Fan	Iris Gao	Eason Gong	Vardy Hai	Edmund He	Liora Li
Miranda Li	Leo Liu	Francis Lv	Dorothy Qian	Dary Song	Jerry Tu	Belinda Wang	Winnie Wang	Elena Wei
James Xu	Mike Yan	Simon Yang	Coco Zhang	Patrick Zhang	Sophia Zhang	Molly Zheng	Carrie Zhou	Rudy Zhu

F5

Abbie Dong	Alan Chen	Angela An	Annette Zhao	Asher Tian	Jove Bai	Camellia Long	Candice Li	Carmen Du
Ricardo Lian	Emma Zhang	Ethan Yu	Ethen Li	Jack Spark Men	Jarry Wang	Cressen Liang	Leo Liu	Lucas Bai
Lycia Liu	Nina Dang	August Mao	Robin Mi	Vicky Yang	Victor Li	Yolanda Wang	Zephyra Hu	

F5

Abbie Dong	Alan Chen	Angela An	Annette Zhao	Asher Tian	Jove Bai	Camellia Long	Candice Li	Carmen Du
Ricardo Lian	Emma Zhang	Ethan Yu	Ethen Li	Jack Spark Men	Jarry Wang	Cressen Liang	Leo Liu	Lucas Bai
Lycia Liu	Nina Dang	August Mao	Robin Mi	Vicky Yang	Victor Li	Yolanda Wang	Zephyra Hu	

Turn and Talk

Let $f(x) = x^2$ with domain $x \geq 0$.

a) Find the range of f.

b) Find the domain and range of f^{-1}.

Let $f(x) = \sqrt{x-4}$.

a) Find the domain and range of f.

b) Find the domain and range of f^{-1}.

Let $f(x) = \dfrac{1}{x-2}$.

a) Find the domain and range of f.

b) State the domain and range of f^{-1}.

$f(x) = x^2, x \geq 0$

Domain of f: $[0, \infty)$

Range of f: $[0, \infty)$

Domain of f^{-1}: $[0, \infty)$

Range of f^{-1}: $[0, \infty)$

$f(x) = \sqrt{x-4}$

Domain of f: $[4, \infty)$

Range of f: $[0, \infty)$

Domain of f^{-1}: $[0, \infty)$

Range of f^{-1}: $[4, \infty)$

$f(x) = \dfrac{1}{x-2}$

Domain of f: $(-\infty, 2) \cup (2, \infty)$

Range of f: $(-\infty, 0) \cup (0, \infty)$

Domain of f^{-1}: $(-\infty, 0) \cup (0, \infty)$

Range of f^{-1}: $(-\infty, 2) \cup (2, \infty)$

Exponential Functions

- exponential function — 指数函数
- base — 底
- exponent — 指数
- growth — 增长
- decay — 衰减
- exponential growth — 指数增长
- exponential decay — 指数衰减
- initial value — 初始值
- growth factor — 增长因子
- decay factor — 衰减因子
- continuous growth — 连续增长

Reminders:

- Q2 Gradebook is closed
- Desmos Project is now due 3rd March

Exponential Functions

For example, if a population starts with P_0 individuals and then grows at an annual rate of 2%, its population after 1 year is

$$P(1) = \text{[redacted]}$$

Its population after 2 years is

$$P(2) = \text{[redacted]}.$$

In general, its population after t years is

$$P(t) = \text{[redacted]}$$

, we graph both $y = x^2$ and $y = 2^x$ to show how the graphs differ.

