F1

Annie Bai	Franklin Chen	Charlie Cheng	Zoey Cheng	Harry Cui	King Deng	Aisling Fu	Ken He	Ivy Jing
Lee Li	Ziheng Li	Lewis Liu	Sam Liu	Ava Peng	Peter Shen	Sylvia Song	Coco Wang	Eileen Wang
Jason Wang	Luna Wang	Cathy Yan	Melody You	Allen Zhang	Lucas Zhang	Lynette Zhang	Ryder Zhang	Flora Zhou
Cyntina Zuo								

F3

Allen Peng	Aurora Yuan	Brittney Wei	Cynthia Liu	Erya Hu	Eva Gai	Felicity Pan	Fiona Ding	Freya Fan
Gordon Yao	Helios Luo	Honey Ruan	Iris Xie	Jasper Wu	Kaven Zhang	Kevin Gao	Leo Yang	Linger Li
Lydia Wei	Maggie Gao	Micheal Zhao	Ray Meng	Rose Jiang	Ross Ma	Roy Liu	Ryan Wang	Sky Bai
Star Su	Stella Xi							

F3

Allen Peng	Aurora Yuan	Brittney Wei	Cynthia Liu	Erya Hu	Eva Gai	Felicity Pan	Fiona Ding	Freya Fan
Gordon Yao	Helios Luo	Honey Ruan	Iris Xie	Jasper Wu	Kaven Zhang	Kevin Gao	Leo Yang	Linger Li
Lydia Wei	Maggie Gao	Micheal Zhao	Ray Meng	Rose Jiang	Ross Ma	Roy Liu	Ryan Wang	Sky Bai
Star Su	Stella Xi							

F2

Angelina Wang	Azura Cheng	Bonnie Zhao	Cecilia Jia	Diana Li	Oscar Liu	Zavier Liu	Tom Ou	Mia Peng
Joshua Hui	Julius Lv	Justin Jia	Lucas Li	Lyra Zhang	Iris Shao	Dylan Suo	PeytonAson Wang	Struck Wang
Selene Hou	Serena Feng	Silas Lv	Simon Wang	Stella Sun	Yola Wang	Mark Xiao	Elsa Ye	Emily Zhang
Zinnia Dong	Flora Zhang							

F5

Abbie Dong	Alan Chen	Angela An	Annette Zhao	Asher Tian	Jove Bai	Camellia Long	Candice Li	Carmen Du
Ricardo Lian	Emma Zhang	Ethan Yu	Ethen Li	Jack Spark Men	Jarry Wang	Cressen Liang	Leo Liu	Lucas Bai
Lycia Liu	Nina Dang	August Mao	Robin Mi	Vicky Yang	Victor Li	Yolanda Wang	Zephyra Hu	

F6

Lucas Chang	Chloe Fang	Bruce Gao	Wyatt Hou	Jackson Jiang	Ivan Lei	Eric Li	Keira Li	Vivian Li
Jeremy Lin	Orange Liu	Serika Ren	Yumiko Shi	Hannah Si	Eric Song	Eric Tan	Silas Wang	Niki Wei
Kiki Wen	Miyu Wu	Tia Wu	Kumi Yuan	Estelle Zhang	Soren Zhang	Viakey Zhang	Jack Zhao	

Turn and Talk

Solve

$$\frac{x+1}{x-2} > 0$$

Turn and Talk

Solve

$$\frac{x+1}{x-2} > 0$$

$$\boxed{(-\infty, -1) \cup (2, \infty)}$$

REVIEW OF FUNCTIONS

function — 函数
equation — 方程
variable — 变量
independent variable — 自变量
dependent variable — 因变量
domain — 定义域
range — 值域
input — 输入
output — 输出
mapping — 映射
graph — 图像
coordinate plane — 坐标平面
ordered pair — 有序对
vertical line test — 垂直线检验
one-to-one function — 一一函数
many-to-one function — 多对一函数
inverse function — 反函数
composite function — 复合函数
piecewise function — 分段函数
linear function — 线性函数

REMINDERS -
Refection – part of Final Exam - Today
Final Exam Feedback Quiz – Jupiter - Today
Desmos Art Project – 2nd Feb

DEFINITION

A **function** f consists of a set of inputs, a set of outputs, and a rule for assigning each input to exactly one output. The set of inputs is called the **domain** of the function. The set of outputs is called the **range** of the function.

https://openstax.org/books/calculus-volume-1/pages/1-1-review-of-functions

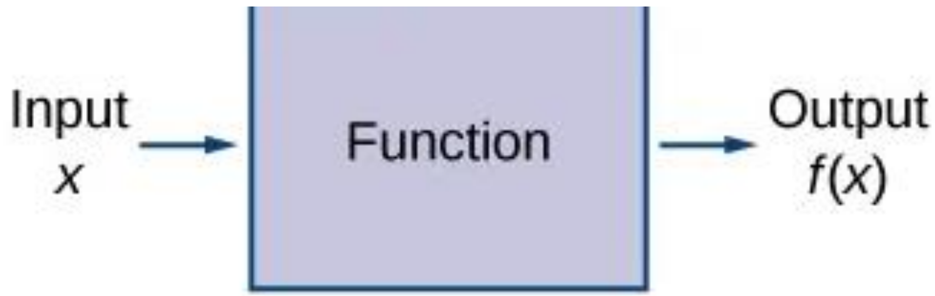
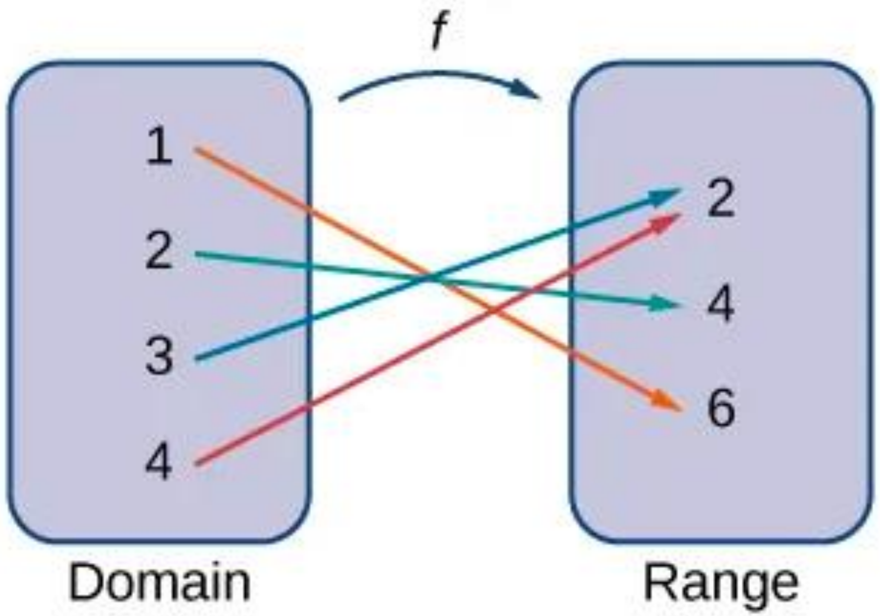


Figure 1.2 A function can be visualized as an input/output device.



