Turn and Talk

1. Differentiate $y = x^2 \sin x$.
2. Differentiate $y = \cos(3x^2 + 1)$.
3. Differentiate $y = \dfrac{\tan x}{x^2 + 1}$.

Turn and Talk

1. $y = x^2 \sin x$

Using the product rule:

$$\frac{dy}{dx} = 2x \sin x + x^2 \cos x$$

2. $y = \cos(3x^2 + 1)$

Using the chain rule:

$$\frac{dy}{dx} = -\sin(3x^2 + 1)(6x)$$

$$\frac{dy}{dx} = -6x \sin(3x^2 + 1)$$

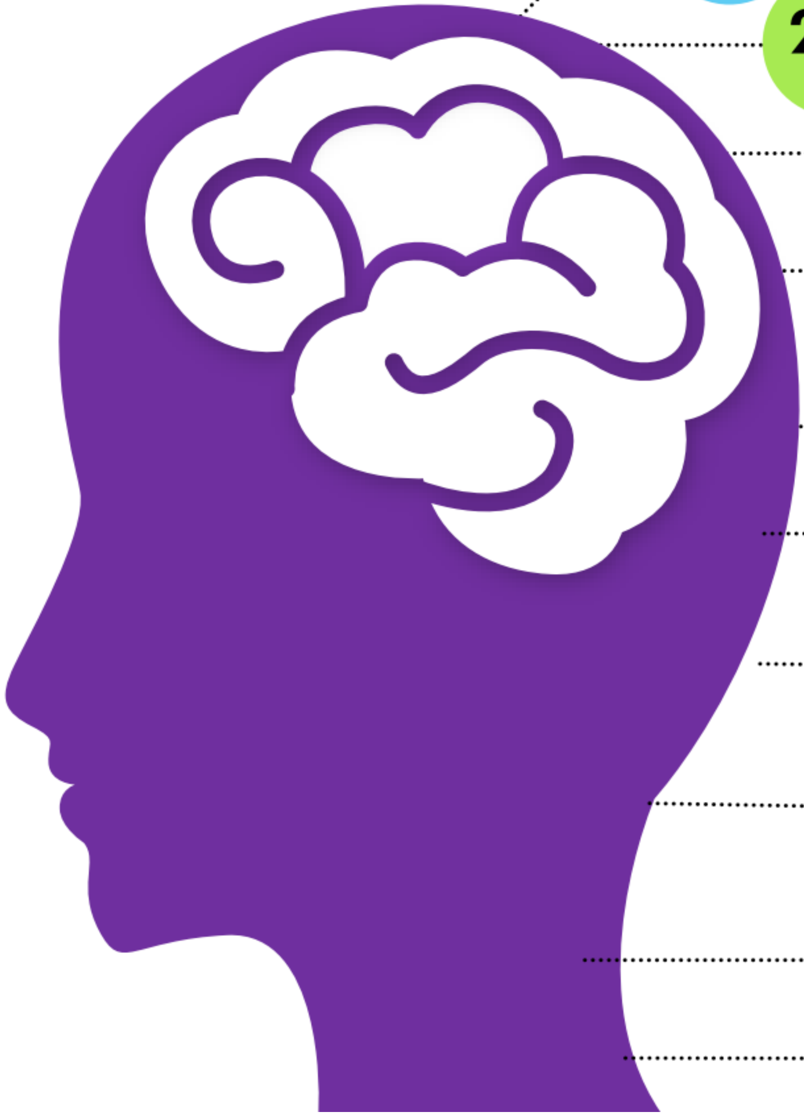
3. $y = \dfrac{\tan x}{x^2 + 1}$

Using the quotient rule:

$$\frac{dy}{dx} = \frac{(x^2 + 1)\sec^2 x - \tan x(2x)}{(x^2 + 1)^2}$$

1. Exponential function — 指数函数
2. Natural exponential function — 自然指数函数
3. Natural logarithm — 自然对数
4. Logarithmic function — 对数函数
5. Base e — 自然常数 e
6. Derivative of e^x — e^x 的导数
7. Derivative of $\ln x$ — $\ln x$ 的导数
8. Chain rule — 链式法则
9. Logarithmic differentiation — 对数求导法
10. Constant multiple rule — 常数倍法则

UNIT 2 KNOWLEDGE - CALCULUS 12 – DIFFERENTIATION: DEFINITION AND FUNDAMENTAL PROPERTIES



2.1
- Defining Average and Instantaneous Rates of Change at a Point ✓

2.2
- Defining the Derivative of a Function and Using Derivative Notation ✓

2.3
- Estimating Derivatives of a Function at a Point ✓

2.4
- Connecting Differentiability and Continuity: Determining When Derivatives Do and Do Not Exist ✓

2.5
- Applying the Power Rule ✓

2.6
- Derivative Rules: Constant, Sum, Difference, and Constant Multiple ✓

2.7
- Derivatives of cos x, sin x, e^x , and ln x

2.8
- The Product Rule

2.9
- The Quotient Rule

2.10
- Finding the Derivatives of Tangent, Cotangent, Secant, and/or Cosecant Functions

What Will We Learn?