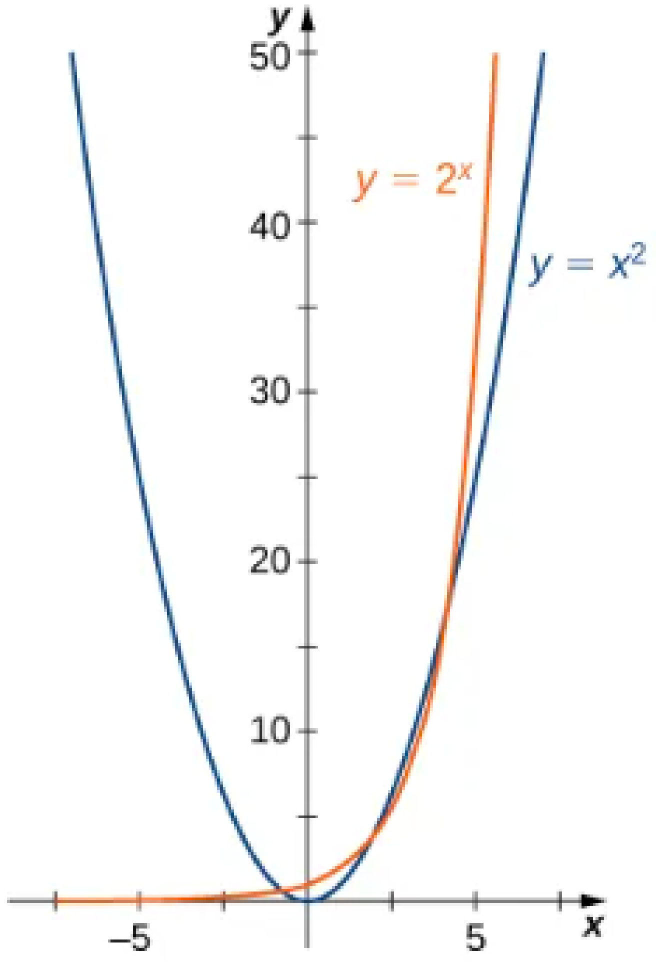


Figure 1.43 Both 2^x and x^2 approach infinity as $x \to \infty$, but 2^x grows more rapidly than x^2. As $x \to -\infty, x^2 \to \infty$, whereas $2^x \to 0$.

Bacterial Growth

Suppose a particular population of bacteria is known to double in size every 4 hours. If a culture starts with 1000 bacteria, the number of bacteria after 4 hours is $n(4) = 1000 \cdot 2$. The number of bacteria after 8 hours is $n(8) = n(4) \cdot 2 = 1000 \cdot 2^2$. In general, the number of bacteria after $4m$ hours is $n(4m) = 1000 \cdot 2^m$. Letting $t = 4m$, we see that the number of bacteria after t hours is $n(t) = 1000 \cdot 2^{t/4}$. Find the number of bacteria after 6 hours, 10 hours, and 24 hours.

The number of bacteria after 6 hours is given by $n\,(6) = 1000 \cdot 2^{6/4} \approx 2828$ bacteria. The number of bacteria after 10 hours is given by $n\,(10) = 1000 \cdot 2^{10/4} \approx 5657$ bacteria. The number of bacteria after 24 hours is given by $n\,(24) = 1000 \cdot 2^6 = 64{,}000$ bacteria.