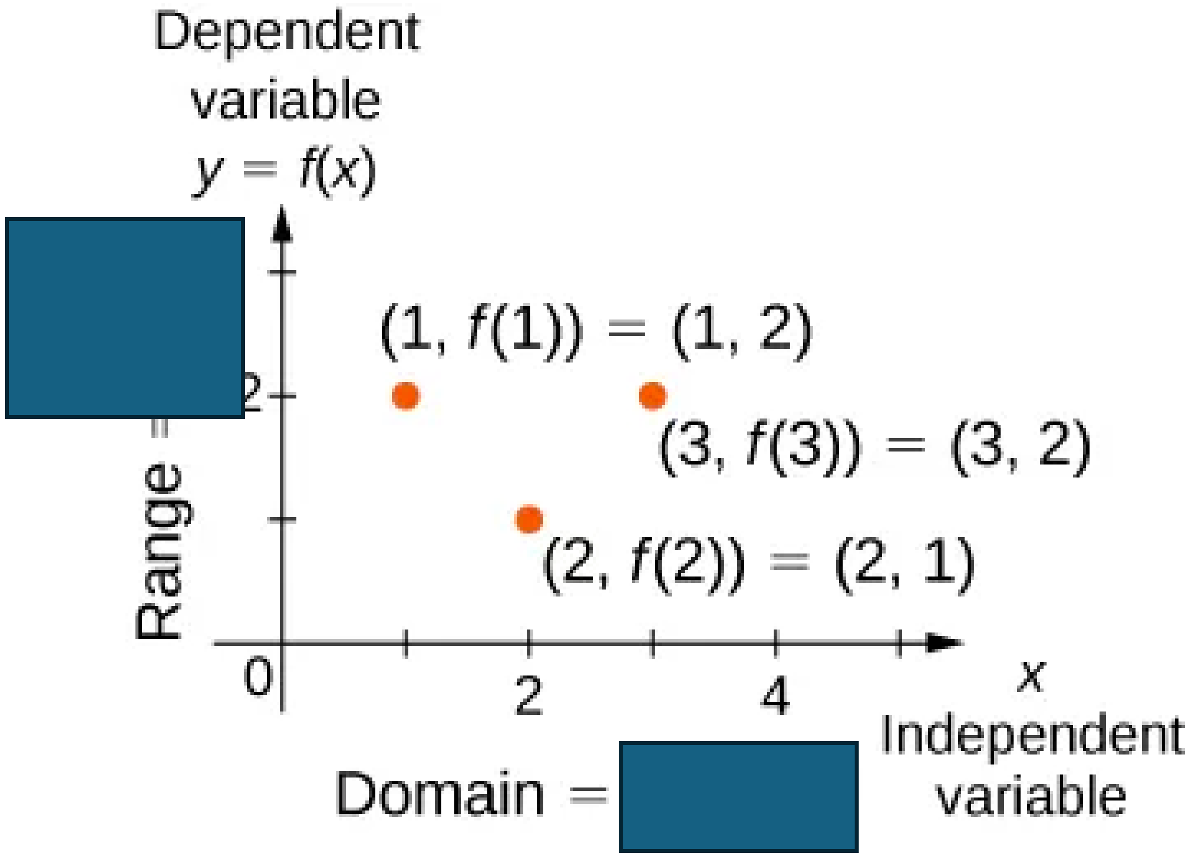


Figure 1.4 In this case, a graph of a function f has a domain of $\{1, 2, 3\}$ and a range of $\{1, 2\}$. The independent variable is x and the dependent variable is y.

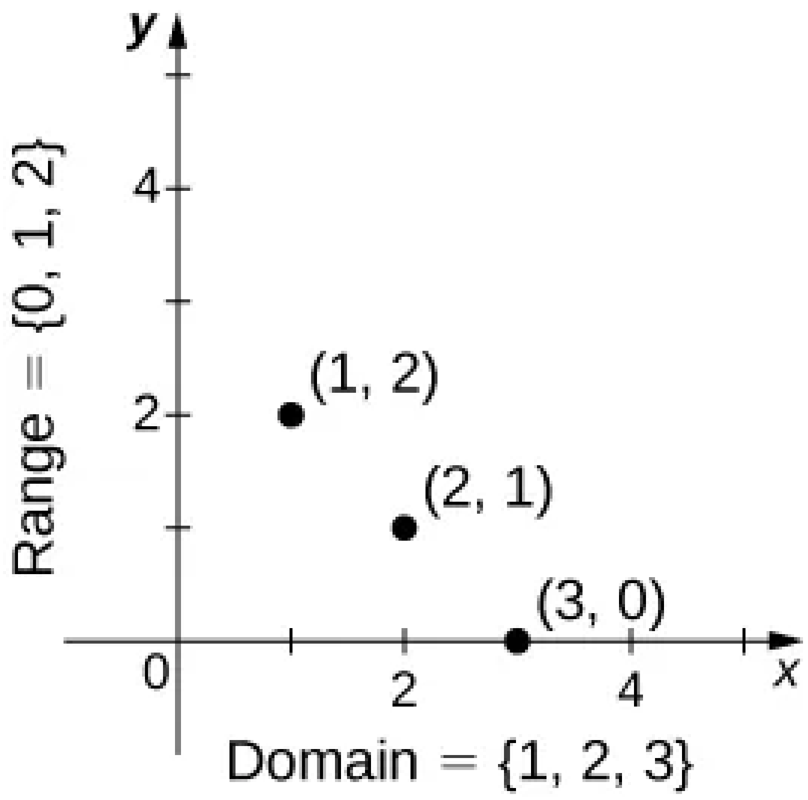
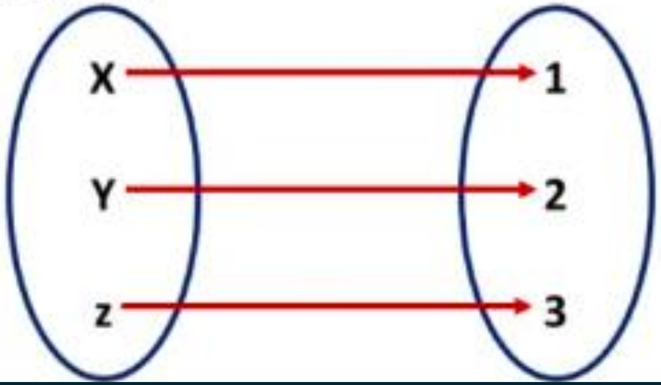
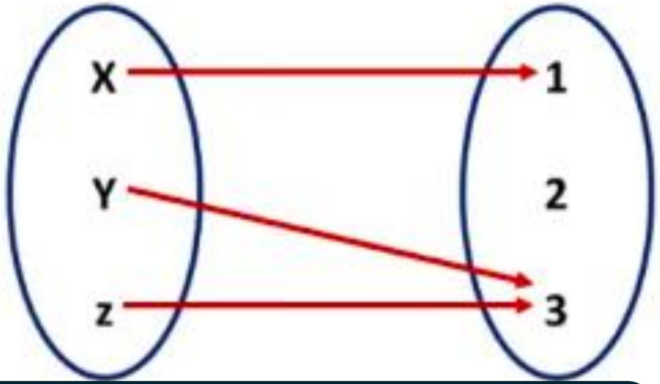