- We'll look at the basic rules for differentiating four of the most common transcendental functions - $\cos x, \sin x, e^x, and \ln x$.

Four Most Common Transcendental Derivative Formulas

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\frac{d}{dx}[\cos x] = -\sin x$$

$$\frac{d}{dx}\left[e^x\right] = e^x$$

$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$

These four functions are the four most common transcendental functions that you will encounter in calculus!

Let’s see a proof!

Derivative of an exponential function

Let $f(x) = e^x$

Then by definition

$$f^1(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{e^{x+h} - e^x}{h} = \lim_{h \to 0} e^x \left(\frac{e^h - 1}{h}\right)$$

$$= e^x \lim_{h \to 0} \frac{(e^h - 1)}{h} = e^x . 1 = e^x$$

Hence, $\frac{d}{dx}\left(e^x\right) = e^x$

The derivative of ln *x*

To find the derivative of $f(x) = \ln x$, we again recall the definition of Euler's number e:

$$\lim_{h\to 0}(1+h)^{1/h} = \mathbf{e}$$

where e is a non-fractional number roughly equal to 2.718281828459045235.

Note that ln *x* is the inverse of the exponential function e^x. Thus $\ln e^x = x$ for all *x*. In particular,

ln e = 1.

Let $f(x) = \ln x$. Then

$$\frac{f(x+h)-f(x)}{h} = \frac{\ln(x+h)-\ln x}{h}$$

$$= \frac{\ln\left(\frac{x+h}{x}\right)}{h}$$

$$= \frac{1}{h}\ln\left(1+\frac{h}{x}\right)$$

Let $\frac{h}{x} = k$. Thus $h = kx$ and the above becomes

$$\frac{f(x+h)-f(x)}{h} = \frac{1}{h}\ln\left(1+\frac{h}{x}\right)$$

$$= \frac{1}{kx}\ln(1+k)$$

$$= \frac{1}{x}\ln(1+k)^{1/k}$$

As $h\to 0$, $k\to 0$. Hence by the definition of e, from the above we have

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \frac{1}{x}\ln \mathbf{e} = \frac{1}{x}$$

$$\boxed{\frac{d}{dx}(\ln x) = \frac{1}{x}}$$

Example: Find the derivative of each of the following.

a) $f(x) = 5e^x$

b) $g(x) = e^x - 4x$

c) $h(x) = 7e^x + \sin x$

d) $j(x) = 3\ln x - \cos x$

Key Takeaways

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\frac{d}{dx}[\cos x] = -\sin x$$

$$\frac{d}{dx}\left[e^x\right] = e^x$$

$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$

These four functions are the four most common transcendental functions that you will encounter in calculus!

Logarithmic Functions

1. Differentiate $y = \ln\left(\frac{x^2+1}{\sqrt{x}}\right)$ and simplify your answer fully.
2. Given $y = x^2 \ln x$, find $\frac{dy}{dx}$, then determine where the gradient is zero for $x > 0$.

1.

$$y = \ln\left(\frac{x^2+1}{\sqrt{x}}\right) = \ln(x^2+1) - \tfrac{1}{2}\ln x$$

$$\frac{dy}{dx} = \frac{2x}{x^2+1} - \frac{1}{2x}$$

2.

$$y = x^2 \ln x$$

$$\frac{dy}{dx} = 2x\ln x + x = x(2\ln x + 1)$$

Stationary point:

$$2\ln x + 1 = 0 \Rightarrow x = e^{-1/2}$$

Here are **3 exponential differentiation questions (in increasing difficulty):**

1. Differentiate $y = 2^x$.
2. Differentiate $y = 3^x \cdot x^2$.
3. Differentiate $y = 5^{x^2+1}$, and simplify your answer in terms of 5^{x^2+1}.

1. $y = 2^x$

$$\frac{dy}{dx} = 2^x \ln 2$$

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2. $y = 3^x \cdot x^2$
   Product rule:

$$\frac{dy}{dx} = 3^x \ln 3 \cdot x^2 + 3^x \cdot 2x$$

$$\frac{dy}{dx} = 3^x (x^2 \ln 3 + 2x)$$

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3. $y = 5^{x^2+1}$
   Chain rule:

$$\frac{dy}{dx} = 5^{x^2+1} \ln 5 \cdot 2x$$

$$\frac{dy}{dx} = 2x \ln 5 \cdot 5^{x^2+1}$$

TRUE or FALSE

1. The derivative of x^x can be found using logarithmic differentiatio
2. The derivative of $\ln(x+1)$ is $\frac{1}{x+1}$.
3. The derivative of 2^x is 2^x.