Graphing Exponential Functions

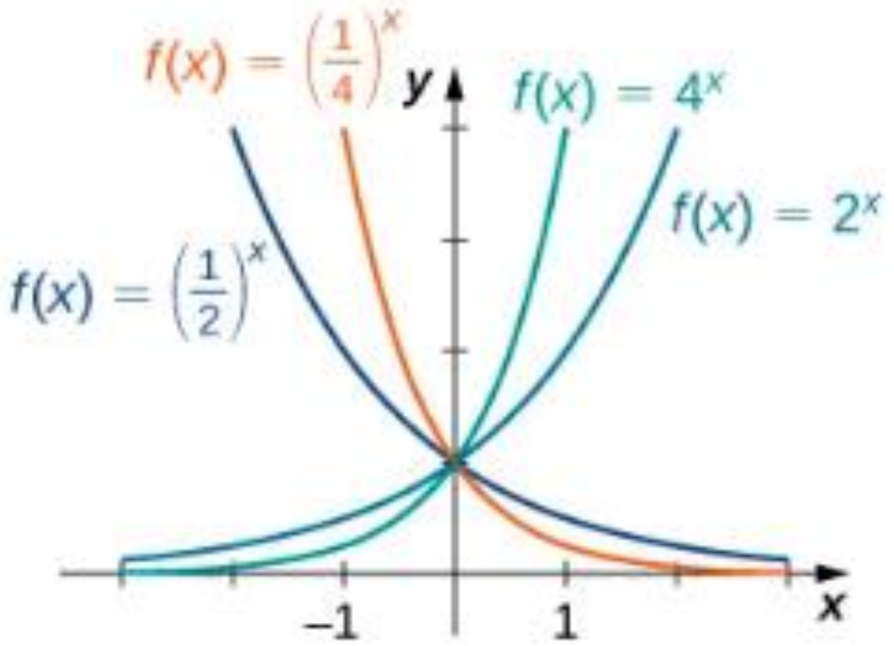


Figure 1.44 If $b > 1$, then b^x is increasing on $(-\infty, \infty)$. If $0 < b < 1$, then b^x is decreasing on $(-\infty, \infty)$.

RULE: LAWS OF EXPONENTS

For any constants $a > 0, b > 0$, and for all *x* and *y*,

1. $b^x \cdot b^y = b^{x+y}$

[redacted]

Using the Laws of Exponents

Use the laws of exponents to simplify each of the following expressions.

a. $\dfrac{\left(2x^{2/3}\right)^3}{\left(4x^{-1/3}\right)^2}$

b. $\dfrac{\left(x^3y^{-1}\right)^2}{\left(xy^2\right)^{-2}}$

a. We can simplify as follows:

$$\frac{\left(2x^{2/3}\right)^3}{\left(4x^{-1/3}\right)^2}=\frac{2^3\left(x^{2/3}\right)^3}{4^2\left(x^{-1/3}\right)^2}=\frac{8x^2}{16x^{-2/3}}=\frac{x^2x^{2/3}}{2}=\frac{x^{8/3}}{2}.$$

b. We can simplify as follows:

$$\frac{\left(x^3y^{-1}\right)^2}{\left(xy^2\right)^{-2}}=\frac{\left(x^3\right)^2\left(y^{-1}\right)^2}{x^{-2}\left(y^2\right)^{-2}}=\frac{x^6y^{-2}}{x^{-2}y^{-4}}=x^6x^2y^{-2}y^4=x^8y^2.$$

The number **e**

money after 1 year is

[redacted]

The amount of money after 2 years is

[redacted]

More generally, the amount after t years is

[redacted]

If the money is compounded 2 times per year, the amount of money after half a year is

[redacted]

The amount of money after 1 year is

[redacted]

After t years, the amount of money in the account is

[redacted]

More generally, if the money is compounded n times per year, the amount of money in the account after t years is given by the function

[redacted]

What happens as $n \to \infty$? To answer this question, we let $m = n/r$ and write

[redacted]

m	10	100	1000	10,000	100,000	1,000,000
$\left(1+\frac{1}{m}\right)^m$	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]

Table 1.12 Values of $\left(1+\frac{1}{m}\right)^m$ as $m \to \infty$

Looking at this table, it appears that $(1 + 1/m)^m$ is approaching a number between 2.7 and 2.8 as $m \to \infty$. In fact, $(1 + 1/m)^m$ does approach some number as $m \to \infty$. We call this **number** e. To six decimal places of accuracy,

$$e \approx 2.718282.$$

Compounding Interest

Suppose $\$500$ is invested in an account at an annual interest rate of $r = 5.5\%$, compounded continuously.

a. Let t denote the number of years after the initial investment and $A(t)$ denote the amount of money in the account at time t. Find a formula for $A(t)$.
b. Find the amount of money in the account after 10 years and after 20 years.