Figure 1.5 Here we see a graph of the function f with domain $\{1, 2, 3\}$ and rule $f(x) =$ [redacted] The graph consists of the points $(x, f(x))$ for all x in the domain.

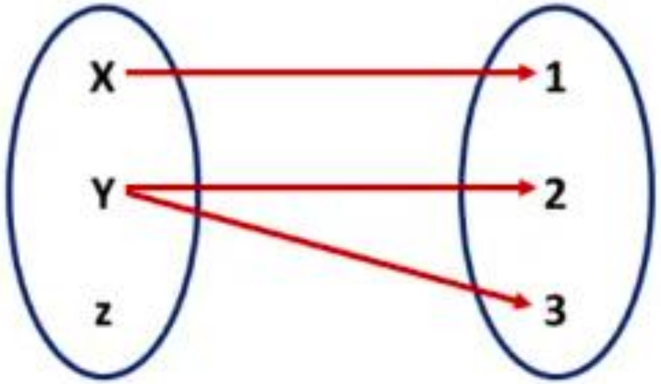
MAPPING OF ELEMENTS

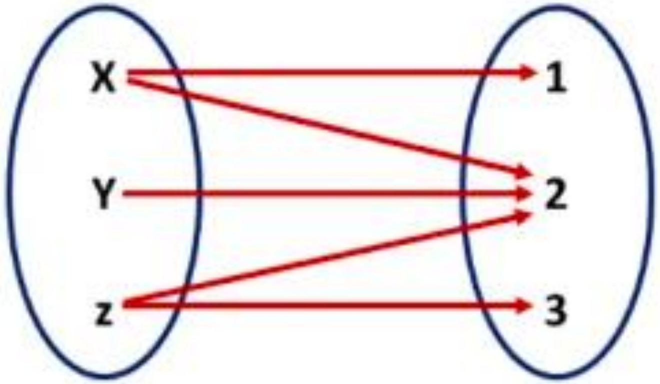




ONE TO ONE

MANY TO ONE





ONE TO MANY

ONE TO MANY

Here are some **equations**. Decide **which define functions** (of x):

1. $y = 2x + 3$
2. $x^2 + y^2 = 1$
3. $y^2 = x$
4. $y = \sqrt{x}$
5. $y = \pm\sqrt{x}$
6. $x = 5$
7. $y = |x|$

1. $y = 2x + 3$ ✅
2. $x^2 + y^2 = 1$ ❌
3. $y^2 = x$ ❌
4. $y = \sqrt{x}$ ✅
5. $y = \pm\sqrt{x}$ ❌
6. $x = 5$ ❌
7. $y = |x|$ ✅

$$\{x|1 < x < 5\}.$$

A set such as this, which contains all numbers greater than a and less than b, can also be denoted using the interval notation (a, b). Therefore,

$$(1,5) = \{x|1 < x < 5\}.$$

The numbers 1 and 5 are called the *endpoints* of this set. If we want to consider the set that includes the endpoints, we would denote this set by writing

$[1,5] =$ [redacted] .

We can use similar notation if we want to include one of the endpoints, but not the other. To denote the set of nonnegative real numbers, we would use the set-builder notation

[redacted]

The smallest number in this set is zero, but this set does not have a largest number. Using interval notation, we would use the symbol ∞, which refers to positive infinity, and we would write the set as

$[0, \infty) =$ [redacted]

Piecewise Functions

Some functions are defined using different equations for different parts of their domain. These types of functions are known as *piecewise-defined functions*. For example, suppose we want to define a function f with a domain that is the set of all real numbers such that $f(x) = 3x + 1$ for $x \geq 2$ and $f(x) = x^2$ for $x < 2$. We denote this function by writing

$$f(x) = \begin{cases} 3x + 1 & x \geq 2 \\ x^2 & x < 2 \end{cases}.$$

EXAMPLE 1.1

Evaluating Functions

For the function $f(x) = 3x^2 + 2x - 1$, evaluate

a. $f(-2)$
b. $f\left(\sqrt{2}\right)$
c. $f(a + h)$

EXAMPLE 1.1

Evaluating Functions

For the function $f(x) = 3x^2 + 2x - 1$, evaluate

a. $f(-2)$
b. $f(\sqrt{2})$
c. $f(a+h)$

a. $f(-2) = 3(-2)^2 + 2(-2) - 1 = 12 - 4 - 1 = 7$
b. $f(\sqrt{2}) = 3(\sqrt{2})^2 + 2\sqrt{2} - 1 = 6 + 2\sqrt{2} - 1 = 5 + 2\sqrt{2}$
c.
$$\begin{aligned} f(a+h) = 3(a+h)^2 + 2(a+h) - 1 &= 3(a^2 + 2ah + h^2) + 2a + 2h - 1 \\ &= 3a^2 + 6ah + 3h^2 + 2a + 2h - 1 \end{aligned}$$