a. If P dollars are invested in an account at an annual interest rate r, compounded continuously, then $A(t) = Pe^{rt}$. Here $P = \$500$ and $r = 0.055$. Therefore, $A(t) = 500e^{0.055t}$.
b. After 10 years, the amount of money in the account is

$$A(10) = 500e^{0.055 \cdot 10} = 500e^{0.55} \approx \$866.63.$$

After 20 years, the amount of money in the account is

$$A(20) = 500e^{0.055 \cdot 20} = 500e^{1.1} \approx \$1,502.08.$$

Logarithmic Functions

$$\log_b(x) = y \text{ if and only if } b^y = x.$$

Logarithmic Functions

- logarithmic function — 对数函数
- logarithm — 对数
- base — 底
- common logarithm — 常用对数
- natural logarithm — 自然对数
- inverse function — 反函数
- log rules (properties) — 对数性质
- change of base — 换底公式
- domain — 定义域
- asymptote — 渐近线

Since the functions $f(x) = e^x$ and $g(x) = \ln(x)$ are inverses of each other,

$$\ln(e^x) = x \text{ and } e^{\ln x} = x,$$

and their graphs are symmetric about the line $y = x$ (Figure 1.46).

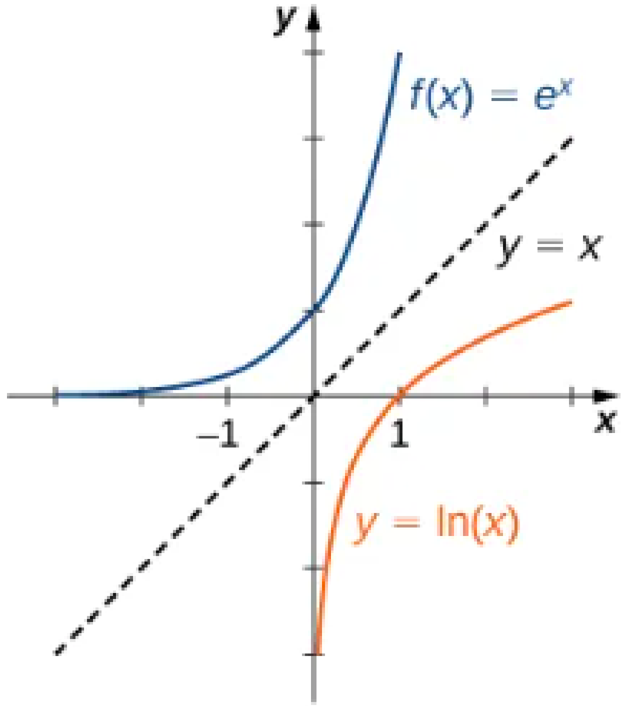
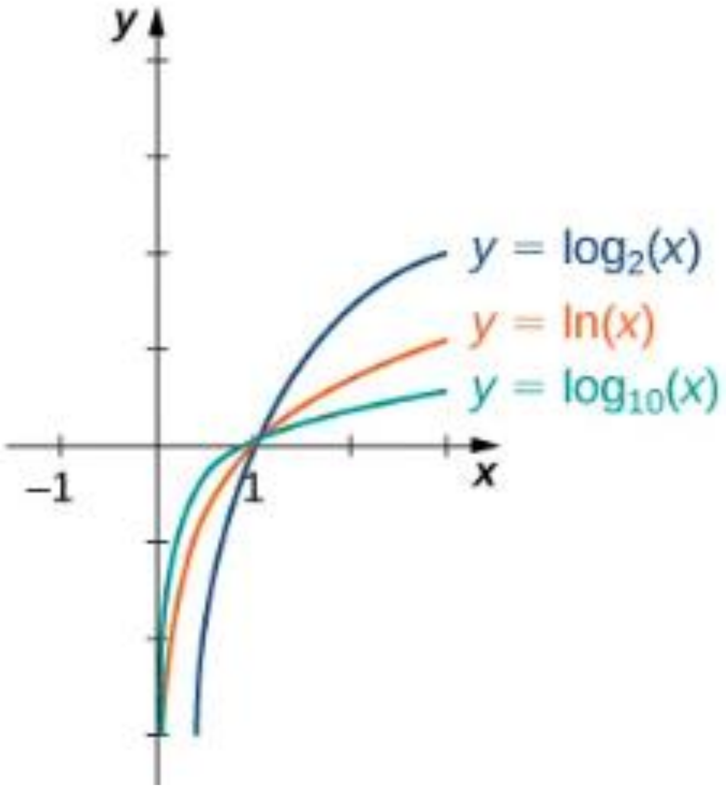


Figure 1.46 The functions $y = e^x$ and $y = \ln(x)$ are inverses of each other, so their graphs are symmetric about the line $y = x$.