Finding Domain and Range

For each of the following functions, determine the i. domain and ii. range.

a. $f(x) = (x-4)^2 + 5$
b. $f(x) = \sqrt{3x+2} - 1$
c. $f(x) = \frac{3}{x-2}$

Finding Domain and Range

For each of the following functions, determine the i. domain and ii. range.

a. $f(x) = (x - 4)^2 + 5$

b. $f(x) = \sqrt{3x + 2} - 1$

c. $f(x) = \frac{3}{x - 2}$

Domain: $oo < x < oo$

Range $y \geq 5$

Domain: $x > -\frac{2}{3}$

Range $y \geq -1$

c. Consider $f(x) = 3/(x - 2)$.

i. Since $3/(x - 2)$ is defined when the denominator is nonzero, the domain is $\{x \mid x \neq 2\}$.

ii. To find the range of f, we need to find the values of y such that there exists a real number x in the domain with the property that

$$\frac{3}{x - 2} = y.$$

Solving this equation for x, we find that

$$x = \frac{3}{y} + 2.$$

Therefore, as long as $y \neq 0$, there exists a real number x in the domain such that $f(x) = y$. Thus, the range is $\{y \mid y \neq 0\}$.

Representing Functions

Typically, a function is represented using one or more of the following tools:

- A table
- A graph
- A formula

Hours after Midnight	Temperature $(^\circ F)$	Hours after Midnight	Temperature $(^\circ F)$
0	58	12	84
1	54	13	85
2	53	14	85
3	52	15	83
4	52	16	82
5	55	17	80
6	60	18	77
7	64	19	74
8	72	20	69
9	75	21	65
10	78	22	60
11	80	23	58

Table 1.1 Temperature as a Function of Time of Day

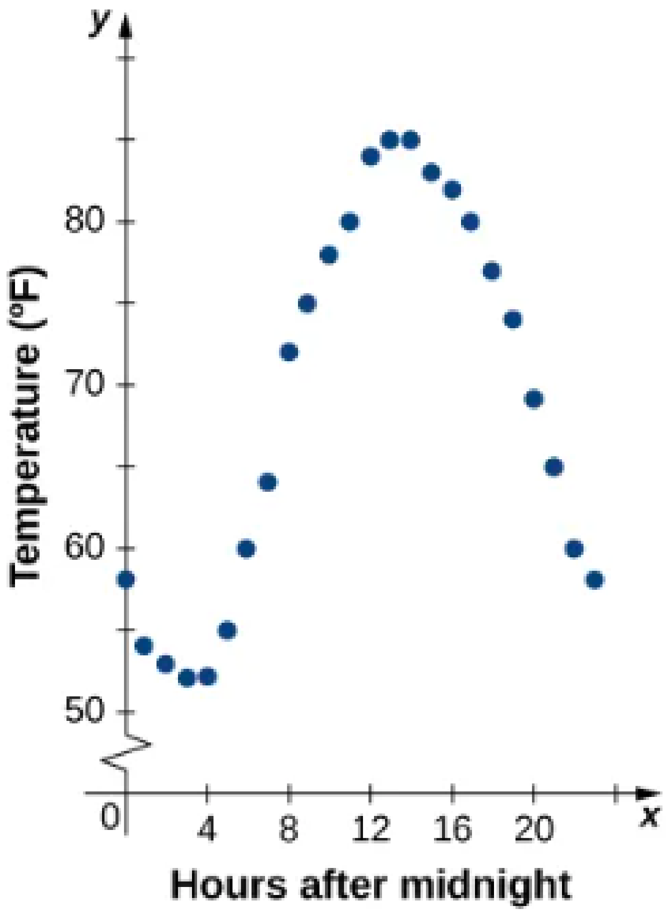
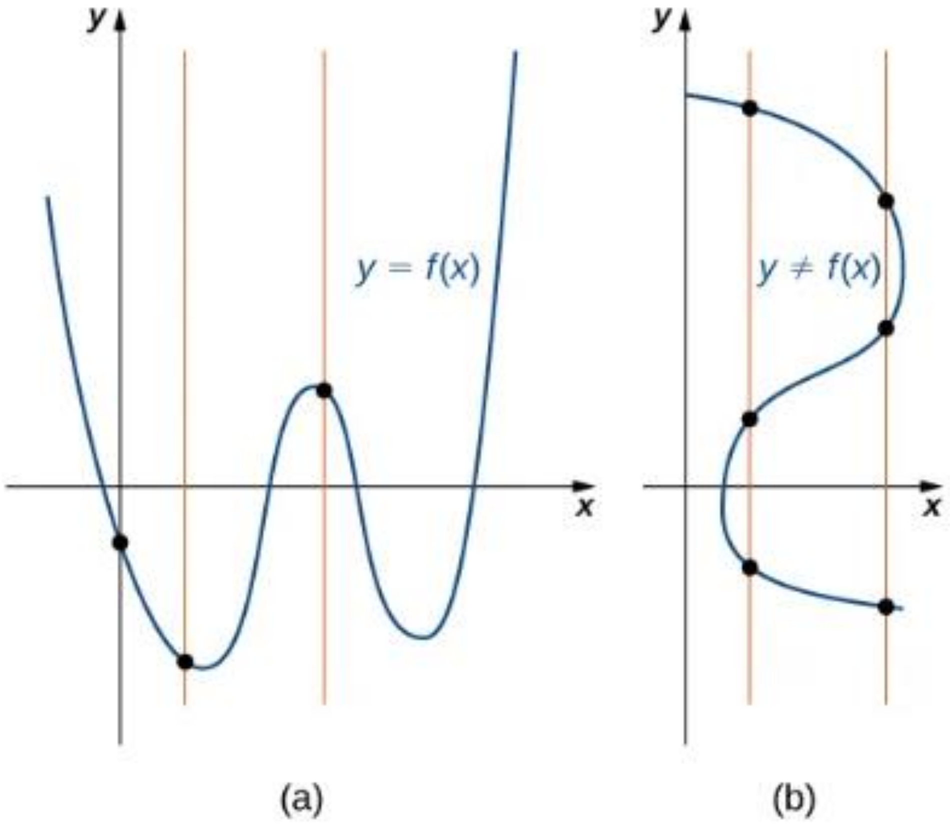


Figure 1.6 The graph of the data from Table 1.1 shows temperature as a function of time.

RULE: VERTICAL LINE TEST

Given a function f, every vertical line that may be drawn intersects the graph of f no more than once. If any vertical line intersects a set of points more than once, the set of points does not represent a function.

We can use this test to determine whether a set of plotted points represents the graph of a function (Figure 1.8).



EXAMPLE 1.3

Finding Zeros and y-Intercepts of a Function

Consider the function $f(x) = -4x + 2$.

a. Find all zeros of f.
b. Find the y-intercept (if any).
c. Sketch a graph of f.

EXAMPLE 1.3

Finding Zeros and y-Intercepts of a Function

Consider the function $f(x) = -4x + 2$.

a. Find all zeros of f.
b. Find the y-intercept (if any).
c. Sketch a graph of f.

a. To find the zeros, solve $f(x) = -4x + 2 = 0$. We discover that f has one zero at $x = 1/2$.
b. The y-intercept is given by $(0, f(0)) = (0, 2)$.
c. Given that f is a linear function of the form $f(x) = mx + b$ that passes through the points $(1/2, 0)$ and $(0, 2)$, we can sketch the graph of f (Figure 1.9).

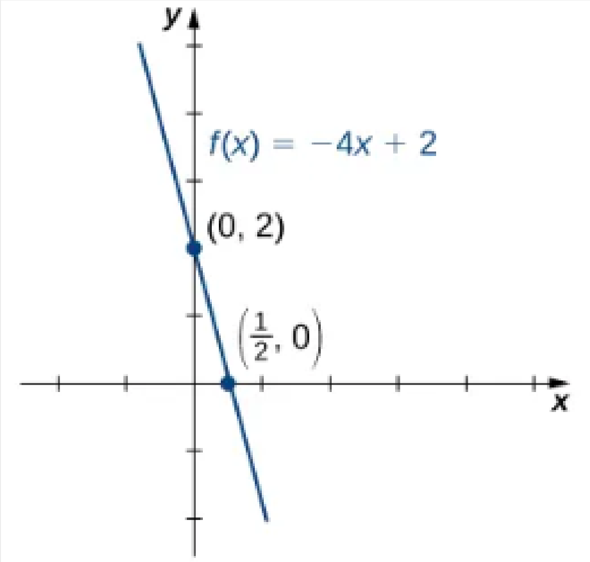


Figure 1.9 The function $f(x) = -4x + 2$ is a line with x-intercept $(1/2, 0)$ and y-intercept $(0, 2)$.

Increasing/ Decreasing Function

DEFINITION

We say that a function f is **increasing on the interval** I if for all $x_1, x_2 \in I$,

[redacted] when $x_1 < x_2$.

We say f is strictly increasing on the interval I if for all $x_1, x_2 \in I$,

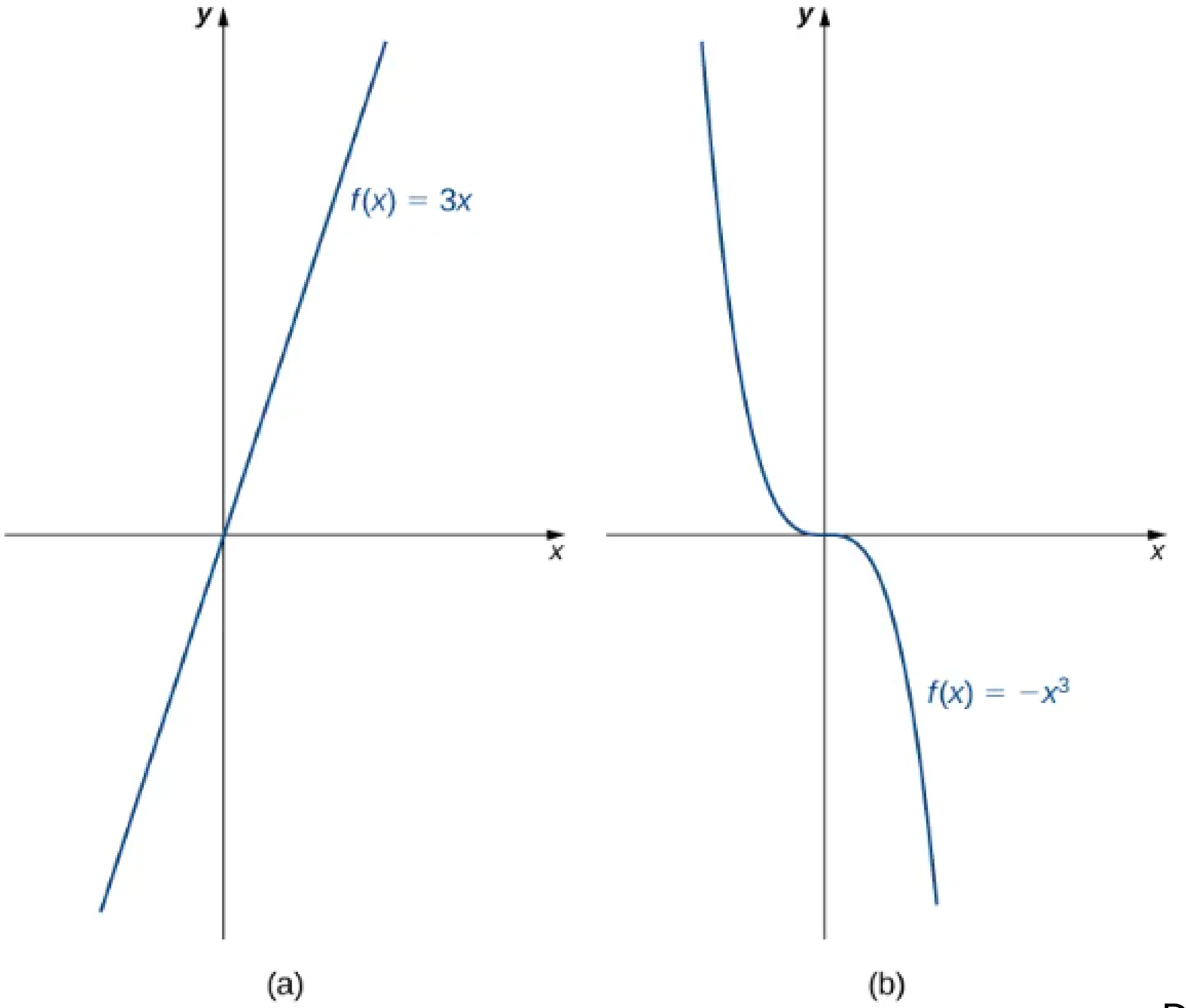
[redacted] when $x_1 < x_2$.