In general, for any base $b > 0, b \neq 1$, the function $g(x) = \log_b(x)$ is symmetric about the line $y = x$ with the function $f(x) = b^x$. Using this fact and the graphs of the exponential functions, we graph functions $\log_b$ for several values of $b > 1$ (Figure 1.47).



RULE: PROPERTIES OF LOGARITHMS

If $a, b, c > 0, b \neq 1$, and r is any real number, then

1. $\log_b(ac) = \log_b(a) + \log_b(c)$ (Product property)
2. $\log_b\left(\frac{a}{c}\right) = \log_b(a) - \log_b(c)$ (Quotient property)
3. $\log_b(a^r) = r\log_b(a)$ (Power property)

Solving Equations Involving Exponential Functions

Solve each of the following equations for x.

a. $5^x = 2$
b. $e^x + 6e^{-x} = 5$

Therefore, $x = \ln 2/\ln 5$.

b. Multiplying both sides of the equation by e^x, we arrive at the equation

$$e^{2x} + 6 = 5e^x.$$

Rewriting this equation as

$$e^{2x} - 5e^x + 6 = 0,$$

we can then rewrite it as a quadratic equation in e^x:

$$(e^x)^2 - 5(e^x) + 6 = 0.$$

Now we can solve the quadratic equation. Factoring this equation, we obtain

$$(e^x - 3)(e^x - 2) = 0.$$

Therefore, the solutions satisfy $e^x = 3$ and $e^x = 2$. Taking the natural logarithm of both sides gives us the solutions $x = \ln 3, \ln 2$.

Solving Equations Involving Logarithmic Functions

Solve each of the following equations for x.

a. $\ln\left(\frac{1}{x}\right) = 4$
b. $\log_{10}\sqrt{x} + \log_{10}x = 2$
c. $\ln(2x) - 3\ln(x^2) = 0$

Therefore, the solution is $x = 1/e^4$.

b. Using the product and power properties of logarithmic functions, rewrite the left-hand side of the equation as

$$\log_{10}\sqrt{x} + \log_{10}x = \log_{10}x\sqrt{x} = \log_{10}x^{3/2} = \frac{3}{2}\log_{10}x.$$

Therefore, the equation can be rewritten as

$$\frac{3}{2}\log_{10}x = 2 \text{ or } \log_{10}x = \frac{4}{3}.$$

The solution is $x = 10^{4/3} = 10\sqrt[3]{10}$.

c. Using the power property of logarithmic functions, we can rewrite the equation as $\ln(2x) - \ln(x^6) = 0$.
Using the quotient property, this becomes

$$\ln\left(\frac{2}{x^5}\right) = 0.$$

Therefore, $2/x^5 = 1$, which implies $x = \sqrt[5]{2}$. We should then check for any extraneous solutions.

RULE: CHANGE-OF-BASE FORMULAS

Let $a > 0, b > 0$, and $a \neq 1, b \neq 1$.

1. $a^x = b^{x \log_b a}$ for any real number x.
   If $b = e$, this equation reduces to $a^x = e^{x \log_e a} = e^{x \ln a}$.
2. $\log_a x = \frac{\log_b x}{\log_b a}$ for any real number $x > 0$.
   If $b = e$, this equation reduces to $\log_a x = \frac{\ln x}{\ln a}$.