We say that a function f is **decreasing on the interval** I if for all $x_1, x_2 \in I$,

[redacted] $x_1 < x_2$.

We say that a function f is strictly decreasing on the interval I if for all $x_1, x_2 \in I$,

[redacted] if $x_1 < x_2$.



DECREASING

INCREASING

Combining Functions

$$(f+g)(x)=f(x)+g(x) \qquad Sum$$

$$(f-g)(x)=f(x)-g(x) \qquad Difference$$

$$(f\cdot g)(x)=f(x)\,g(x) \qquad Product$$

$$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)} \text{ for } g(x)\neq 0 \qquad Quotient$$

EXAMPLE 1.6

Combining Functions Using Mathematical Operations

Given the functions $f(x) = 2x - 3$ and $g(x) = x^2 - 1$, find each of the following functions and state its domain.

a. $(f + g)(x)$
b. $(f - g)(x)$
c. $(f \cdot g)(x)$
d. $\left(\frac{f}{g}\right)(x)$

EXAMPLE 1.6

Combining Functions Using Mathematical Operations

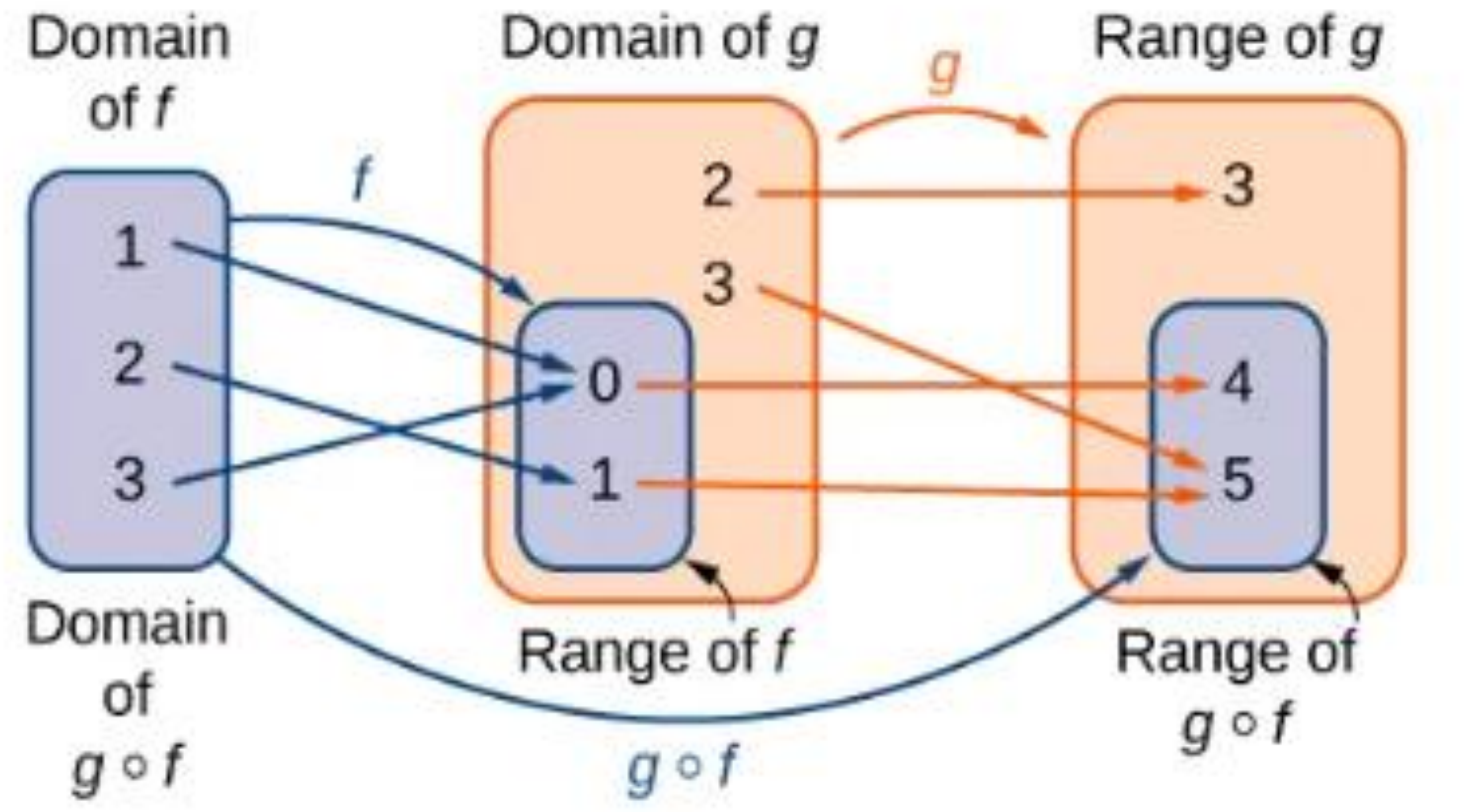
Given the functions $f(x) = 2x - 3$ and $g(x) = x^2 - 1$, find each of the following functions and state its domain.

a. $(f + g)(x)$
b. $(f - g)(x)$
c. $(f \cdot g)(x)$
d. $\left(\frac{f}{g}\right)(x)$

Solution

a. $(f + g)(x) = (2x - 3) + (x^2 - 1) = x^2 + 2x - 4$. The domain of this function is the interval $(-\infty, \infty)$.
b. $(f - g)(x) = (2x - 3) - (x^2 - 1) = -x^2 + 2x - 2$. The domain of this function is the interval $(-\infty, \infty)$.
c. $(f \cdot g)(x) = (2x - 3)(x^2 - 1) = 2x^3 - 3x^2 - 2x + 3$. The domain of this function is the interval $(-\infty, \infty)$.
d. $\left(\frac{f}{g}\right)(x) = \frac{2x-3}{x^2-1}$. The domain of this function is $\{x | x \neq \pm 1\}$.

Composite Functions



Compositions of Functions Defined by Formulas

Consider the functions $f(x) = x^2 + 1$ and $g(x) = 1/x$.

a. Find $(g \circ f)(x)$ and state its domain and range.
b. Evaluate $(g \circ f)(4), (g \circ f)(-1/2)$.
c. Find $(f \circ g)(x)$ and state its domain and range.
d. Evaluate $(f \circ g)(4), (f \circ g)(-1/2)$.

Compositions of Functions Defined by Formulas

Consider the functions $f(x) = x^2 + 1$ and $g(x) = 1/x$.

a. Find $(g \circ f)(x)$ and state its domain and range.
b. Evaluate $(g \circ f)(4), (g \circ f)(-1/2)$.
c. Find $(f \circ g)(x)$ and state its domain and range.
d. Evaluate $(f \circ g)(4), (f \circ g)(-1/2)$.

$$g(f(x)) = \frac{1}{x^2 + 1}$$

Domain: $oo < x < oo$

Range $0 < g(f(x)) < 1$

b. $(g \circ f)(4) = g(f(4)) = g(4^2 + 1) = g(17) = \frac{1}{17}$
$(g \circ f)\left(-\frac{1}{2}\right) = g\left(f\left(-\frac{1}{2}\right)\right) = g\left(\left(-\frac{1}{2}\right)^2 + 1\right) = g\left(\frac{5}{4}\right) = \frac{4}{5}$

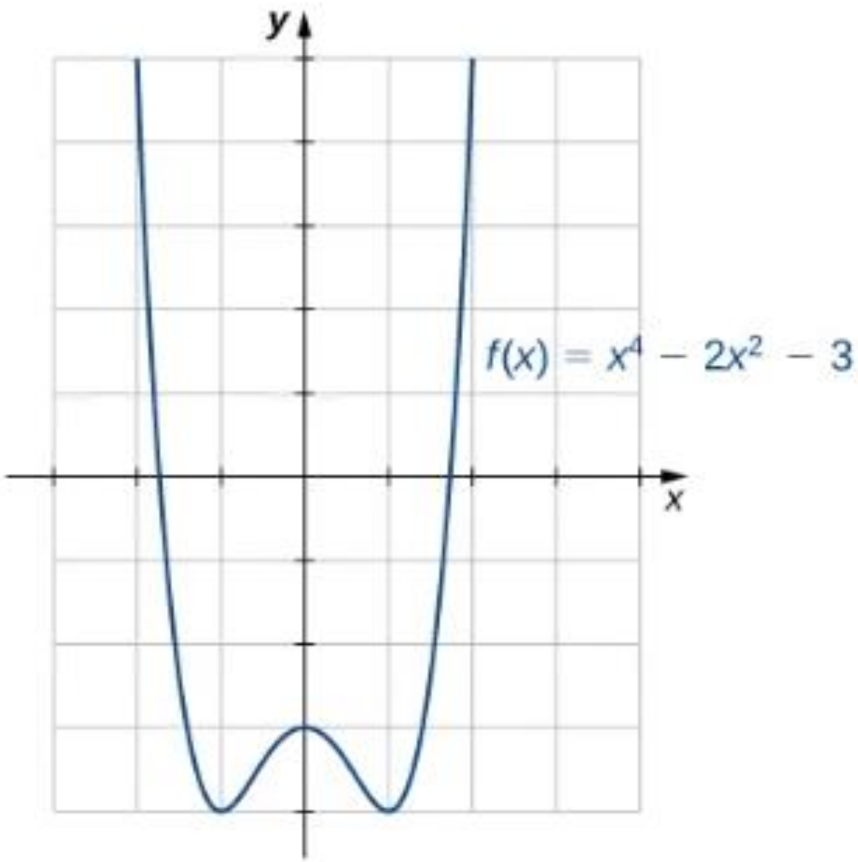
Since $1/\sqrt{y-1}$ is a real number if and only if $y > 1$, the range of f is the set $\{y | y > 1\}$.
d. $(f \circ g)(4) = f(g(4)) = f\left(\frac{1}{4}\right) = \left(\frac{1}{4}\right)^2 + 1 = \frac{17}{16}$
$(f \circ g)\left(-\frac{1}{2}\right) = f\left(g\left(-\frac{1}{2}\right)\right) = f(-2) = (-2)^2 + 1 = 5$

Symmetry of Functions

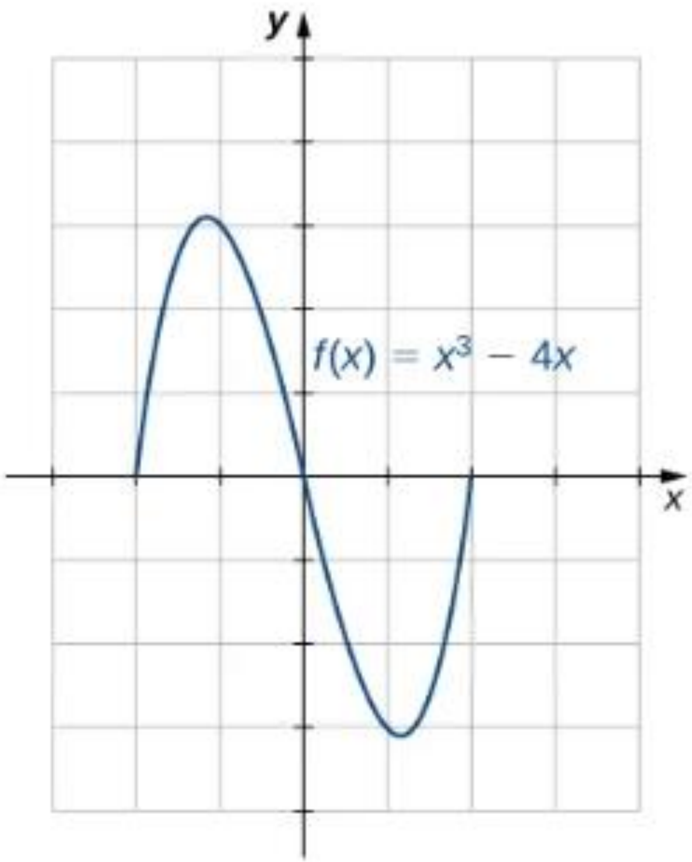
DEFINITION

If $f(x) = f(-x)$ for all x in the domain of f, then f is an **even function.** An even function is symmetric about the *y*-axis.