Use a calculating utility to evaluate $\log_3 7$ with the change-of-base formula presented earli

Use the second equation with $a = 3$ and $e = 3$:

$\log_3 7 = \frac{\ln 7}{\ln 3} \approx 1.77124.$

Hyperbolic Functions

Hyperbolic cosine

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

Hyperbolic sine

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

Hyperbolic tangent

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Hyperbolic cosecant

$$\operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}$$

Hyperbolic secant

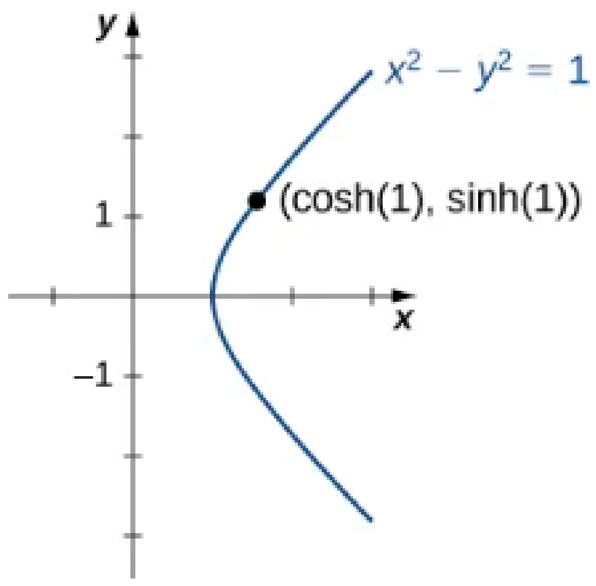
$$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$$

Hyperbolic cotangent

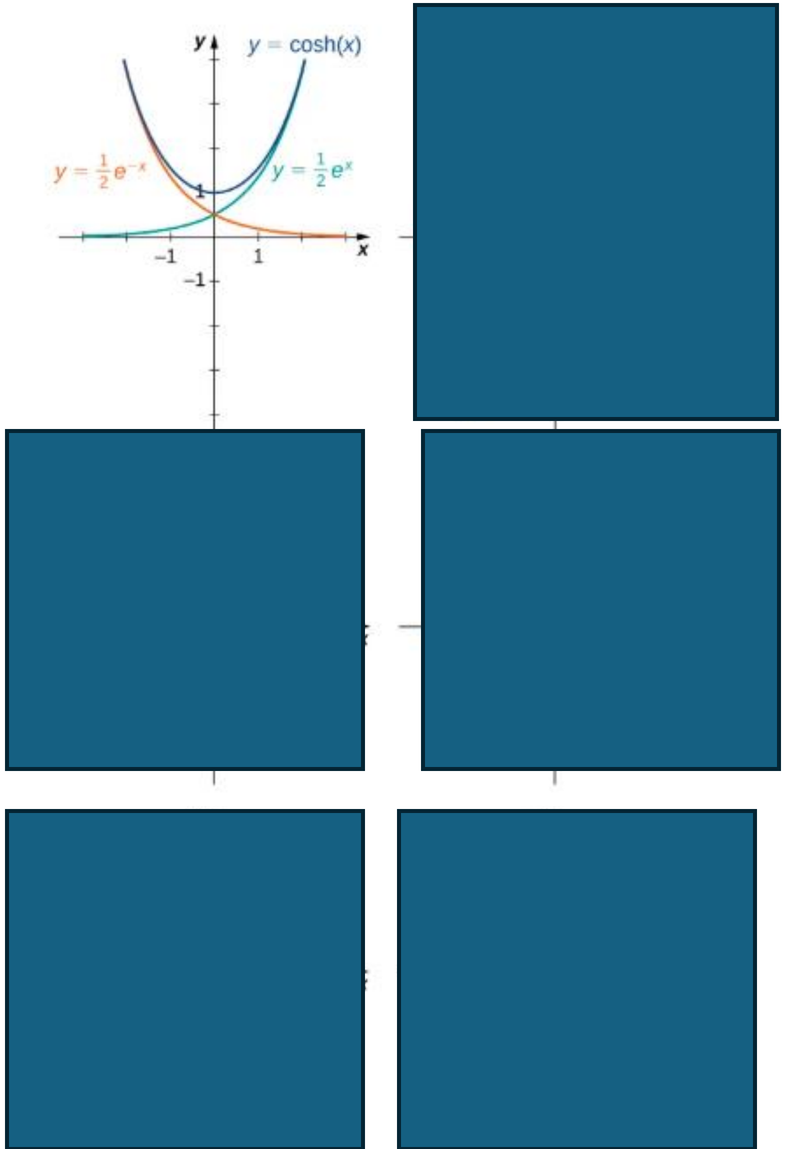
$$\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$\cosh^2 t - \sinh^2 t = \frac{e^{2t} + 2 + e^{-2t}}{4} - \frac{e^{2t} - 2 + e^{-2t}}{4} = 1.$$