If $f(-x) = -f(x)$ for all x in the domain of f, then f is an **odd function.** An odd function is symmetric about the origin.



(a) Symmetry about the *y*-axis



(b) Symmetry about the origin

Even and Odd Functions

Determine whether each of the following functions is even, odd, or neither.

a. $f(x) = -5x^4 + 7x^2 - 2$
b. $f(x) = 2x^5 - 4x + 5$
c. $f(x) = \frac{3x}{x^2+1}$

Even and Odd Functions

Determine whether each of the following functions is even, odd, or neither.

a. $f(x) = -5x^4 + 7x^2 - 2$
b. $f(x) = 2x^5 - 4x + 5$
c. $f(x) = \frac{3x}{x^2+1}$

Solution

To determine whether a function is even or odd, we evaluate $f(-x)$ and compare it to *f*(*x*) and $-f(x)$.

a. $f(-x) = -5(-x)^4 + 7(-x)^2 - 2 = -5x^4 + 7x^2 - 2 = f(x)$. Therefore, f is even.
b. $f(-x) = 2(-x)^5 - 4(-x) + 5 = -2x^5 + 4x + 5$. Now, $f(-x) \neq f(x)$. Furthermore, noting that $-f(x) = -2x^5 + 4x - 5$, we see that $f(-x) \neq -f(x)$. Therefore, f is neither even nor odd.
c. $f(-x) = 3(-x)/((-x)^2 + 1) = -3x/(x^2 + 1) = -[3x/(x^2 + 1)] = -f(x)$. Therefore, f is odd.

Desmos Art Project- 2nd Feb

1. Upload an image into **Desmos** and adjust its position and scale.
2. Trace the image using a variety of **functions** (polynomial, rational, radical, trigonometric, exponential, logarithmic).
3. Use **domain and range restrictions** to control each curve.
4. Combine multiple equations to accurately represent the image.
5. Clearly label your equations and submit your completed Desmos graph.

$$\frac{(x+4)^2}{1.3225}+\frac{(y+7.3)^2}{.36}=1$$

$$(y+6.8)^2=-.8(x-1.1)\ \{-.1<x<1.$$

$$(x+.5)^2=2(y+7.9)\ \{-.7<x<0\}$$

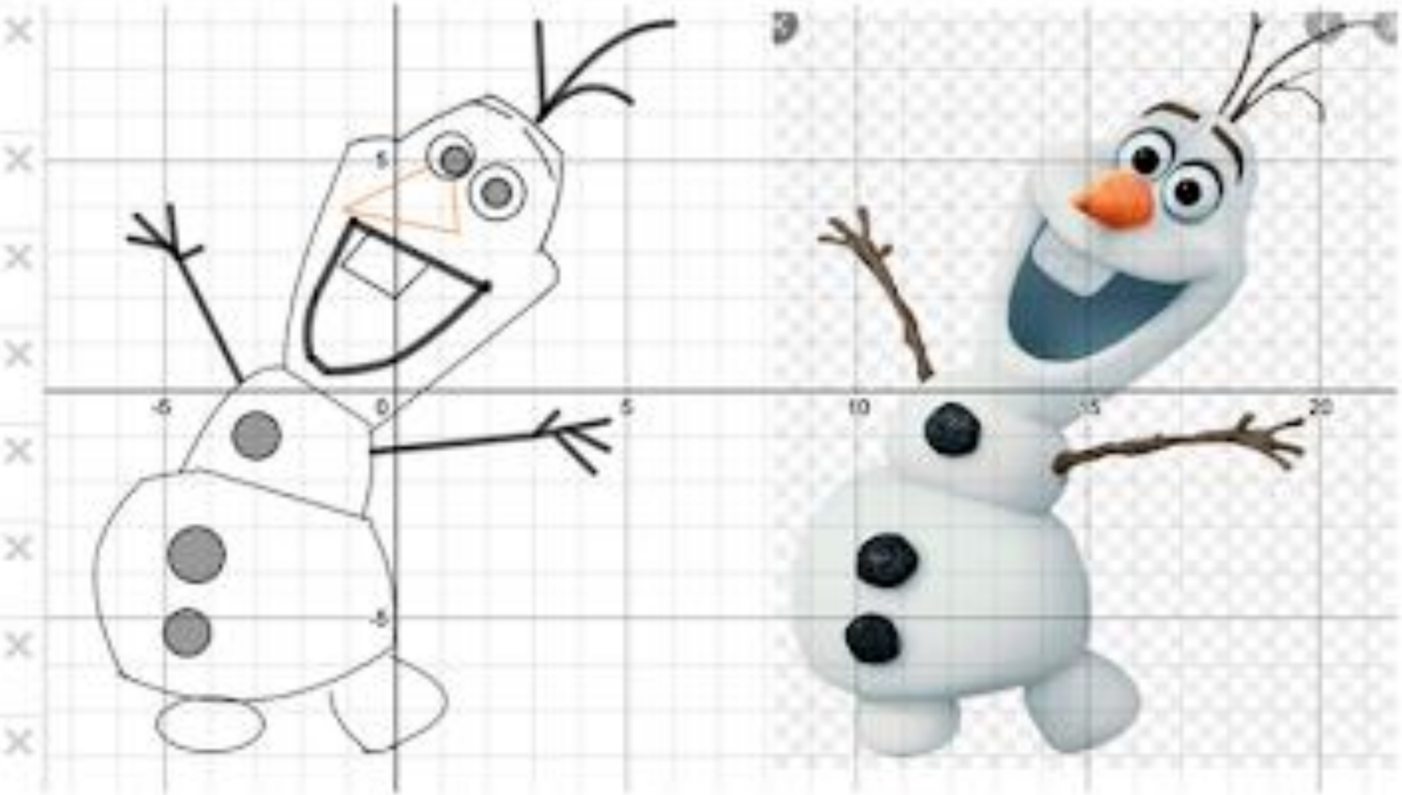
$$(y+6.6)^2=2(x+1.4)\ \{-6.6>y>-7$$

$$(x+3.5)^2=10(y+6.8)\ \{-6<x<-0$$

$$(y+4)^2=8(x+6.5)\ \{-6.3<y<-4\}$$

$$(y+4)^2=4(x+6.5)\ \{-4<y<-2\}$$

$$(y+5)^2=-8(x)\ \{-5.5<y<-2.9\}$$



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