This identity is the analog of the trigonometric identity $\cos^2 t + \sin^2 t = 1$. Here, given a value t, the point $(x, y) = (\cosh t, \sinh t)$ lies on the unit hyperbola $x^2 - y^2 = 1$ (Figure 1.50).



Graphs of Hyperbolic Functions



RULE: IDENTITIES INVOLVING HYPERBOLIC FUNCTIONS

1. $\cosh(-x) = \cosh x$
2. $\sinh(-x) = -\sinh x$
3. $\cosh x + \sinh x = e^x$
4. $\cosh x - \sinh x = e^{-x}$
5. $\cosh^2 x - \sinh^2 x = 1$
6. $1 - \tanh^2 x = \operatorname{sech}^2 x$
7. $\coth^2 x - 1 = \operatorname{csch}^2 x$
8. $\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$
9. $\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$

Evaluating Hyperbolic Functions

a. Simplify $\sinh(5\ln x)$.

b. If $\sinh x = 3/4$, find the values of the remaining five hyperbolic functions.

a. Using the definition of the $\sinh$ function, we write

$$\sinh(5\ln x) = \frac{e^{5\ln x} - e^{-5\ln x}}{2} = \frac{e^{\ln(x^5)} - e^{\ln(x^{-5})}}{2} = \frac{x^5 - x^{-5}}{2}.$$

b. Using the identity $\cosh^2 x - \sinh^2 x = 1$, we see that

$$\cosh^2 x = 1 + \left(\frac{3}{4}\right)^2 = \frac{25}{16}.$$

Since $\cosh x \geq 1$ for all x, we must have $\cosh x = 5/4$. Then, using the definitions for the other hyperbolic functions, we conclude that $\tanh x = 3/5, \operatorname{csch} x = 4/3, \operatorname{sech} x = 4/5$, and $\coth x = 5/3$.

Inverse Hyperbolic Functions

DEFINITION

Inverse Hyperbolic Functions

$$\sinh^{-1} x = \operatorname{arcsinh} x = \ln\left(x + \sqrt{x^2+1}\right)$$

$$\cosh^{-1} x = \operatorname{arccosh} x = \ln\left(x + \sqrt{x^2-1}\right)$$

$$\tanh^{-1} x = \operatorname{arctanh} x = \tfrac{1}{2}\ln\left(\tfrac{1+x}{1-x}\right)$$

$$\coth^{-1} x = \operatorname{arccoth} x = \tfrac{1}{2}\ln\left(\tfrac{x+1}{x-1}\right)$$

$$\operatorname{sech}^{-1} x = \operatorname{arcsech} x = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right)$$

$$\operatorname{csch}^{-1} x = \operatorname{arccsch} x = \ln\left(\frac{1}{x} + \frac{\sqrt{1+x^2}}{|x|}\right)$$

Evaluating Inverse Hyperbolic Functions

Evaluate each of the following expressions.

$$\sinh^{-1}(2)$$

$$\tanh^{-1}(1/4)$$

$$\sinh^{-1}(2) = \ln\left(2 + \sqrt{2^2 + 1}\right) = \ln\left(2 + \sqrt{5}\right) \approx 1.4436$$

$$\tanh^{-1}(1/4) = \frac{1}{2}\ln\left(\frac{1+1/4}{1-1/4}\right) = \frac{1}{2}\ln\left(\frac{5/4}{3/4}\right) = \frac{1}{2}\ln\left(\frac{5}{3}\right) \approx 0.2554$$